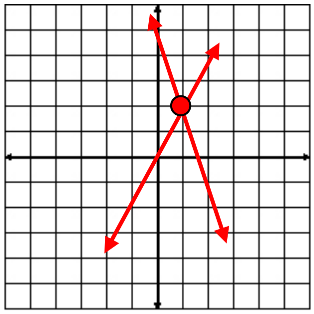
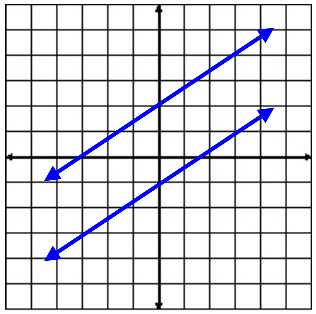
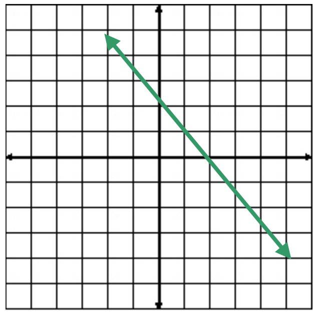
SYSTEMS OF EQUATIONS







Unit Overview

This unit is about solving systems of equations graphically and algebraically. Systems of equations are used in real-world situations where two variables need to be determined. Some examples are maximum profit of sales, cost, and even in predicting the weather.

Graphing Systems of Equations

Two equations in two variables are called a system of equations. The solution to a system of equations is the point on a coordinate plane where the two equations intersect.

There are three possible solutions for a system of equations.

1) The lines may intersect at one point; therefore, the solutions would be an ordered pair **(*x*, *y*)**.

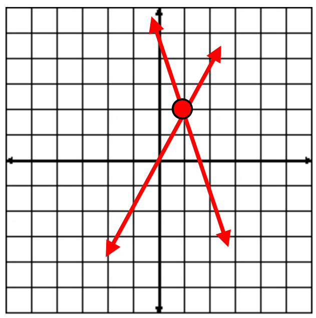
2) The lines may be parallel and not intersect at all; therefore, the solution would be **no solution**.

3) The lines may be identical and would lie on top of each other when graphed; so, the solution would be **many solutions**.

Each of the three possible solutions is shown in the three graphs below.

$y = 2x$

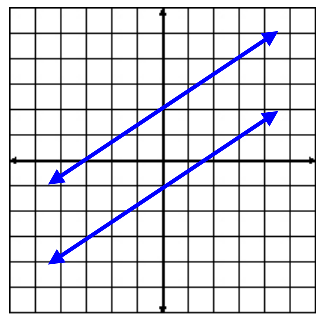
$y = -3x + 5$



One Solution

$y = \frac{2}{3}x + 2$

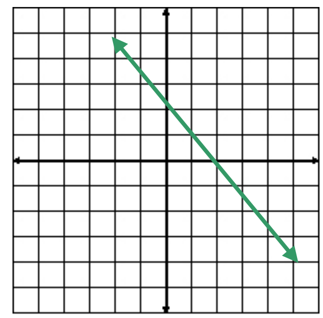
$y = \frac{2}{3}x - 1$



No Solution

$y = -x + 2$

$y = -x + 2$



Many Solutions

QuickTime Systems of Equations and Graphs (03:19)

Solving a System of Equations by Graphing

To solve a system of equations by graphing, first solve each equation for y and make sure they are in the slope-intercept form:

$$\text{“}y = mx + b\text{”}$$

Plot the y-intercept and use the slope ratio of $\frac{\text{rise}}{\text{run}}$ to plot more points. After graphing each equation, determine the point of intersection (x, y) as the solution.

Example #1: Use graphing to solve the system of equations shown below.

$3x + y = 4$
$2x - y = 6$

Step 1: Solve each equation for y.

$$\begin{array}{rcl} 3x + y &=& 4 \\ -3x & & -3x \\ y &=& -3x + 4 \end{array}$$

$$\begin{array}{rcl} 2x - y &=& 6 \\ -2x & & -2x \\ -y &=& -2x + 6 \end{array} \rightarrow \frac{-1y}{-1} = \frac{-2x+6}{-1}$$

$$y = 2x - 6$$

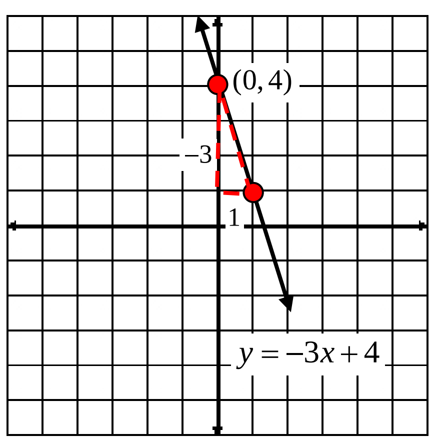
Step 2: Determine the y-intercept and the slope. $y = mx + b$

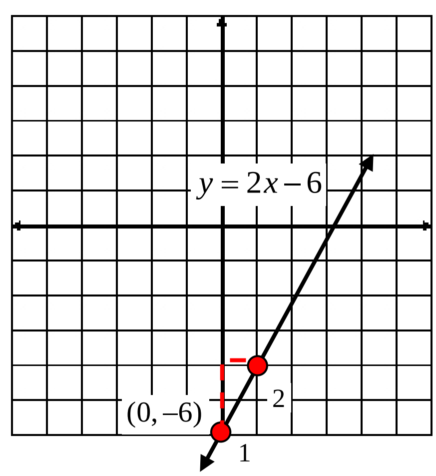
$$y = -3x + 4 \qquad\qquad y = 2x - 6$$

y-intercept = (0, 4) $\qquad$ y-intercept = (0, –6)

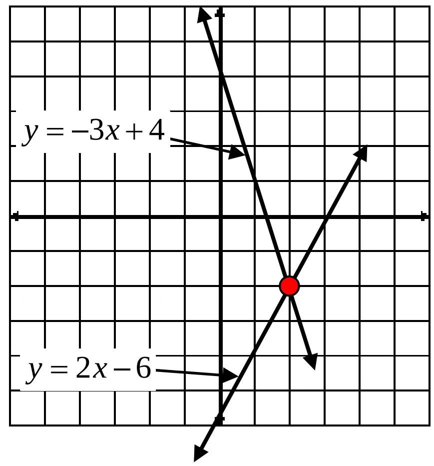
$\text{slope} = \frac{-3}{1}$ $\qquad$ $\text{slope} = \frac{2}{1}$

Step 3: Plot the y-intercept and use the slope to graph each equation.





Step 4: Now look at both lines in the same graph and find the point of intersection.



The point at which this system of equations intersects is (2, –2); therefore, the solution is (2, –2).

Another way to state the solution is to say that $x = 2$ and $y = -2$.

Note: Please observe that if you put the ordered pair into each of the original (or transformed) equations, it is a solution for the equations (makes it true)!

Let's try!

First equation:

Let $x = 2$ and $y = -2$.

$3x + y = 4$
$3(2) + -2 = 4$
$6 + -2 = 4$
$4 = 4$ *True*

Second equation:

Let $x = 2$ and $y = -2$.

$y = 2x - 6$
$-2 = 2(2) - 6$
$-2 = 4 - 6$
$-2 = -2$ *True*

This check shows that indeed, (2, –2), the solution of the system, is a point on the graphs of both equations. It is the point where the two graphs cross each other (intersect).

Application

Let's look at a simple application of how graphing a system of equations can help solve real-world problems.

In the next example, write an equation to describe each train's travels, and then graph both equations to find the solution.

Example #2: A train leaves a station and travels east at 72 KPM. Three hours later a second train leaves on a parallel track and travels east at 120 KPH. When will the second train overtake the first train. (*Note*: This problem is set up for ideal conditions.)

First, since this problem is about distance, rate, and time, we will use $d = r\,t$ to set up the equations.

We'll use a table to organize our data.

	Distance	Rate	Time
Train 1	d	72	t
Train 2	d	120	$t - 3$*

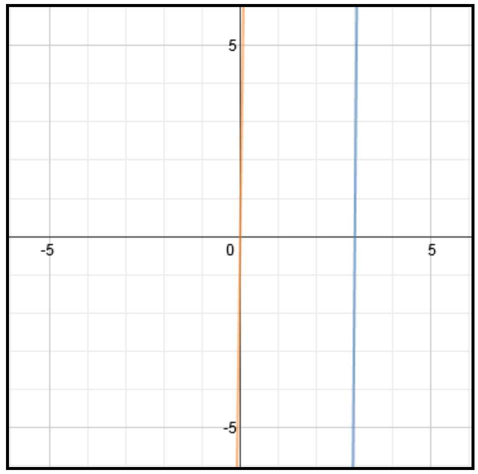
**Note*: Train 2 will have 3 less hours of travel time, so "$t - 3$" represents its time.

The equation for Train 1 is $d = 72t$.

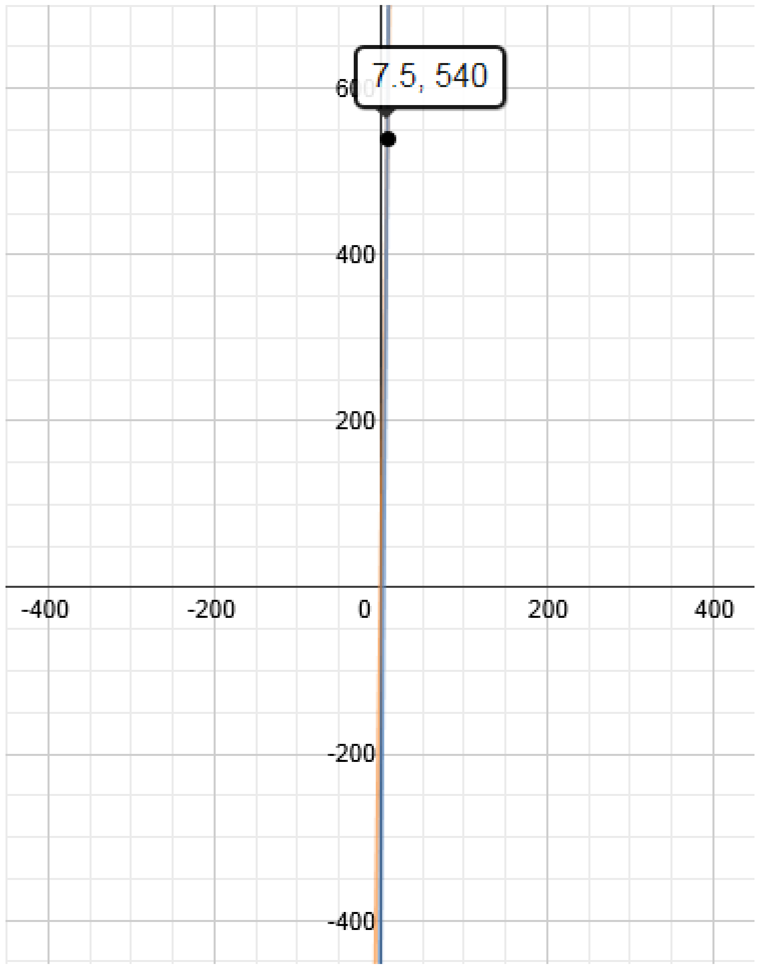
The equation for Train 2 is $d = 120(t - 3)$.

Now, its time to graph the two equations. Use a graphing calculator or click *here* for a link to an online grapher.

The first graph doesn't really tell us much. The orange and blue lines look parallel. So, we must zoom "out" to see if they truly are parallel or if they intersect.



After zooming "out" many times, we find that the two graphs intersect at (7.5, 540).



This means that after 7.5 hours, the two trains will be side by side after they have traveled 540 miles!

Let's check our answer to be sure this is true. Substitute the ordered pair (7.5, 540) in each equation to see if it tests true.

	Distance	Rate	Time	
Train 1	$d = rt$	540 = 72(7.5)	540 = 540	***True***
Train 2	$d = rt$	540 = 120(7.5 – 3)	540 = 540	***True***

Indeed, (7.5, 540) tests true for both equations! ✓

QuickTime Pair of Linear Equations--Balloons (02:42)

Stop! **Go to Questions #1-8 about this section, then return to continue on to the next section.**

Solving Systems by Substitution

Another way to solve a system of equations is algebraically using substitution. To solve a system of equations by substitution:

1) Solve one equation for a variable (it is much easier to solve an equation for a variable that has a coefficient of 1).

2) Substitute this value into the other equation and find the value of one variable.

3) Substitute the value found in step 2 into either of the equations to solve for the second variable.

Follow the examples below:

One Solution

Example #1: Solve the system below by using substitution. Use the steps to plan a solution.

$$x + y = 2$$

$$3x + y = 8$$

Know
Neither equation is solved for either variable.

In this particular case, we have choices as to which variable to solve for as both x and y have one as their coefficients in the first equation.

Need to Know
The solution or ordered pair for the system.

Plan
Solve one of the equations for one of the variables.

1) Solve the first equation for either x or y or solve the second equation for y. (Again, it is easier to solve an equation for a variable that has a coefficient of 1). **We'll solve the first equation for *y*.**

$x + y = 2$ (subtract x from both sides)

$y = 2 - x$

2) Substitute $2 - x$ into the second equation for y.

$3x + y = 8$ (second equation)

$3x + (2 - x) = 8$ (substitute)

$3x + 2 - x = 8$ (remove parentheses)

$3x - x + 2 = 8$ (put like terms together)

$2x + 2 = 8$ (combine like terms)

$2x = 6$ (subtract 2 from both sides)

$x = 3$ (divide both sides by 2)

3) Substitute $x = 3$ back into either of the equations to solve for y. We'll use the first equation.

$x + y = 2$ (first equation)

$3 + y = 2$ (substitute)

$y = -1$ (solve for y)

The solution to this system is $(3, -1)$.

Now, let's look at this same problem again, but solve the first equation for *x* to begin. Do you think the solution will be the same?

1) Solve the first equation for either *x* or *y* or solve the second equation for *y*. This time we'll choose to **solve the first equation for *x*.**

$x + y = 2$ (subtract y from both sides)

$x = 2 - y$

2) Substitute 2 – *y* into the second equation for *x*.

$3x + y = 8$ (second equation)

$3(2 - y) + y = 8$ (substitute)

$6 - 3y + y = 8$ (distribute)

$6 - 2y = 8$ (combine like terms)

$-2y = 2$ (subtract 6 from both sides)

$y = -1$ (divide both sides by -2)

3) Substitute *y* = –1 back into either of the equations to solve for *x*. We'll use the first equation.

$x + y = 2$ (first equation)

$x + (-1) = 2$ (substitute)

$x - 1 = 2$ (remove parentheses)

$x = 3$ (solve for *x*)

The solution to this system is (3, –1).

Are the two solutions the same? Yes!

Example #2: Solve the following system using substitution.

$$11x + 2y = 160$$

$$x - 4y = -21$$

In this particular case, the only variable with a coefficient of 1 is x in the second equation.

1) Solve the second equation for x.

$x - 4y = -21$	(add $4y$ to both sides)
$x = -21 + 4y$	

2) Substitute $-21 + 4y$ into the first equation for x.

$11x + 2y = 160$	(first equation)
$11(-21 + 4y) + 2y = 160$	(substitute)
$-231 + 44y + 2y = 160$	(distribute)
$-231 + 46y = 160$	(combine like terms)
$46y = 391$	(add 231 to both sides)
$y = 8.5$	(divide both sides by 46)

3) Substitute $y = 8.5$ back into either of the equations to solve for x. We'll substitute into the first equation.

$x - 4y = -21$	(first equation)
$x - 4(8.5) = -21$	(substitute)
$x - 34 = -21$	(multiply)
$x = 13$	(solve for x)

The solution to this system is (13, 8.5).

QuickTime Solving Systems of Equations with Substitution (04:24)

Just as the systems of equations that were graphed had a possibility of 3 solutions, so do the systems of equations that are solved by substitution.

No Solution

Example #3: Solve the system below by using substitution.

$$2x + y = 9$$

$$y + 2x = 7$$

1) Solve the first equation for *y*.

$2x + y = 9$

$y = 9 - 2x$ (subtract $2x$ from both sides)

2) Substitute 9 – 2*x* into the second equation for *y*.

$y + 2x = 7$ (second equation)

$9 - 2x + 2x = 7$ (substitute)

$9 = 7$ (false statement, 9 does NOT equal 7)

Since this is NOT a true statement and the variable (*x*) has been eliminated completely; this system does not have a solution. So you will say that the solution is **no solution**.

Many Solutions

Example #4: Solve the following system using substitution.

$$2x - y = -4$$

$$y - 2x = 4$$

1) Solve the second equation for *y*.

$y - 2x = 4$ (second equation)

$y = 4 + 2x$ (add $2x$ to both sides)

2) Substitute 4 + 2*x* into the first equation for *y*.

$2x - y = -4$	(first equation)
$2x - (4 + 2x) = -4$	(substitute)
$2x - 4 - 2x = -4$	(remove parentheses, switch the signs of the terms within as the parentheses has a minus sign in front of it)
$2x - 2x - 4 = -4$	(put like terms together)
$-4 = -4$	(collect like terms)

Since this is a true statement and the variable (*x*) has been eliminated completely; this means that the solution is **many solutions**.

QuickTime Systems of Linear Equations Can Have One Solution, No Solution, or Infinite Solutions (09:12)

Application

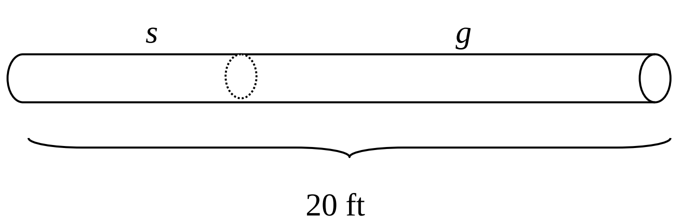
Let's look at a simple application of how using sustitution with a system of equations can help solve real-world problems.

In the next example, write an equation to describe each train's travels, and then graph both equations to find the solution.

Example #5: An installer of underground irrigation systems wants to cut a 20-foot length of tubing into two pieces. The longer piece is to be 2 feet longer than twice the shorter piece. Find the length of each piece.

First, sketch a picture to model the scenario and label it.

Let *s* = shorter piece. Let *g* represent the longer piece.



After labeling the picture, we can see that one equation will be $s + g = 20$.

We can write a second equation from the statement:

$$\underbrace{\text{"The longer piece}}_{g}\ \underbrace{\text{is to be}}_{=}\ \underbrace{\text{2 feet longer than twice the shorter piece."}}_{2+2s}$$

Now, let's put it all together and solve.

$$s + g = 20$$
$$g = 2 + 2s$$

$s + 2 + 2s = 20$	Substitute.
$3s + 2 = 20$	Collect like terms.
$3s = 18$	Subtract 2 from both sides.
$s = 6$	Divide.

The shorter piece is 6 feet.

The longer piece is found by substituting 6 in for either equation.

$$g = 2 + 2s$$
$$g = 2 + 2(6)$$
$$g = 2 + 12$$
$$g = 14$$

The longer piece is 14 feet.

Check the answers using the other equation.

$$s + g = 20$$
$$6 + 14 = 20$$
$$20 = 20 \checkmark$$

Stop! **Go to Questions #9-30 to complete this unit.**