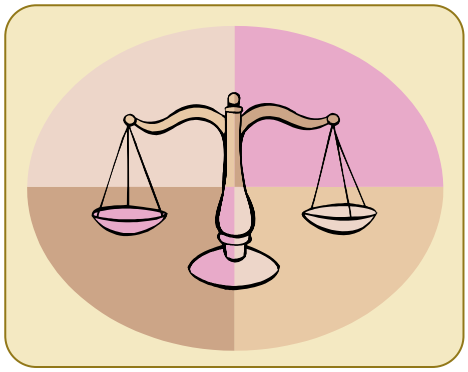
EXPRESSIONS, EQUATIONS, AND INEQUALITIES



Unit Overview

In this unit, you will learn about one of the most important and widely used properties in algebra, the distributive property. Throughout the unit, you will use this property to simplify expressions and solve equations and inequalities. The unit will conclude with graphing inequality solutions.

Distributive Property

One of the most important properties in algebra is the **distributive property.** This property ties addition or subtraction together with multiplication. The distributive property allows you to write expressions in different forms and is given with the following definition.

Distributive Property
The sum or difference of two numbers multiplied by a number is the sum or difference of the product of each number and the number used to multiply. $2(3 + 6) = 6 + 12$ For any number x, y, and z, $x(y + z) = xy + xz$ *The expression $x(y + z)$ is read "x times the quantity of $y + z$"*

The Distributive Properties of Multiplication over Addition (01:26)

To rewrite an algebraic expression using the distributive property make sure that you multiply each term inside the parentheses by the number on the outside. Take a look at the following examples.

Example #1: $3(m - 8)$

$= 3(m) - 3(8)$

$= 3m - 24$ *Notice that the m and the 8 were both multiplied by the 3 located on the outside of the parentheses.

This problem is complete because $3m$ and 24 are not **like terms** and cannot be combined.

In any algebraic expression, the numbers and variables are called **terms.** Therefore, in the expression $3m - 24$ from above, the $3m$ and the 24 are considered the terms of the expression. If the terms contain the same variable with the same exponent, they are considered **like terms**.

QuickTime Equations (02:19)

Examples of like terms:

$4xy$ and $2xy$ $6m$ and $9m$ $2x^2$ and $5x^2$

*Notice in the last example that both x's have an exponent of 2. This makes them like terms.

Examples of terms that are **not** like terms:

$4x$ and $2xy$ $7m$ and 10 $4x^3y$ and $12xy^3$

*Notice that the last example does not represent like terms because the exponent on the x in the first term is 3, whereas the exponent on the y in the second term is 3. If they are going to be like terms, each variable **must** have the same exponent.

Example #2: $5(2q - 7r - 9)$

$= 5(2q) - 5(7r) - 5(9)$

$= 10q - 35r - 45$ *Notice that the $2q$, the $7r$ and the 9 were multiplied by the 5 located on the outside of the parentheses.

Example #3: $\frac{3}{4}(12x+4y-16z)$

$$=\frac{3}{4}(12x)+\frac{3}{4}(4y)-\frac{3}{4}(16z)$$

$$=\frac{3}{\cancel{4}}(\cancel{12}^{3}x)+\frac{3}{\cancel{4}}(\cancel{4}^{1}y)-\frac{3}{\cancel{4}}(\cancel{16}^{4}z)$$ *Use canceling.

$$=9x+3y-12z$$

Stop! **Go to Questions #1-5 about this section, then return to continue on to the next section.**

Simplifying Expressions

Expressions are in **simplest form** when there are no parentheses and no like terms. Like terms can be combined by adding or subtracting the numbers in front of the variables. These numbers are called the **coefficients** of the term and will be referred to as such throughout the course.

Example: The coefficient of $4x$ is 4, $\frac{2}{3}mn$ is $\frac{2}{3}$ and so on.

Examples of simplifying:

1) $10y - y$	The coefficient of y is understood to be "1".
$10y - 1y$	
$(10 - 1)y$	
$9y$	This is in simplest form.
2) $3x + 4 + 8x$	$3x$ and $8x$ are like terms, combine them.
$3x + 8x + 4$	
$(3 + 8)x + 4$	
$11x + 4$	This is in simplest form.
3) $6(b + 3) + 7b$	Use the distributive property to eliminate the parentheses.
$6b + 18 + 7b$	Combine $6b$ and $7b$ as they are like terms.
$(6 + 7)b + 18$	
$13b + 18$	
4) $2(x + y) + 3(2x + 3y)$	Use the distributive property.
$2x + 2y + 6x + 9y$	Rearrange so the x's and the y's are beside each other.
$2x + 6x + 2y + 9y$	Combine like terms.
$(2 + 6)x + (2 + 9)y$	
$8x + 11y$	
5) $6x + 4y - 3x + 12y$	Rearrange so the x's and the y's are beside each other.
$6x - 3x + 4y + 12y$	Combine like terms.
$(6 - 3)x + (4 + 12)y$	
$3x + 16y$	

*Notice that when the –6 is distributed over the –2, the result is +12.

6) $7(x^2 + 2y) - 5x^2$ Use the distributive property.

$7x^2 + 14y - 5x^2$ Combine like terms.*

$7x^2 - 5x^2 + 14y$

$(7-5)x^2 + 14y$

$2x^2 + 14y$

*Notice that the exponent (2) did not change. When combining like terms, the exponent STAYS THE SAME.

7) $2.3s + 5.7r - 1.1s + 3.6r$ Combine like terms.

$(2.3 - 1.1)s + (5.7 + 3.6)r$

$1.2s + 9.3r$

Stop! **Go to Questions #6-10 about this section, then return to continue on to the next section.**

Solving One-Step Equations

At this point in our unit, we will begin to solve simple equations.

Click here to view a video that will help you understand using variables and keeping the equations balanced when solving them.

A mathematical sentence such as $374 + x = 795$ is called an **equation** because it contains an equal sign.

The **solution** to an equation is the value of the variable that results in a true statement. The process of finding this solution is called **solving the equation** which uses opposite operations in order to **isolate** the variable. When we say opposite operations, we mean the opposite of adding is subtracting, the opposite of multiplying is dividing, and visa versa.

At this point, it should be mentioned that an equation must be balanced. This means that whatever is done on one side of the equation must also be done on the other side of the equal sign.

Introduction (03:06)

Let's take a look at a few examples of solving equations.

Example #1: Solve $x + 4 = 17$

-Since we want x by itself, we have to perform the opposite of + 4 on both sides of the equal sign, opposite of +4 is –4.

-Performing opposite operations can be done different ways; below you will see that this can be done vertically or horizontally; you make the choice.

$$\begin{array}{r} x + 4 = 17 \\ \underline{\quad -4 \quad -4} \\ x + 0 = 13 \\ x = 13 \end{array} \qquad \textit{or} \qquad \begin{array}{r} x + 4 - 4 = 17 - 4 \\ x + 0 = 13 \\ x = 13 \end{array}$$

-Check the solution by replacing x in the equation with the solution 13.

Check: $13 + 4 = 17$
$17 = 17$ ✓

Since this is true, **13** is the correct solution.

QuickTime Equations Using Variables--Paris Population (01:32)

Example #2: Solve $\frac{n}{4} = 7$

-The opposite of dividing by 4 is multiplying by 4, multiply both sides by 4.

$$4\left(\frac{n}{4}\right) = (7)4$$

$$\frac{\cancel{4}n}{\cancel{4}} = 28$$ Cancel the 4's to isolate the variable.

$$n = 28$$

-Check the solution by replacing *n* in the equation with the solution 28.

Check: $\frac{28}{4} = 7$

$7 = 7$ ✓

Since this is true, **28** is the correct solution.

Example #3: Solve $y - 26 = 38$

-The opposite of –26 is +26, add 26 to both sides of the equation.

$$\begin{array}{r} y - 26 = 38 \\ +26 \quad +26 \\ \hline y = 64 \end{array}$$

-Check the solution by replacing *y* in the equation with the solution 64.

Check: $64 - 26 = 38$

$38 = 38$ ✓

Since this is true, **64** is the correct solution.

Example #4: Solve $5n = 75$

-The opposite of multiplying by 5 is dividing by 5.

$$\frac{5n}{5} = \frac{75}{5}$$

$$n = 15$$

-Check the solution by replacing *n* in the equation with the solution 15.

Check: $5(15) = 75$
$75 = 75$ ✓

Since this is true, **15** is the correct solution.

Example #5: Solve $144 = 9x$

-The opposite of multiplying *x* by 9 is dividing by 9.

$$\frac{144}{9} = \frac{9x}{9}$$

$16 = x$ (16 = *x*, therefore *x* = 16)

$x = 16$

-Check the solution by replacing *x* in the equation with the solution 16.

Check: $144 = 9(16)$
$144 = 144$ ✓

Since this is true, **16** is the correct solution.

Stop! **Go to Questions #11-17 about this section, then return to continue on to the next section.**

Literal Equations and Formulas

Sometimes, in math and science classes, it is necessary to rewrite an equation or a formula to highlight another quantity of interest. When given formulas with more than one variable, we solve for the stated variable.

For example, let's look at Ohm's Law:

$$V = IR$$

where V represents **volts**, I represents **amperes**, and R represents **resistance**.

This formula is written with the voltage (V) as the quantity of interest. If the problems at hand were to find the voltage, given numerical values for amperes and resistance, then this formula would be easy to follow and finding the actual voltage would be a simple substitute and multiply problem.

However, if the quantity of interest was finding amperes (I), then the formula would change to $I = \frac{V}{R}$.

Let's take a look to see how this idea simplifies the work in the long term. We will continue to use inverse operations to solve for the new quantity when more than one variable is within the equation.

Example #1: Given the formula $d = rt$ where d represents distance, r represents rate or speed, and t represents time, solve the formula for t, so that t is the quantity of interest.

Read the formula to yourself mathematically. How is it read?

Click here to check your answer.

The formula is read *d* equals *r* times *t*.

Locate the variable. $d = r\,t$ What variable are you solving for?

Click here to check your answer.

t

Now ask yourself: "What is being done to this variable that needs undone?"

Click here to check your answer.

The variable *t* is being multiplied by *r*.

What is the inverse of multiplication?

Click here to check your answer.

Division

What do we need to do to both sides of the equation to "undo" the multiplication?

Click here to check your answer.

Divide both sides by *r*.

Now, let's look at the math for our steps.

$d = rt$

$\frac{d}{r} = \frac{rt}{r}$ Divide both sides by r.

$\frac{d}{r} = \frac{\cancel{r}^1 t}{\cancel{r}^1}$ Cancel the r's.

$\frac{d}{r} = \frac{1t}{1}$ Simplify

$\frac{d}{r} = t$ Simplify ($\frac{1t}{1} = 1t = t$)

We say the 1 is "understood" and just write t.

$t = \frac{d}{r}$ ($\frac{d}{r} = t$, therefore, $t = \frac{d}{r}$)

*Note: When we have just t (or $1t$) on one side of the equation, we can say that we isolated "t." This is a common expression used in algebra.

Solving a literal equation such as this one would be useful in science class if you were examining the time elapsed for various objects knowing the distance and rate. You would use the inverse operation one time then plug the values into your newly defined formula.

Now let's revisit Ohm's Law.

Example #2: Solve Ohm's Law $V = IR$ for I.

$$V = IR$$

$$\frac{V}{R} = \frac{IR}{R}$$ Divide both sides by R.

$$\frac{V}{R} = \frac{I\cancel{R}^{1}}{\cancel{R}^{1}}$$ Cancel the R's.

$$\frac{V}{R} = \frac{I}{1}$$ Simplify

$$\frac{V}{R} = I$$ Simplify $\left(\frac{I}{1} = I\right)$

$$I = \frac{V}{R}$$ $\left(\frac{V}{R} = I,\text{ therefore, } I = \frac{V}{R}\right)$

When the literal equation is in this form, finding the amperage (I) would be a simple substitute and divide problem.

Stop! **Go to Questions #18-21 about this section, then return to continue on to the next section.**

Solving One-Step Inequalities

An **inequality** is a mathematical sentence that contains one of the following symbols:

$<$ “less than”

$>$ “greater than”

$\leq$ “less than or equal to”

$\geq$ “greater than or equal to”

QuickTime Inequalities--Bridge Capacity (02:27)

Just like equations, inequalities are solved by using opposite operations. The one exception is that **when multiplying or dividing by a negative number, flip the inequality sign**. We will address this issue in the next unit when we study integers.

Let’s take a look at a few examples of solving inequalities. ***Note: it will be easier to understand the solution set if you write your answer with the variable on the left.** (You will see what this means in example #2.)

Example #1: Solve $n - 7 < 22$

-The opposite of – 7 is + 7.

$$\begin{array}{r} n\ -7 < 22 \\ +7 \quad +7 \\ \hline n < 29 \end{array}$$

This solution means that all numbers (n) less than 29 are a solution to the inequality.

Example #2: Solve $48 \geq 6y$

-The opposite of multiplying by 6 is dividing by 6.

$$\frac{48}{6} \geq \frac{6y}{6}$$

$$8 \geq y$$

Rewrite this solution with the variable y on the left. When doing this, make sure the inequality sign is pointing to the same term as in the original solution (in this case the y).

$y \leq 8$ *Notice the inequality is still pointing to the variable.

This solution means that all numbers *y*, that are less than or equal to 8, are a solution.

Example #3: Solve $\frac{z}{11} \leq 22$

-The opposite of dividing by 11 is multiplying by 11.

$$\boxed{\begin{aligned} (\cancel{11})\frac{z}{\cancel{11}} &\leq 22(11) \\ z &\leq 242 \end{aligned}}$$

This solution means that all numbers *z*, that are less than or equal to 242, are a solution.

Stop! **Go to Questions #22-25 about this section, then return to continue on to the next section.**

Graphing Inequality Solutions

The solution set of an inequality can be graphed on a number line in the following manner:

-If < or >, use an open circle (○) on the number line because the solution set **does not** include the solution (it includes only values less than or more than the solution).

-If ≤ or ≥ use a closed circle (●) on the number line because the solution set **does** include the solution.

Once you have determined if you are going to use an open circle or a closed circle, you will shade the part of the number line that includes the solution set.

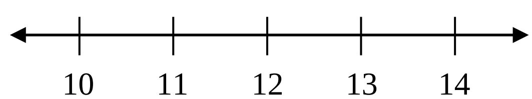
Let's take a look at a couple of examples:

Example #1: Solve and graph.

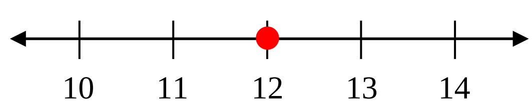
$$\begin{array}{r} m + 3 \leq 15 \\ \underline{\quad -3 \quad -3} \\ m \leq 12 \end{array}$$

This solution tells us that all values "less than or equal" to 12 will result in a true statement. Follow the steps below to graph this inequality solution.

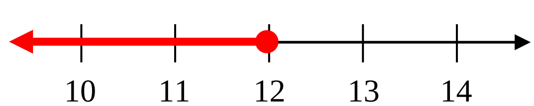
a) Draw a number line and label it with a few numbers.



b) Determine which circle to use. In this case, ≤ means to use a closed circle. Place a closed circle on the 12, since this was the solution to the inequality.



c) Determine which direction to shade on the number line. In this case, we want numbers less than 12; so, shade to the left of 12.



Inequality on a Number Line--Circus Snacks (02:00)

Example #2: Solve and graph.

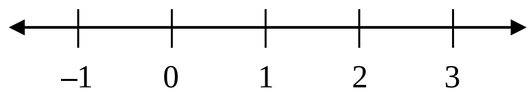
$8 < 4n$

$\frac{8}{4} < \frac{4n}{4}$

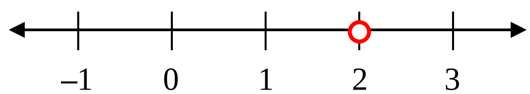
$2 < n$ *rewrite with the variable on the left

$n > 2$

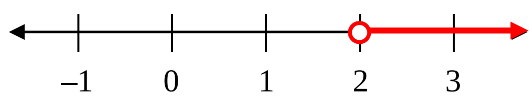
a) Draw a number line



b) Use an open circle around the solution because the inequality is >.



c) Shade the number line to the right, as the solution states that all values greater than 2 will be in the solution.



Stop! **Go to Questions #26-28 to complete this unit.**