# QUADRATICS

## Unit Overview

In this unit, you will learn how to graph quadratic functions, also known as parabolas. A quadratic function models the height of a projectile, such as the path that a golf ball would take through the air. You will also learn how to solve quadratic equations using square roots.

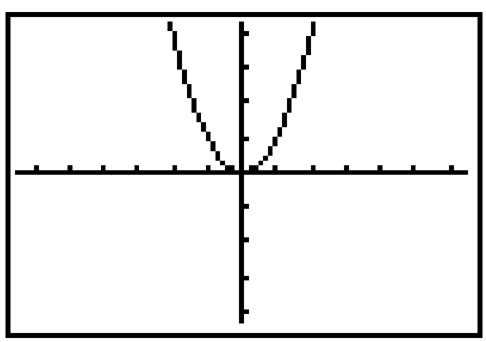
## Graphing Quadratic Functions

### Quadratic Function

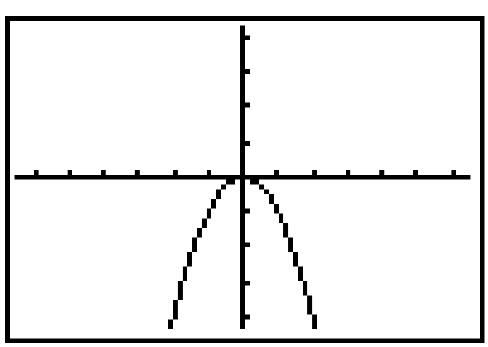
A quadratic function is a function of the form $y = ax^2 + bx + c$ where *a, b,* and *c* are real numbers and $a \neq 0$.

The graph of a quadratic function is a curve known as a **parabola** and is shown below. The lowest point on this parabola is the **minimum value** of the function, the point (0, 0) and is called the **vertex** of the parabola. In a parabola, there is a vertical line called the axis of symmetry drawn through the vertex that reflects the parabola across the line of $x = h$, or in other words, splits the parabola into two equal parts. In the case below, the axis of symmetry would be the $y$-axis or $x = 0$.

$y = x^2$



If "$a$" is negative ($y = -x^2$), the parabola opens down, and therefore, has a maximum value at the vertex (0, 0).



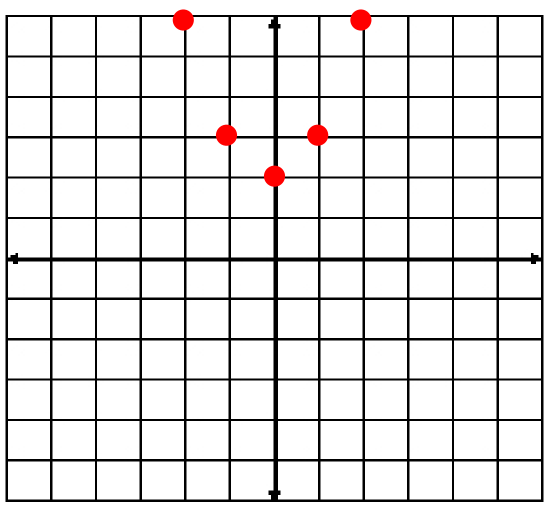
To graph a quadratic equation, you will need to make a table of values. Study the example below.

*Example #1*: Graph $y = x^2 + 2$

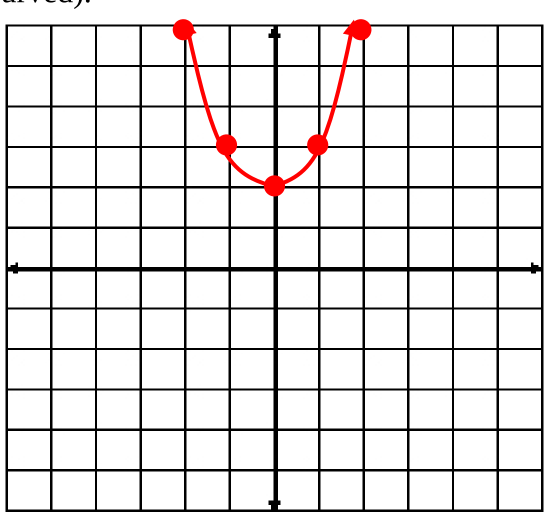
a.) Make a table of values using positive and negative *x*-values.

| $x$ | $x^2 + 2$ | $y$ |
|---|---|---|
| $-2$ | $(-2)^2 + 2$ | 6 |
| $-1$ | $(-1)^2 + 2$ | 3 |
| 0 | $(0)^2 + 2$ | 2 |
| 1 | $(1)^2 + 2$ | 3 |
| 2 | $(2)^2 + 2$ | 6 |

b.) Graph the points from above on a coordinate plane.

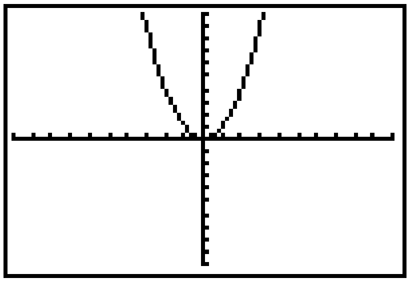


c.) Connect the points using a curve (remember that parabolas are curved).

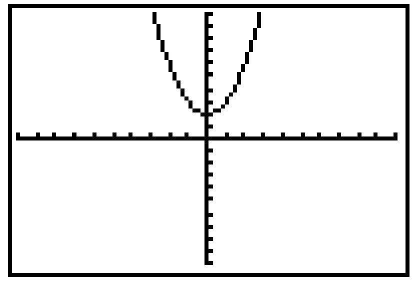


*Notice that the vertex of this parabola is located at (0, 2) as opposed to (0, 0) as shown in the first part of the unit. Let's take a look at both functions and decide why this happened.

a.) $y = x^2$



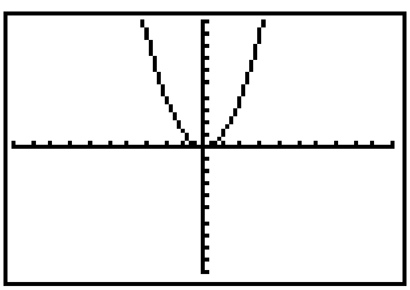
b.) $y = x^2 + 2$



If you compare both equations, you will notice that in the equation in example "a," there is no constant term, whereas in the equation in example "b," there is a constant of +2. The graph of the equation in example "b" has moved up the *y*-axis 2 units; so you can conclude that a constant moves the graph vertically along the *y*-axis.

Let's use $y = x^2$ as our parent function with a vertex at (0, 0) and compare it with $y = x^2 - 3$. What do you think will happen in this case?

$y = x^2$



$y = x^2 - 3$

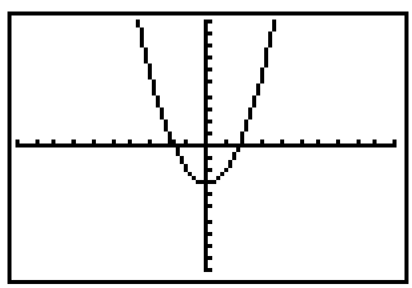


***Table of Values***

| ***x*** | ***y*** |
|---|---|
| –2 | 1 |
| –1 | –2 |
| 0 | –3 |
| 1 | –2 |
| 2 | 1 |

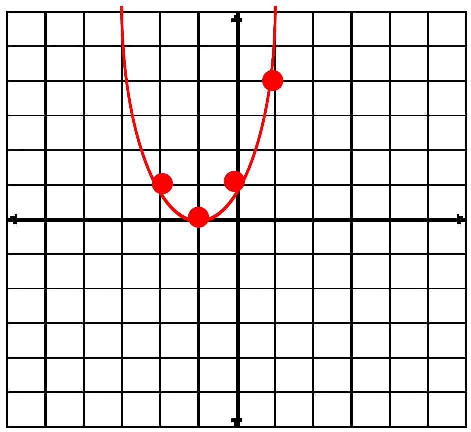
***Due to the pixels in the graphing calculator, it may look like the second equation passes through (2, 0) and (–2, 0). This is not the case, and at this point, we are only concerned about what the constant does to the graph.**

By examining the two equations, you can see that by subtracting 3 the graph has moved down 3 units.

*Example #2*: Graph $y=(x+1)^2$

Make a table of values and graph the points.

| ***x*** | ***y*** |
|---|---|
| –2 | 1 |
| –1 | 0 |
| 0 | 1 |
| 1 | 4 |



Notice that, in this example, we do not have the same number of points plotted on each side of the vertex. If you want, you can substitute more values for *x* to determine how wide the parabola gets on the left.

Notice that when the constant term is in a quantity (inside the parentheses) with the variable, the vertex moved horizontally along the *x*-axis. In the case above, it moved one unit to the **left**. What do you suppose will happen to the vertex of $y=(x-2)^2$? If you answered that the vertex would move 2 units to the right, you are correct. Notice that subtracting the constant inside the parentheses moves the graph right and adding moves it left. This may seem opposite of what you would think would happen. This concept now brings us to the vertex form of a quadratic ($y=-3(x-8)^2+10$), which will be explained next.

***Stop!*** **Go to Questions #1-4 about this section, then return to continue on to the next section.**

## Vertex Form of a Quadratic

### Vertex Form

The vertex form of a quadratic function is $y = a(x-h)^2 + k$, where ($h$, $k$) is the vertex and $x = h$ is the axis of symmetry. When "$a$" is positive, the parabola opens up and the vertex is the minimum value. When "$a$" is negative, the parabola opens down and the vertex is the maximum value.

You are now ready to identify the direction of opening, vertex, and axis of symmetry of quadratic functions in vertex form.

*Example*: Identify the direction of opening, vertex, and axis of symmetry for the following quadratic functions.

a.) $y = 2(x-3)^2 + 5$

direction of opening: up
because "$a$" (2) is +.

vertex: (3, 5)

b.) $y = -3(x+4)^2 - 1$

direction of opening: down
because "$a$" (–3) is –.

vertex: (–4, –1)

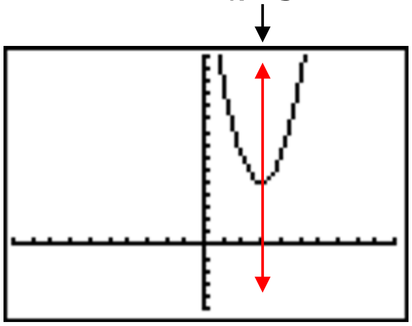
**The $x$-value is the number subtracted inside parentheses from the $x$ variable, the $y$-value is the constant added outside parentheses.

axis of symmetry: $x = 3$

axis of symmetry: $x = -4$

This value is found the same way the $x$-value for the vertex is found.

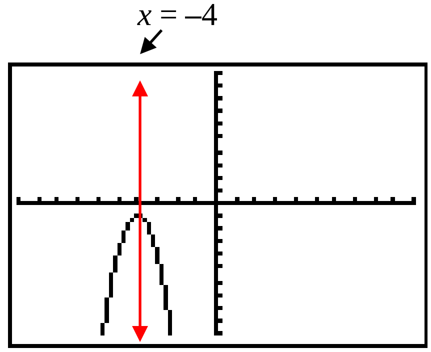
axis of symmetry
$x = 3$



$y = 2(x - 3)^2 + 5$

*Notice that the graph is shifted 3 units to the RIGHT of the *y*-axis and 5 units UP parallel to the *x*-axis.

axis of symmetry
$x = -4$



$y = -3(x + 4)^2 - 1$

*Notice that the graph is shifted 4 units to the LEFT of the *y*-axis and 1 unit DOWN parallel to the *x*-axis.

***Stop!*** **Go to Questions #5-13 about this section, then return to continue on to the next section.**

## Finding the Vertex Form of a Quadratic Using the Zeros

In a previous unit, we talked about the zeros of a quadratic function (where the graph of a quadratic crosses the *x*-axis). We are going to use this concept to determine what the vertex of a parabola is, and then you will write the vertex form of a given quadratic.

*Example 1*: Use the zeros to find the vertex of $y = x^2 - 8x + 15$, and then write the vertex form of the equation.

1) Find the zeros by setting the function equal to 0 and factoring the trinomial.

$$x^2 - 8x + 15 = 0$$
$$(x-3)(x-5) = 0$$

2) Set each binomial equal to 0 and solve for *x*.

$$x - 3 = 0 \qquad x - 5 = 0$$
$$x = 3 \qquad x = 5$$

The zeros of this quadratic function are 3 and 5.

*Because a parabola is symmetrical, the vertex is half way between the zeros.*

3) Find the *x*-coordinate of the vertex by finding the midpoint of the zeros.

$$x = \frac{3+5}{2}$$

$$x = \frac{8}{2} = 4$$ the *x*-coordinate of the vertex is 4.

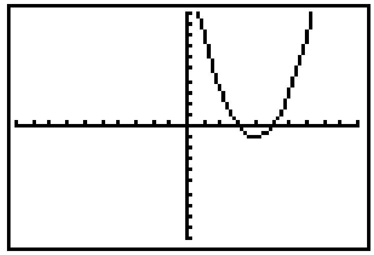
4) Substitute 4 in the equation for *x* and solve to find *y*, which will represent the *y*-coordinate of the vertex.

$$y = x^2 - 8x + 15$$
$$y = (4)^2 - 8(4) + 15$$
$$y = 16 - 32 + 15$$
$$y = -1$$

5) The vertex of $y = x^2 - 8x + 15$ is located at (4, –1).

6) Replace 4 and –1 for *h* and *k* in the vertex form $y = a(x-h)^2 + k$

$y = (x-4)^2 - 1$



*The general equation shows + *k*, but remember that adding –1 is the same as subtracting 1.

***Stop!*** **Go to Questions #14-19 about this section, then return to continue on to the next section.**

## Solving Equations Using Square Roots

The equation $x^2 = 16$ can be read as a question, "What number multiplied by itself is 16?" There are two possible solutions to this, 4 times itself is 16 and (–4) times itself is 16.

The positive square root of the number is called the principal square root. In general, if $x^2 = y$ and $y > 0$, then $x = \sqrt{y}$ or $x = -\sqrt{y}$. This can be written as $x = \pm\sqrt{y}$ and read as "$x$ = plus or minus the square root of $y$."

*Example #1*: Solve $x^2 = 36$

Take the square root of both sides to solve.

$$\sqrt{x^2} = \sqrt{36}$$

$$x = \pm 6$$

*Example #2*: Solve $y^2 = 18$

$$\sqrt{y^2} = \sqrt{18}$$

$$y = \pm\sqrt{18}^{*}$$

***For now, we will leave radical answers under the radical sign. You will learn how to simplify this in a later unit.**

*Example #3*: Solve $16x^2 = 49$

$$x^2 = \frac{49}{16}$$

$$\sqrt{x^2} = \sqrt{\frac{49}{16}}$$

$$x = \pm\sqrt{\frac{49}{16}}$$

When taking the square root of a fraction, you may take the square root of the numerator and denominator separately.

$$x = \pm\frac{7}{4}$$

If an equation contains a quantity that is squared and includes the variable, first isolate the quantity, take the square root of both sides and solve. Follow the example given below.

*Example #4*: Solve $(x-3)^2-16=0$

$$(x-3)^2=16$$

$$\sqrt{(x-3)^2}=\sqrt{16}$$

$$(x-3)=\pm\sqrt{16}$$

$$(x-3)=\pm 4$$

There are actually two equations that you must solve.

$x-3=4$ *and* $x-3=-4$

$x=7$ $\qquad$ $x=-1$

*Check both solutions in the original equation.

| | |
|---|---|
| $(7-3)^2-16=0$ | $(-1-3)^2-16=0$ |
| $(4)^2-16=0$ | $(-4)^2-16=0$ |
| $16-16=0$ ✓ | $16-16-0$ ✓ |

Both solutions check out, so the answer to this equation is $x=7$ and $x=-1$.

*Example #5*: Solve $5(x+3)^2=125$

-isolate the quantity $5(x+3)^2=125$

$$(x+3)^2=\frac{125}{5}$$

-divide $(x+3)^2=25$

-square root both sides $\sqrt{(x+3)^2}=\sqrt{25}$

$$(x+3)=\pm 5$$

$$x+3=\pm 5$$

$$x+3=5 \quad \textit{and} \quad x+3=-5$$
$$x=2 \qquad\qquad x=-8$$

Check your answers:

$$5(2+3)^2=125 \qquad 5(-8+3)^2=125$$
$$5(5)^2=125 \qquad 5(-5)^2=125$$
$$5(25)=125 \qquad 5(25)=125$$
$$125=125\ \checkmark \qquad 125=125\ \checkmark$$

Both solutions check.

*Example #6*: Solve $36\left(x-\frac{2}{3}\right)^2-81=0$

-isolate the quantity $36\left(x-\frac{2}{3}\right)^2=81$

$$\left(x-\frac{2}{3}\right)^2=\frac{81}{36}$$

-square root both sides $\sqrt{\left(x-\frac{2}{3}\right)^2}=\sqrt{\frac{81}{36}}$

$$\left(x-\frac{2}{3}\right)=\pm\sqrt{\frac{81}{36}}$$

$$x-\frac{2}{3}=\pm\frac{9}{6}$$

$$x-\frac{2}{3}=\frac{9}{6} \quad \textit{and} \quad x-\frac{2}{3}=\frac{-9}{6}$$

$$x=\frac{9}{6}+\frac{2}{3} \qquad x=-\frac{9}{6}+\frac{2}{3}$$

$$=\frac{9}{6}+\frac{4}{6} \qquad =-\frac{9}{6}+\frac{4}{6}$$

*Remember when adding fractions to use a common denominator.

$$x=\frac{13}{6} \qquad x=\frac{-5}{6}$$

Check your answers:

$$36\left(\frac{13}{6}-\frac{2}{3}\right)^2-81=0 \qquad 36\left(\frac{-5}{6}-\frac{2}{3}\right)^2-81=0$$

$$36\left(\frac{3}{2}\right)^2-81=0 \qquad 36\left(\frac{-3}{2}\right)^2-81=0$$

$$36\left(\frac{9}{4}\right)-81=0 \qquad 36\left(\frac{9}{4}\right)-81=0$$

$$81-81=0\ \checkmark \qquad 81-81=0\ \checkmark$$

Both solutions check.

***Stop!* Go to Questions #20-25 to complete this unit.**