Course Overview

In this course, students connect physical, verbal, and symbolic representations of the real number system; investigate properties including closure; demonstrate fluency in computations with real numbers; solve and graph linear equations and inequalities. Students use formulas to solve problems including exponential growth and decay; add, subtract, multiply, and divide monomials and polynomials; and solve quadratic equations with real roots by graphing, formula, and factoring. Students define functions, determine slope, calculate distance, and draw graphs of linear equations using slope, *y*-intercept, parallel, and perpendicular lines; determine the characteristics of linear, quadratic, and exponential functions; solve systems of linear equations involving two variables graphically and symbolically; simplify and compute with rational and radical expressions; model and solve problem situations involving direct and indirect variation.

In Algebra I, you will begin your journey to learn mathematical and theoretical concepts which lay the foundation to take more advanced math classes, both in high school and beyond. Mathematics knowledge is built in steps and Algebra I is one of its building blocks. With mastery of Algebra I skills, you will have a solid foundation to pursue many different paths and further your knowledge of mathematics.

EXPRESSIONS, VARIABLES, AND PROPERTIES

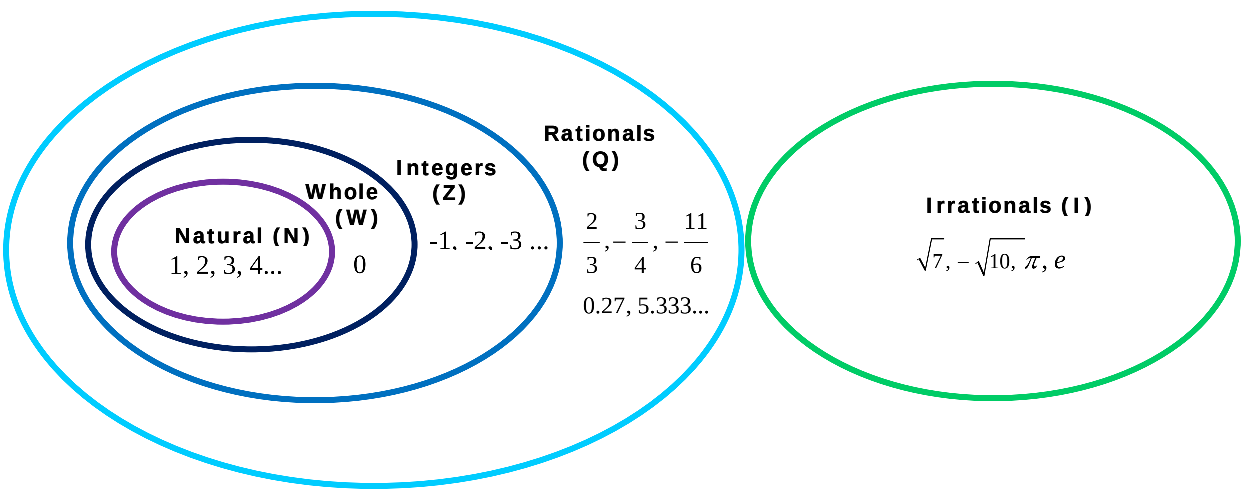
Unit Overview

This unit will lay the foundation for the studies of Algebra. In this unit, you will be introduced to algebraic terminology, sets of numbers, and most importantly, identify the properties of addition and multiplication, which are extensively used in solving algebraic equations.

Sets of Numbers

Throughout Algebra, you will be working with different sets of numbers. Below, is a Venn Diagram that defines all of the sets we will be using in this coursework:

real numbers = rational numbers + irrational numbers



Natural Numbers (N): 1, 2, 3, 4…

Whole Numbers (W): 0, 1, 2, 3…

Integers (Z): Whole numbers and their opposites

Rational Numbers (Q): Any number that can be expressed as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Irrational Numbers (I): Any number that cannot be expressed as a fraction $\frac{a}{b}$

Real Numbers (R): All numbers previously listed here.

QuickTime Numbers and Numberlines (05:28)

Stop! **Go to Questions #1-4 about this section, then return to continue on to the next section.**

Order of Operations

In order to find the numerical value (**evaluate**) of any combination of numbers and operations (**expression**) correctly, mathematicians have established the order of operations which tells us which operations to do first in any mathematical problem.

P (parentheses)

E (exponents – powers)

M (multiply) } work left to right
D (divide)

A (add) } work left to right
S (subtract)

This "saying" may be used to help remember the order of operations.

Please
Excuse
My **D**ear
Aunt **S**ally

*Note: Multiplication and division are at the same level, meaning multiplication does NOT take priority over division. Work these two operations as they occur, left to right. The same is true about addition and subtraction. Work the two operations as they occur, left to right.

Thus, if "multiplication and division" or "addition and subtraction" are the only two operations in the expression, work the problem from left to right!

QuickTime Order of Operations (10:50)

Examples:

1) $6 \times 4 + 2$	1) Multiply 6×4
$24 + 2$	2) Add $24 + 2$
26	
2) $4(6+3)-5\cdot 2$	1) Parentheses $(6 + 3)$
$4(9)-5\cdot 2$	2) Multiply $4(9)$
$36-5\cdot 2$	3) Multiply $5\cdot 2$
$36 - 10$	4) Subtract
26	
3) $5[(3+12)-2(4)]$	1) Work within [ ]
	a. Multiply $2\cdot 4$
$5[(3+12)-8]$	b. Add $3 + 12$

	$5[15-8]$	c. Subtract 15 – 8
	$5[7]$	2) Multiply 5[7]
	35	

4)	$4[3(3+2)^2]$	1) Work within [ ]
		a. Parentheses (3 + 2)
	$4[3(5)^2]$	b. Powers $(5)^2$
	$4[3\cdot 25]$	c. Multiply $3\cdot 25$
	$4[75]$	2) Multiply 4[75]
	300	

5)	$4\cdot 5-18\div 6+2\cdot 3$	1) Multiply and divide left to right
	$20-3+6$	2) Add and subtract left to right
	$17+6$	3) Add
	23	

Practice:

To evaluate the expression 6[9 + 2(15 – 8)] , what is the first step?

"Click here" to check your answer.

6[9 + 2(7)] Parentheses first.

What will the next step be?

"Click here" to check your answer.

6(9 +14) Multiply before adding.

What will the next step be?

"Click here" to check your answer.

6(23) Parentheses first.

What is the final answer?

"Click here" to check your answer.

You will continue to use the **order of operations** throughout the remainder of this unit and throughout any other mathematics courses you continue to take.

Stop! **Go to Questions #5-9 about this section, then return to continue on to the next section.**

Introduction to Variables and Expressions

Did you know?

Most countries in the world use the Celsius scale to measure temperature. Two critical temperatures on the Celsius scale are $0°$ freezing and $100°$ boiling. On the other hand, in the United States we use the Fahrenheit scale most of the time. The same two critical temperatures on the Fahrenheit scale measure $32°$ freezing and $212°$ boiling. It is possible to convert between temperature scales by using algebra. If the Celsius temperature is multiplied by $\frac{9}{5}$ and then added to 32, the Fahrenheit temperature can be determined.

Algebra can be thought of as a language of symbols. For example, we already know the symbols for addition (+) and multiplication (× or ·), so we could write the temperature relationship from above as follows:

$$\frac{9}{5} \cdot \textbf{Celsius} + 32$$

In arithmetic, we could write the same expression as:

$$\frac{9}{5} \cdot \square + 32$$

where □ represents the Celsius temperature and is serving as a place holder.

In algebra, when a problem has missing or "unknown" information, the place holders used are called **variables**. Variables are letters such as *x*, *n*, or *a* that are used to represent the unknown value. (You may use any letter as a variable; these were just a few examples.)

***When choosing a variable to represent an unknown value, make sure not to use the letter "o" because it could be mistaken for the number zero.**

Let's take a look at how to represent an expression using algebra and a variable.

$\frac{9}{5}\cdot$ **Celsius** + 32	Words and symbols
$\frac{9}{5}\cdot \square + 32$	Arithmetic
$\frac{9}{5}C+32$	Algebra (variable)

$\frac{9}{5}\cdot C+32$ is called an **algebraic expression** because it contains a combination of variables, numbers and at least one operation.

* Note it is important to understand that expressions do not contain an equal sign!

Algebraic expressions can be **evaluated** by replacing the variable with numbers.

For example, if given the expression $a + b - 24$ and asked to evaluate it for the given values $a = 19$ and $b = 20$, you would:

a.) replace *a* and *b* with the given values
b.) evaluate the expression using the order of operations

Let's try the example above. You will be given this type of problem in the following form.

Example #1: Evaluate $a + b - 24$ if $a = 19$ and $b = 20$.

$a + b - 24$	
$19 + 20 - 24$	1) replace *a* and *b* with the given values
$39 - 24$	2) evaluate using the order of operations
15	

Example #2: Evaluate $5a + bc - c$ if $a = 4$, $b = 2$, and $c = 3$.

$5a + bc - c$	
$5(4) + (2)(3) - 3$	1) replace *a*, *b*, and *c* with the given values
$20 + (2)(3) - 3$	2) multiply 5(4)
$20 + 6 - 3$	3) multiply 2(3)
$26 - 3$	4) add 20 + 6
23	5) subtract 26 – 3

Example #3: Evaluate $\frac{7ab}{2c-1}$ if $a = 5$, $b = 6$, and $c = 2$.

$\frac{7ab}{2c-1}$	
$\frac{7(5)(6)}{2(2)-1}$	1) replace a, b, and c with the given values
$\frac{35(6)}{2(2)-1}$	2) multiply 7(5)
$\frac{210}{2(2)-1}$	3) multiply 35(6)
$\frac{210}{4-1}$	4) multiply 2(2)
$\frac{210}{3}$	5) subtract $4-1$
70	6) divide 210 by 3

Practice:

What must be done first to evaluate the following expression?

$$12b - \frac{2c-4}{a+4} \quad \text{if } a = 2, b = 3, \text{and } c = 8$$

"Click here" to check your answer.

Substitute 2 in for "*a*", 3 in for "*b*", and 8 in for "*c*".

What is the next step in calculating $12(3) - \frac{2(8)-4}{2+4}$?

"Click here" to check your answer.

Multiply.

What is the next step in calculating $36-\frac{2(8)-4}{2+4}$?

"Click here" to check your answer.

Multiply in the numerator of the fraction.

What is the next step in calculating $36-\frac{16-4}{2+4}$?

"Click here" to check your answer.

Simplify the numerator and denominator of the fraction.

What is the next step in calculating $36-\frac{12}{6}$?

"Click here" to check your answer.

Divide before subtracting.

What is the final answer for $36-2$?

"Click here" to check your answer.

34

As you become more familiar with evaluating expressions, you will be able to perform more than one operation per step. For example, in the example above it would have been okay in step 2 to also multiply 2 and 3. Again you will be able to do this after more practice.

Stop! **Go to Questions #10-14 about this section, then return to continue on to the next section.**

Parts of an Algebraic Expression

As we work with algebraic expressions, it is important to examine the parts of the expression. Review the example below and the description of its parts.

Terms: The terms are the individual parts of the expression.

$$5x + 6xz + 13$$

There are three terms in this expression. One is 5*x* and second is 6*xz*.

What is the third term?

"Click here" to check your answer.

13

Variable Term: The variable terms are the terms that contain variables.

Which terms of the expression 5*x* + 6*xz* + 13 are variable terms?

"Click here" to check your answer.

5*x* and 6*xz*

Constant Term: In simple definitions, a constant is a number (no variable).

The constant in the expression 5*x* + 6*xz* + 13 is 13.

Factors: When a term contains multiplication, the term has "factors."

For example, in the expression 5*x* + 6*xz* + 13 is 13 the factors of the term 6*xz* are 6, *x*, and *z.*

Coefficient: A coefficient is a numerical factor of a product.

In the expression 5*x* + 6*xz* + 13, the coefficent of the term 5*x* is 5.

Name another coefficient in the expression 5*x* + 6*xz* + 13.

"Click here" to check your answer.

6

Throughout this course and any of the more advanced mathematics courses you will take, it will be necessary to interpret verbal sentences into algebraic sentences. For this, you will need to know the words and phrases that suggest the operation to use.

The chart below lists some of the most common phrases that will be used.

Addition	**Subtraction**	**Multiplication**	**Division**
sum	difference	product	quotient
plus	minus	times	divided by
increased by	decreased by	of	ratio
more than	less than	twice (× 2)	per
total	subtract	multiplied by	average

Example #1: "eight more points than Rachel's score"

a.) Let r represent Rachel's score
b.) "More than" suggests addition
c.) The algebraic expression is $r + 8$ or $8 + r$

Example #2: "four times as much money as Pete"

a.) Let p represent Pete's money
b.) "Times" suggests multiplication
c.) The algebraic expression is $p \times 4$, $4 \times p$, or more commonly seen in algebra as $4p$.

Example #3: "twice the quantity of a number plus seven"

a.) Let n represent the number
b.) "Twice" suggests two times as much
c.) "Quantity" suggests parentheses $(n + 7)$

The algebraic expression is $2 \times (n + 7)$, or more commonly seen in algebra as $2(n + 7)$.

*At this point, it should be noted that anytime a variable or variables are multiplied with a number, it will be written with the number first and then the variable or variables following. Study the examples given below:

6*x* means 6 times *x* OR *x* times 6.

7*y* means 7 times *y* OR *y* times 7.

3*xyz* means 3 times *x* times *y* times *z* in any order.

Stop! **Go to Questions #15-20 about this section, then return to continue on to the next section.**

Properties of Real Numbers

In the previous section, you learned how to translate verbal phrases into algebraic sentences. You were able to do this because of mathematical properties. In this section, you will study these properties that you will use in future units to solve equations.

Commutative Properties of Addition and Multiplication	
The **order** in which numbers are added does not make a difference in the sum. $6 + 4 = 4 + 6$ For any numbers *x* and *y*, $x + y = y + x$	The **order** in which numbers are multiplied does not make a difference in the product. $6 \times 4 = 4 \times 6$ For any numbers *x* and *y*, $xy = yx$

QuickTime The Commutative Properties of Addition and Multiplication (02:21)

Associative Properties of Addition and Multiplication	
The way in which numbers are **grouped** does not change the sum. $(2 + 3) + 4 = 2 + (3 + 4)$ For any numbers *x*, *y*, and *z*, $(x + y) + z = x + (y + z)$	The way in which numbers are **grouped** does not change the product. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ For any numbers *x*, *y*, and *z*, $(xy)z = x(yz)$

QuickTime Three's a Crowd: Working with Two Numbers at a Time (00:51)

QuickTime The Associative Properties of Addition and Multiplication (00:46)

Identity Properties of Addition and Multiplication	
The sum of a number and zero is that number. $3 + 0 = 3$ For any number *n*, $n + 0 = n$	The product of a number and one is the number. $5 \cdot 1 = 5$ For any number *n*, $n \cdot 1 = n$

Multiplicative Property of Zero
The product of a number and zero is zero. $7 \cdot 0 = 0$ For any number *n*, $n \cdot 0 = 0$

Operation: An operation is a process such as addition, subtraction, multiplication, division, or square root that is performed in a specified sequence and in accordance with specific rules.

Closure Property

Given any set of numbers and an operation to perform, if the solution after the operation is a number that remains in that set of numbers, then the set is said to be **closed**.

For example, given the whole numbers, is the set of whole numbers *closed* for addition?

We are given a set of whole numbers {0, 1, 2, 3, 4, 5...} to work with.

Let's test a problem to see if the whole numbers are closed for addition. Is the answer to 7 + 6 a whole number?

Yes, 13 is a whole number. Can you think of any examples when you are adding whole numbers that the result will *not* be a whole number?

Whenever you add two whole numbers, the result will be a whole number; thus, the whole numbers are *closed* for addtion.

Now, let's test the whole numbers to see if they are closed for subtraction. You only need to find one "counterexample" to show that the set is ***not* closed**.

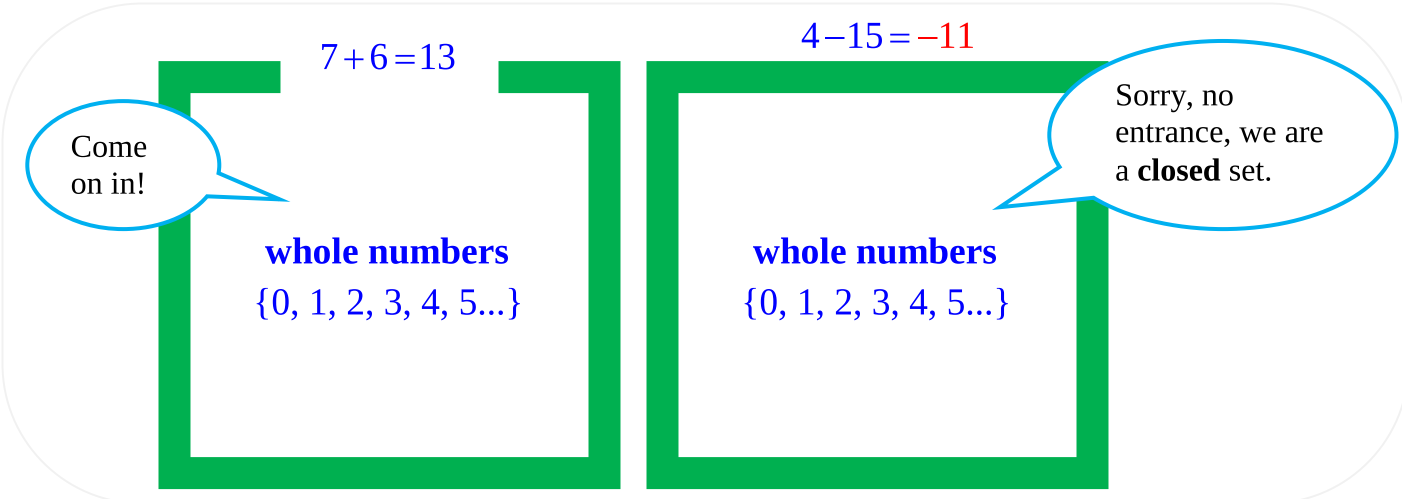
A **counterexample** is an example where the *operation* does not hold true for the given set.

For example, is the answer to 4 –15 a whole number?

The answer is –11 which is NOT a whole number. Thus, the set of whole numbers are **NOT *closed*** for subtraction.

*You only need **one counterexample** to show that the set is not closed.

Study the figures below to review the closure property.



These properties will be helpful when you start solving equations later on in the course. Right now, you will be asked in the assignment to identify properties illustrated by algebraic expressions, so let's practice deciding which property is shown.

Examples:

1) $4 + (9 + 5) = (4 + 9) + 5$ — Associative Property of Addition

2) $y + 14 = 14 + y$ — Commutative Property of Addition

3) $(9 + n) + 6 = 6 + (9 + n)$ — Commutative Property of Addition

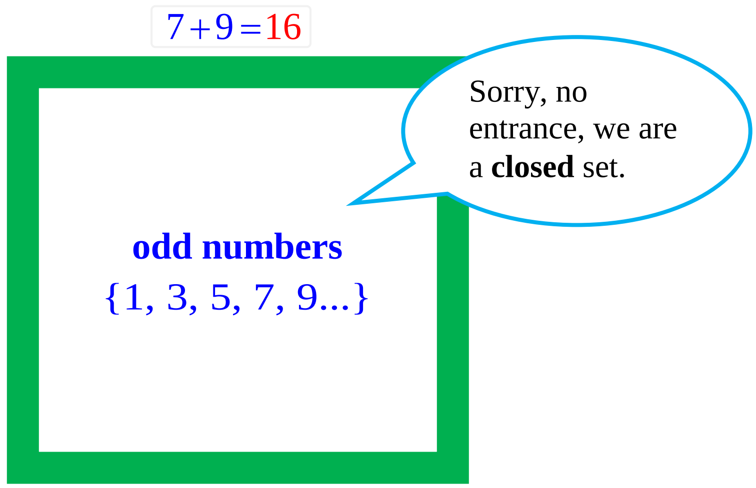
4) $5m \cdot 0 = 0$ — Multiplicative Property of Zero

5) $0 + 17 = 17$ — Identity Property of Addition

6) $8xyz \cdot 1 = 8xyz$ — Identity Property of Multiplication

7) Are odd numbers closed for addition?

When two odd numbers are added, the result is an even number. Thus, the set of odd numbers are *NOT* closed for addition.



QuickTime Algebraic Properties (10:14)

Stop! **Go to Questions #21-28 to complete this unit.**