QUOTIENTS OF MONOMIALS

$$\frac{a^m}{a^n} = a^{m-n} \qquad a^{-n} = \frac{1}{a^n}$$

Unit Overview

In this unit, you will learn how to simplify ratios involving monomials. This is useful in biology, physics, and using scientific notation. The unit will conclude with negative exponents which are used to represent very small numbers.

Dividing Monomials

When simplifying quotients, you can do so by first expressing the powers in terms of their factors. Take a look at the example below and see if you can derive a rule on how to simplify monomial quotients.

Example #1: $\frac{x^6}{x^4} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2$

Did you figure out a rule for dividing monomials? If you said that you can subtract the exponents, you are correct. Study the property below.

Quotient-of-Powers Property

For all nonzero real numbers *a* and all integers *m* and *n*,

$$\frac{a^m}{a^n} = a^{m-n}$$

Example #2: Use the property stated above to simplify the following monomials.

a.) $\frac{2^7}{2^4} = 2^{7-4} = 2^3 = 8$

b.) $\frac{a^x}{a} = a^{x-1}$

c.) $\frac{a^{x+y}}{a^z} = a^{x+y-z}$

d.) $\frac{a^{x+1}}{a} = a^{x+1-1} = a^x$

e.) $\frac{p^8q^5r^2}{p^3q^2r} = p^{8-3}q^{5-2}r^{2-1} = p^5q^3r^1 = p^5q^3r$

Note: When there are coefficients, you simply reduce the fraction!

f.) $\frac{-6t^7}{8t} = \frac{-3t^7}{4t} = \frac{-3t^{7-1}}{4} = \frac{-3t^6}{4} = -\frac{3}{4}t^6$

Exponent rules can also be used when scientific notation is involved:

g) $\frac{5\times10^3}{2\times10^2} = 2.5\times10^1$

Power-of-a-Fraction Property

For all real numbers x and y, where $y \neq 0$, and all integers n,

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Notice in the property above that the exponent is given to the term in the numerator and the term in the denominator. Let's take a look at some examples involving this property.

Example #3: Simplify $\left(\frac{3}{2}\right)^3$.

$$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Example #4: Simplify $\left(\frac{x^3}{y^2}\right)^4$.

$$\left(\frac{x^3}{y^2}\right)^4 = \frac{(x^3)^4}{(y^2)^4} = \frac{x^{12}}{y^8}$$ (Multiply the exponents.)

Example #5: Simplify $\left(\frac{2w^5}{5y^4}\right)^3$

Notice the coefficients are actually raised to the power and not multiplied by the exponent as are the exponents.

$$\left(\frac{2w^5}{5y^4}\right)^3 = \frac{(2w^5)^3}{(5y^4)^3} = \frac{(2)^3(w^5)^3}{(5)^3(y^4)^3} = \frac{8w^{15}}{125y^{12}}$$

Example #6: Simplify $\left(\frac{t^3u^5}{v^{2x}}\right)^{3x}$

$$\left(\frac{t^3u^5}{v^{2x}}\right)^{3x} = \frac{(t^3u^5)^{3x}}{(v^{2x})^{3x}} = \frac{(t^3)^{3x}(u^5)^{3x}}{(v^{2x})^{3x}} = \frac{t^{9x}u^{15x}}{v^{6x^2}}$$

Example #7: Simplify Scientific Notation

$$\frac{(3\times10^4)^3}{(2\times10^2)^2} = \frac{27\times10^{12}}{4\times10^4} = 6.75\times10^8$$

Note: Make sure your final answer is always in scientific notation with the number between 1 and 10 times ten to a power.

QuickTime Same Base: The Second Law of Exponents (Quotient of Powers Property) (04:03)

***Stop!* Go to Questions #1-15 about this section, then return to continue on to the next section.**

Negative Exponents

Negative exponents are used to represent very small numbers. Study the property defining negative exponents below.

Negative Exponents

For all nonzero numbers *a* and all integers *n*,

$$a^{-n} = \frac{1}{a^n} \text{ OR } \frac{1}{a^{-n}} = \frac{a^n}{1} = a^n$$

Example #1: Simplify using positive exponents.

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Example #2: Simplify using positive exponents.

$$x^{-4}y^3 = \frac{1}{x^4} \cdot \frac{y^3}{1} = \frac{y^3}{x^4}$$

*All properties from this unit and any previous units will also apply to negative exponents.

Example #3: Simplify using positive exponents.

$3^{-3} \cdot 3^2$ -like bases, add the exponents

$= 3^{-3+2}$

$= 3^{-1}$ -negative exponent

$= \frac{1}{3}$ -negative exponent property

Example #4: Simplify using positive exponents.

$$\frac{12^2}{12^{-1}}$$ -like bases, subtract the exponents

$$=12^{2-(-1)}$$

$$=12^3$$

$$= 1728$$

Example #5: Simplify using positive exponents.

$$\frac{4^5}{4^7}$$

$$=4^{5-7}$$

$$=4^{-2}$$ -negative exponent property

$$=\frac{1}{4^2}=\frac{1}{16}$$

Example #6: Simplify using positive exponents.

$$\frac{2}{3^{-2}}$$

$$=\frac{2}{1}\cdot\frac{1}{3^{-2}}$$

$$=\frac{2}{1}\cdot\frac{3^2}{1}$$

$$=2\cdot 3^2=2\cdot 9=18$$

*Notice in example #6 that 3^{-2}, which is in the denominator, is moved to the numerator to make the exponent positive. Study the statement given below.

Example #7: Simplify using positive exponents.

$$\frac{c^3 d^5}{c^7 d^3}$$

$$= c^{3-7} d^{5-3} = c^{-4} d^2 = \frac{1}{c^4} \cdot d^2 = \frac{d^2}{c^4}$$

If an exponent is **negative** in the **numerator**, the base will be moved to the **denominator** to make the exponent positive. If an exponent is **negative** in the **denominator**, the base will be moved to the **numerator** to make the exponent positive.

Let's take a look at an example of the statement above.

Example #8: Simplify using positive exponents.

$$\frac{x^{-3} y^4}{z^{-3}}$$

$$= \frac{y^4 z^3}{x^3}$$

Since the exponent on the base *x* is negative in the numerator, you will move the base *x* to the denominator to make the exponent positive. Also, since the exponent on *z* is negative in the denominator, you will move the base *z* to the numerator to make it positive.

Example #9: Simplify using positive exponents.

$$\frac{m^4 n^{-3}}{m^{-5} n^4 p}$$

*This type of monomial can be simplified a couple of different ways. We will take a look at two ways below.

a.) Using exponent properties.

$$\frac{m^4 n^{-3}}{m^{-5} n^4 p}$$

$$= m^{4-(-5)} \cdot n^{-3-4} \cdot \frac{1}{p}$$

$$= m^9 \cdot n^{-7} \cdot \frac{1}{p}$$

$$= \frac{m^9}{n^7 p}$$

b.) Moving bases then using properties.

$$\frac{m^4 n^{-3}}{m^{-5} n^4 p}$$

$$= \frac{m^4 m^5}{n^3 n^4 p}$$

$$= \frac{m^{4+5}}{n^{3+4} p}$$

$$= \frac{m^9}{n^7 p}$$

Example #10: Simplify using positive exponents.

$$\frac{(5q^{-2})(12q^7)}{4q^{-3}}$$

$$= \frac{5 \cdot 12 q^{-2} \cdot q^7}{4q^{-3}}$$

$$= \frac{5 \cdot \cancel{12}^{\,3}}{\cancel{4}^{\,1}} \cdot \frac{q^{-2+7}}{q^{-3}}$$

$$= 15q^{-2+7-(-3)}$$

$$= 15q^8$$

Definition of Negative Exponent (04:45)

Simplifying Expressions with Negative Exponents (08:13)

Zero as an Exponent

Zero as an Exponent

For any nonzero number *a*,

$$a^0 = 1$$

The property illustrated above shows us that any number to the zero power is equal to 1.

Example #1: $4^0 = 1$

Example #2: $x^{-3} \cdot x^3 = x^{-3+3} = x^0 = 1$

Example #3: $\dfrac{x^4 y^3}{x^6 y^3} = x^{4-6} y^{3-3} = x^{-2} y^0$

$$= \frac{1}{x^2} \cdot 1 = \frac{1}{x^2}$$

Example #4: $\dfrac{5a^4 b^5}{15a^4 b^8}$

$$= \frac{1}{3} a^{4-4} b^{5-8}$$

$$= \frac{1}{3} a^0 b^{-3}$$

$$= \frac{1}{3} \cdot 1 \cdot \frac{1}{b^3} = \frac{1}{3b^3}$$

Exponent Property Summary		
Product of Powers $a^m \cdot a^n = a^{m+n}$	**Quotient of Powers** $\frac{a^m}{a^n} = a^{m-n}$	**Power of a Power** $(a^m)^n = a^{m \times n}$
Powers of a Product $(ab)^n = a^n b^n$	**Power of a Fraction** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	**Negative Exponents** $a^{-m} = \frac{1}{a^m}$ or $\frac{1}{a^{-m}} = a^m$
	Zero Exponent $a^0 = 1$	

QuickTime Definition of Zero Exponent (04:12)

Stop! **Go to Questions #16-32 to complete this unit.**