FUNCTIONS AND RELATIONS

Unit Overview

This unit is about functions and relations. The graph of a function or relation can often show patterns that data in a chart do not show.

Functions and Relations

A **relation** is a set of ordered pairs. The first coordinate of the ordered pairs, *x*, form a set called the **domain**, and the second coordinate of the ordered pairs, *y*, form a set called the **range**. A **function** is a special type of relation because each element of the domain is paired with exactly one element of the range. In other words, all *x*-values are different.

Consider the following relations:

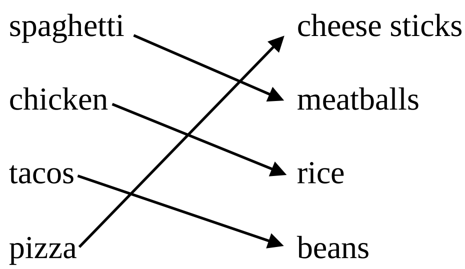
a.) {(spaghetti, meatballs), (chicken, rice), (tacos, beans), (pizza, cheese sticks)}

b.) {(cheeseburger, french fries), (hot dog, chips), (sloppy joe, chips), (cream chicken, corn)}

c.) {(apple pie, ice cream), (chocolate sundae, whip topping), (apple pie, milk), (chocolate cake, milkshake)}

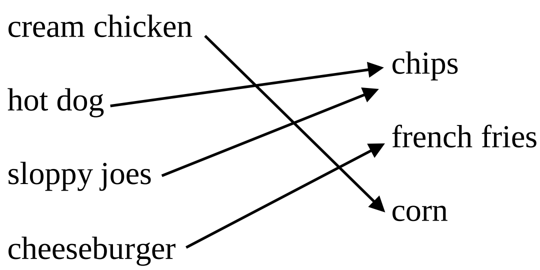
We are going to represent each relation by first drawing a diagram. This diagram will allow you to determine if the relation is a function or not.

a.)



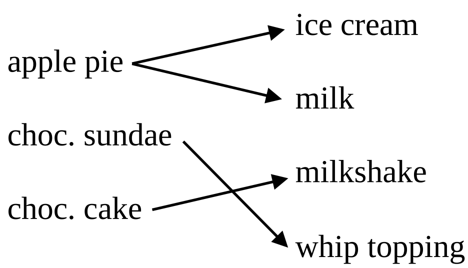
The relation in "*a*" is a function because each element of the first set is paired with exactly one in the second set.

b.)



The relation in "*b*" is a function because each element of the first set is paired with exactly one element in the second set.

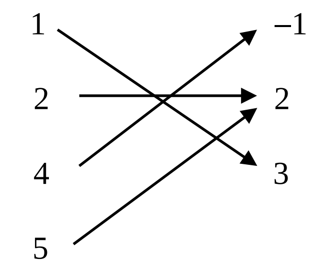
c.)



The relation in "*c*" is **NOT** a function because an element from the first set is used twice with elements in the second set.

Example #1: Determine if the relation is a function. Explain why or why not.

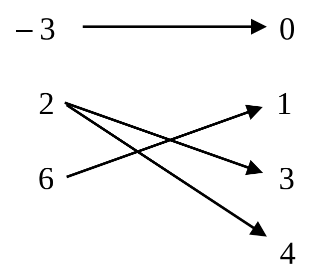
$$\{(5,2),(4,-1),(1,3),(2,2)\}$$



The relation **IS** a function because each element of the first set is paired with exactly one element in the second set.

Example #2: Determine if the relation is a function. Explain why or why not.

$$\{(6,1),(2,4),(-3,0),(2,3)\}$$



The relation is **NOT** a function because an element from the first set (2) is used twice with elements in the second set.

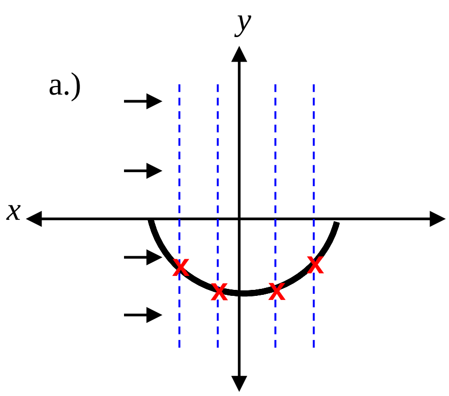
Stop! **Go to Questions #1-7 about this section, then return to continue on to the next section.**

Vertical Line Test

Another way to determine if a relation is a function is by using the "vertical line test." After graphing a relation on a coordinate plane, use a vertical line (pencil, ruler, edge of paper) to run through the graph, usually starting on the left and sliding right. If the vertical line **does not touch more than one point at a time, the relation is a function. If the vertical line touches more than one point at a time the relation is not a function.**

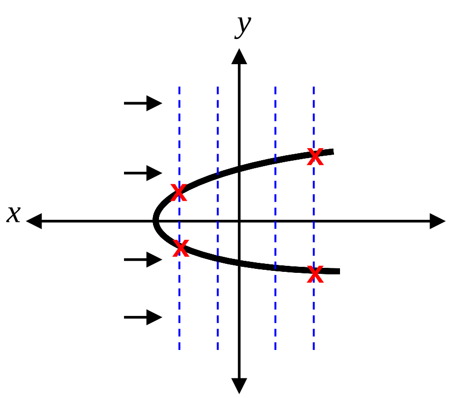
Example: Use the vertical line test to determine if each relation is a function.

a.)



By looking at the blue vertical lines that are drawn through the graph, they only intersect the graph at one point at a time, the red **x**'s; therefore, this graph is a function.

b.)



By looking at the blue vertical lines that are drawn through the graph, they intersect the graph at more than one point at a time, the red **x**'s; therefore, this graph is **NOT** a function.

Stop! **Go to Questions #8-12 about this section, then return to continue on to the next section.**

Function Notation

The statement $y = x$ is a linear function. Another way to write $y = x$ in function notation is where the dependent variable, y, is written as $f(x)$. This is read "f of x," or $g(x)$, or $h(x)$. Notice that the independent variable x is inside the parentheses. Therefore, $\boldsymbol{y = x}$ can be written as $\boldsymbol{f(x) = x}$ in function notation. The graph of f is the graph of the equation $y = f(x)$. In other words, instead of using y, we are now writing $f(x)$ which indicates the same as y. The x in $f(x)$ indicates the value that will be substituted into the expression on the other side of the = sign.

Let's take a look at another function, $f(x) = x^2 + 2$. The expression on the right, $x^2 + 2$, is the **function rule**. The function rule is used to evaluate functions for certain domain elements. This is the same as finding y in $y = x^2+2$.

For example, if asked to evaluate the function $f(x) = x^2 + 2$ for the value $f(3)$, then replace all x's in the function rule with 3 and evaluate the statement.

Example #1: Find $f(3)$ if $f(x) = x^2 + 2$.

$$f(3) = 3^2 + 2$$

$$f(3) = 9 + 2$$

$$f(3) = 11$$

This is saying that when the value of x is 3, $f(x)$ or y is 11.

Try this activity: Function Machine - NVLM

*You may have to update or download Java for this to work properly. If Java is running properly, you will get a pop up asking you to click Run or Cancel. Be sure to click Run.

This activity shows a "function machine." You put numbers into it (inputs) and it gives a number in return (outputs). The idea is that you see when you put in a number (x), you receive an output ($f(x)$). Drag the numbers in to the machine and study the outputs. See if you can figure out the rule and enter the output values for the rest of the numbers in the table.

Example #2: If $g(x) = x^2 - 3x + 4$, what is $g(-2)$?

* $g(-2)$ indicates to replace all x's with –2 in the function rule $x^2 - 3x + 4$, and then evaluate.

$$g(x) = x^2 - 3x + 4$$
$$g(-2) = (-2)^2 - 3(-2) + 4$$
$$g(-2) = 4 + 6 + 4$$
$$g(-2) = 14$$

This is saying that when the value of x is –2, then $g(x)$ or y is 14.

Example #3: If $h(x) = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{7}{8}$, what is $h(\frac{1}{2})$?

* $h(\frac{1}{2})$ indicates to replace all x's with $\frac{1}{2}$ in the function rule $\frac{1}{2}x^2 - \frac{3}{4}x + \frac{7}{8}$, and then evaluate.

$$h(x) = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{7}{8}$$
$$h(\frac{1}{2}) = \frac{1}{2}(\frac{1}{2})^2 - \frac{3}{4}(\frac{1}{2}) + \frac{7}{8}$$
$$h(\frac{1}{2}) = \frac{1}{2}(\frac{1}{4}) - \frac{3}{4}(\frac{1}{2}) + \frac{7}{8}$$
$$h(\frac{1}{2}) = \frac{1}{8} - \frac{3}{8} + \frac{7}{8}$$
$$h(\frac{1}{2}) = \frac{5}{8}$$

This is saying that when the value of x is 1/2, then $h(x)$ or y is 5/8.

Example #4: Consider the function $f(x) = x^3$ where the domain consists of integers –3 to 3. Make a list of the ordered pairs of the function.

a.) Make a chart listing all domain values.

x	x^3	$f(x)$ or y
–3		
–2		
–1		
0		
1		
2		
3		

b.) Using the values of the domain, find the values of the range by substituting the domain values into the given function.

x	x^3	$f(x)$ or y
–3	$(-3)^3$	–27
–2	$(-2)^3$	–8
–1	$(-1)^3$	–1
0	$(0)^3$	0
1	1^3	1
2	$(2)^3$	8
3	$(3)^3$	27

c.) List the set of ordered pairs that were found from the chart above.

{(–3, –27), (–2, –8), (–1, –1), (0, 0), (1, 1), (2, 8), (3, 27)}

Example #5: Consider the function $f(x) = 3x^2 - 5$ where the domain consists of integers –2 to 4. Make a list of the ordered pairs of the function.

Make a chart listing the domain values and use the values of the domain to find the values of the range by substituting the domain values into the given function.

x	$3x^2 - 5$	$f(x)$ or y
–2	$3(-2)^2 - 5$	7
–1	$3(-1)^2 - 5$	–2
0	$3(0)^2 - 5$	–5
1	$3(1)^2 - 5$	–2
2	$3(2)^2 - 5$	–7
3	$3(3)^2 - 5$	22
4	$3(4)^2 - 5$	43

The ordered pairs are: {(–2, 7), (–1, –2), (0, –5), (1, –2), (2, –7), (3, 22), (4, 43)}.

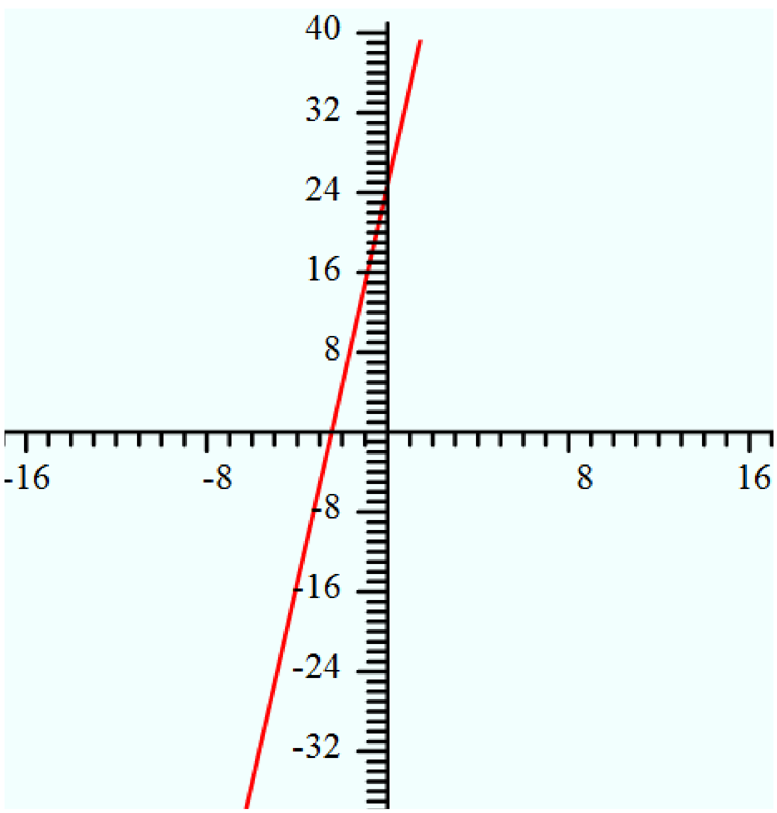
Example #6: Consider the following problem. Marcy is a personal shopper. She is paid to shop for others who may be too busy or may not be able to do so due to an illness or injury. She charges a fee of $25 to cover her initial expenses of things like gas and delivery. However, she also charges $10 per hour for the shopping itself. So, the longer your list, the longer it will take her to shop, and thus, the more you would pay for her assistance. How much does it cost to hire Mary for her shopping services?

First, make a table. Remember that we are creating a function rule for this problem. Marcy charges $10/hr, so we would need an input of how many hours Marcy will work. Let x = # hours shopping (worked). Let $f(x)$ = total $ Marcy charges the customer.

What will our function rule be? Remember that Marcy charges each customer $25 regardless of the number of hours. So our function rule is $f(x) = 10x + 25$. This means $10 times the number of hours plus the $25 fee. Put this in a chart to show possible inputs and outputs.

x	$10x+25$	$f(x)$ or y
1	10(1) +25	35
2	10(2) + 25	45
3	10(3) + 25	55
4	10(4) + 25	65
5	10(4) + 25	75

We could find many more points for this function. We could even consider values such as 2.5 (Marcy works 2 1/2 hours.) Consider the domain and range for this function. If we graph $f(x)$ $10x+25$, we would get:



But what does this graph show? If $f(x)$ indicates what Marcy earns, should $f(x)$ ever be negative? What is the least she could earn? What is the most? What is the least number of hours she could work? The most? Let's consider this. In order to take a job, Marcy would have to work greater than zero hours. She may have a minimum she requires but we do know that she must work more than zero.

Therefore, our inputs must be > 0. So what is our domain? Remember that domain is the input values. For this problem, we must say that our domain is $x > 0$. What about the output values (or y)? Well at zero hours, Marcy would charge her fee of $25 but have no hours to charge for. Therefore, the minimum y value would be $25. Notice on the graph this is the y-intercept. The y-intercept is the value when $x = 0$. Our range is, therefore, $f(x) > 25$.

So, while we could use our function rule to find values when $x < 0$, it would not fit the constraints of this problem. We must also use the information we know to think about output values that make sense. If you got negative values, does that make sense with the problem? So, the graph above reflects the entire function without considering the domain and range necessary for the problem. When that is taken into account, we would only plot values that start with zero.

Try this: Tanya goes to the local grocery store and decides to purchase a salad from the salad bar. The price of the salad is $5.99/lb. Write a function to describe the price of the salad where x is the number of pounds. (Remember that lb. is the abbreviation for pound.)

"Click here" to check your answer.

$\boldsymbol{f(x) = 5.99x}$

What is the domain for this function?

"Click here" to check your answer.

$\boldsymbol{x > 0}$

She cannot buy less than 0 pounds of salad.

What is the range for this function?

"Click here" to check your answer.

$f(x) > 0$

Price cannot be less than zero. However, if not much salad is purchased, the price can still be between 0 and 5.99 (if less than one pound of salad is purchased.)

Try this: Ramon is going on a trip. He fills his fuel tank which holds 14 gallons of fuel. He drives 90 miles and sees that he has 9 gallons of fuel left. Write a function rule that relates the number of miles he can travel to the number of gallons remaining in the tank.

First: Figure his gas mileage (miles/gallon).

"Click here" to check your answer.

Gas mileage = 18mpg

(driven 90 miles and used 14 – 9 = 5 gallons of gas, 90/5 = 18 miles per gallon)

Second: How many miles can be driven on a full tank of gas?

"Click here" to check your answer.

14*18= 252miles

Third: Write the function rule. Consider this: Is amount of gas in the gas tank increasing or decreasing?

"Click here" to check your answer.

$f(x) = -18x + 252$

$f(x)$ is the output or miles that can be traveled with the remaining fuel. x represents number of gallons of fuel used. As the number of miles traveled increases, the amount of fuel in the car decreases. Hence, why the slope is a negative 18. Each gallon used by the car is 18 miles driven. The *y*-intercept of 252 represents the number of miles that can be driven on a full tank of gas (14 gallons in the tank × 18 miles per gallon).

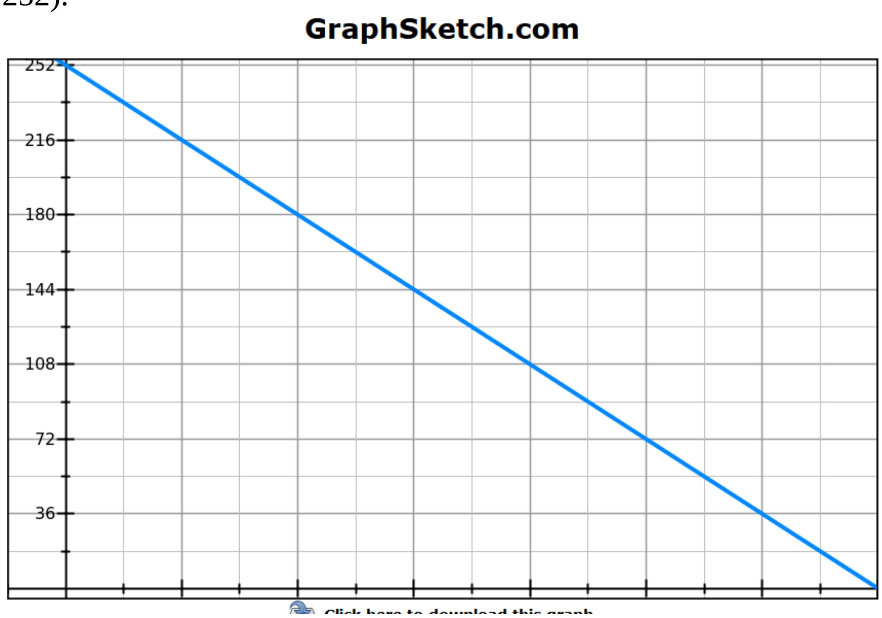
Fourth: What are the domain and range of this problem?

"Click here" to check your answer.

f **(*x*) = –18*x* + 252**

f **(*x*) is the output or miles traveled. Therefore, the range can be as little as 0 miles or as much as 252 (the maximum that can be driven on a tank of gas.) *x* represents number of gallons of fuel. This can be as little as 0 or as much as can fill the tank, or 14 gallons. Domain $0 \le x \le 14$, range $0 \le f(x) \le 252$.**

Notice the graph of this function. While we could graph further points, the constraints of this problem narrow the graph to the domain and range found above. Therefore, we have only plotted in the first quadrant. The graph ends at the *x*-axis at (14, 0) and the the *y*-axis at point (0, 252).



Stop! **Go to Questions #13-32 to complete this unit.**