

# RATIONAL EXPRESSIONS AND FUNCTIONS

## **Unit Overview**

In this unit, you will learn about rational expressions and functions which are fractions that contain two algebraic expressions. Rational functions can model real world problems, such as calculating the average cost of something.

## Rational Expressions

If you recall from a previous unit, a rational number is any number in the form of  $\frac{a}{b}$  where “a” and “b” are integers and  $b \neq 0$ . A rational expression is an expression that can be written as a quotient,  $\frac{P}{Q}$ , where “P” and “Q” are algebraic expressions and  $Q \neq 0$ . The following expressions are rational expressions.

a.)  $\frac{x}{x-3}$

b.)  $\frac{x^2+2}{x^2+4}$

c.)  $\frac{3-2x}{2}$

As you study the definition of a rational expression, you should notice that  $Q \neq 0$ . This is because division by 0 is undefined.

Because of this mathematical rule, restrictions are made on the denominator of a rational expression. You will find that the restrictions help to identify the domain. In general, the domain of a rational function consists of all real numbers that give a nonzero denominator.

## Finding the Domain of a Rational Function

To find the domain of a rational function:

- 1) Factor the denominator.
- 2) Use the zero product property – set all factors **not** equal to zero.
- 3) Solve.

\*The values you find will be the values that produce a zero in the denominator and they must be excluded from the domain.

*Example #1:* Find the domain of  $y = \frac{2}{x+2}$ .

- a.) The denominator cannot be factored, so set it  $\neq$  to 0.

$$x+2 \neq 0$$

- b.) Solve.

$$x+2 \neq 0$$

$$x \neq -2$$

In this example, **all real numbers except  $-2$**  is the domain because  $-2$  will produce a 0 in the denominator.

*Example #2:* Find the domain of  $y = \frac{2x+3}{x^2+4x+3}$ .

a) Factor the denominator.

$$x^2 + 4x + 3 = (x + 3)(x + 1)$$

b) Set each factor  $\neq 0$ .

$$x + 3 \neq 0 \quad x + 1 \neq 0$$

c) Solve.

$$x \neq -3 \text{ and } x \neq -1$$

The domain of  $y = \frac{2x+3}{x^2+4x+3}$  is **all real numbers except  $-3$  and  $-1$** , again because these are the two values that would produce a zero in the denominator. This means that the expression is undefined for these values of  $x$ .

***Stop!* Go to Questions #1-7 about this section, then return to continue on to the next section.**

## Evaluating a Rational Expression or Function

To evaluate a rational expression or function, replace  $x$  with the given values. You will perform the operations within the expression using the order of operations. If, when evaluating an expression the denominator equals zero, you must say that the expression is undefined for that particular value.

*Example #1:* Evaluate  $y = \frac{1}{x-4}$  if  $x = -2$ .

a.) Replace  $x$  with  $-2$ .

$$y = \frac{1}{-2-4}$$

b.) Perform the operations.

$$y = \frac{1}{-6} \quad \text{or} \quad y = \frac{-1}{6} \quad \text{or} \quad y = -\frac{1}{6}$$

*Example #2:* Evaluate  $y = \frac{2x(x+1)}{x^2-1}$  if  $x = 1$

a.) Replace **all**  $x$ 's with 1.

$$y = \frac{2(1)(1+1)}{(1)^2-1}$$

b.) Perform the operations.

$$y = \frac{2(1)(2)}{1-1} = \frac{4}{0}$$

This expression is **undefined for the value  $x = 1$**  because it produced a **0 in the denominator**.

**Stop!** Go to Questions #8-12 about this section, then return to continue on to the next section.

## Simplifying Rational Expressions

By simplifying rational expressions, you will be able to evaluate it much easier.

To simplify:

- 1) Factor the numerator and the denominator.
- 2) State any restrictions on the domain (what values will produce a zero in the denominator).
- 3) Cancel any common factors.

*Example #1:* Simplify  $\frac{4a+12}{4a}$ . State any restrictions on the variable.

- a.) Factor the numerator and denominator.

$$\frac{4a+12}{4a} = \frac{4(a+3)}{4a}$$

- b.) State the restrictions on the variable (set denominator  $\neq 0$ ).

$$4a \neq 0$$

$$a \neq 0$$

- c.) Cancel any common factors.

$$\frac{\cancel{4}(a+3)}{\cancel{4}a} = \frac{a+3}{a}$$

\*You **cannot** cancel the  $a$ 's because the " $a$ " in the numerator is attached to the 3 with a (+) sign.

The expression  $\frac{a+3}{a}$  is the simplified form of  $\frac{4a+12}{4a}$  and  $a \neq 0$ .

*Example #2:* Simplify  $\frac{x^2 + 5x + 6}{x^2 + 4x + 4}$ . State any restrictions on the variable.

a.) Factor the numerator and the denominator.

$$\frac{x^2 + 5x + 6}{x^2 + 4x + 4} = \frac{(x+3)(x+2)}{(x+2)(x+2)}$$

b.) State the restrictions on the variable by setting each factor of the denominator  $\neq 0$  and solving.

$$\begin{array}{l} x+2 \neq 0 \quad x+2 \neq 0 \\ x \neq -2 \end{array}$$

\*Since both factors are the same you only have to solve one of them.

c.) Cancel common factors.

$$\frac{(x+3)\cancel{(x+2)}}{(x+2)\cancel{(x+2)}}$$

The expression  $\frac{x+3}{x+2}$  is the simplified form of  $\frac{x^2 + 5x + 6}{x^2 + 4x + 4}$  and  $x \neq -2$ .

Example #3: Simplify  $\frac{6x^2 - 13x - 5}{4x^2 - 4x - 15}$ . State any restrictions on the variable.

a.) Factor the numerator and the denominator.

$$\frac{6x^2 - 13x - 5}{4x^2 - 4x - 15} = \frac{(3x+1)(2x-5)}{(2x+3)(2x-5)}$$

b.) State the restrictions on the variable by setting each factor of the denominator  $\neq 0$  and solving.

$$\begin{array}{ll} 2x+3 \neq 0 & 2x-5 \neq 0 \\ 2x \neq -3 & 2x \neq 5 \\ x \neq \frac{-3}{2} & x \neq \frac{5}{2} \end{array}$$

c.) Cancel common factors.

$$\frac{(3x+1)\cancel{(2x-5)}}{(2x+3)\cancel{(2x-5)}}$$

The expression  $\frac{3x+1}{2x+3}$  is the simplified form of  $\frac{6x^2 - 13x - 5}{4x^2 - 4x - 15}$  and  $x \neq \frac{-3}{2}$  or  $x \neq \frac{5}{2}$ .

**Stop!** Go to Questions #13-32 to complete this unit.