QUADRATICS WITH COMPLEX ROOTS

Unit Overview

In this unit, you will study complex roots of quadratic equations. Complex roots arise when using the quadratic formula results in taking the square root of a negative number.

In units 19 and 20, we studied quadratic equations and how to solve them. However, sometimes we may get an equation that we cannot factor. It may be helpful to review the quadratic formula as well as the discriminant. *To review the quadratic formula, click here.* (*Unit 20, Quadratic Formula*)

Consider: $x^2 + 2x + 5 = 0$

Can you find 2 numbers that are factors of 5 and add to 2? No, this is not possible. So, we must use the quadratic formula. Remember:

Quadratic Formula

If a quadratic is in the form of $ax^2 + bx + c = y$, then the quadratic formula can be used to solve the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our example, define *a*, *b*, and *c*. *a* = 1, *b* = 2, and *c* = 5. Plugging this into our formula, we get:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$$

Now, while we could try to simplify further, I want you to stop at this point and consider the discriminant (the part under the square root or radical). What number is under the radical sign?

Be careful, make sure you see that it is **NEGATIVE** 16. What is the square root of –16? 4^2 is **positive** 16. $(-4)^2$ is also **positive** 16. There is no number that gives a negative 16. Therefore, there are no real solutions to this equation. To deal with this problem, we have defined the $\sqrt{-1} = i$ where *i* stands for imaginary. We call the square root of negative numbers, the imaginary numbers.

We use this information to understand that, sometimes, the quadratic formula yields non-real solutions. We sometimes get imaginary solutions. We can discern the type of solutions by looking at the discriminant. The **discriminant is the $b^2 - 4ac$ value under the radical sign**.

Finding the Discriminant

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the discriminant is the part under the radical sign or $b^2 - 4ac$.

Quick review. Use the quadratic formula to find the discriminant of the following equation: $\boldsymbol{g(x) = x^2 + 10x + 35}$.

"Click here" to check your answer.

$a = 1$ $b = 10$ $c = 35$

$b^2 - 4ac = 10^2 - 4(1)(35) = 100 - 140 = -40$

Now that you can find the discriminant, you will be able to easily determine the type of solutions to the equation.

Solutions based on Discriminant

$b^2 - 4ac > 0$ or positive yields 2 real solutions

$b^2 - 4ac = 0$ yields 1 real solution

$b^2 - 4ac < 0$ or negative yields 2 imaginary solutions (NO real solutions)

To solve quadratic equations that have imaginary solutions, you solve the same way you would normally.

Example #1: $x^2 + 2x + 17 = 0$

Use the quadratic formula to solve.

$a = 1$, $b = 2$, c = 17

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(17)}}{2(1)} = \frac{-2 \pm \sqrt{4-68}}{2} = \frac{-2 \pm \sqrt{-64}}{2}$$

At this point, you will need to know how to simplify $\sqrt{-64}$. To do this, factor the same way you would when taking the square root of a positive. Make sure to make one factor –1.

$$\sqrt{-64} = \sqrt{-1} \cdot \sqrt{-64} = i \cdot 8 = 8i$$

Now, finish by substituting 8*i* for $\sqrt{-64}$.

$x = \frac{-2 \pm 8i}{2} = -1 \pm 4i$ ***Don't forget to divide each term by 2!**

The answer can be written this way. However, to differentiate the two answers, one is –1 + 4*i*, while the other is = –1 – 4*i*. This is known as a complex number.

Complex Numbers

Complex numbers are a combination of a real number and an imaginary number. They take the form ***a* + *bi*** where a is a real number and bi is an imaginary number. b is a coefficient.

Example #2: $x^2 + 4x + 5 = 0$

$a = 1$, $b = 4$, $c = 5$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \sqrt{4}\sqrt{-1}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

Note, this has 2 answers $-2 + i$ and $-2 - i$. Because the discriminant was negative, the solutions are both imaginary.

Example #3: Find the **number** and **type** of solutions for: $3x^2 - 10x + 3 = 0$

The number and type of solutions are determined by the discriminant, $b^2 - 4ac$.

$a = 3$, $b = -10$, $c = 3$

$$b^2 - 4ac = (-10)^2 - 4(3)(3) = 100 - 36 = 64$$

Because our answer is > 0 or positive, this equation has 2 real solutions. Therefore, 2 is the number of solutions, and real is the type of solutions.

Example #4: Find the **number** and **type** of solutions for: $x^2 + 2x + 1 = 0$.

$a = 1$, $b = 2$, $c = 1$

$$b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$$

Because our answer = 0, this equation has exactly 1 real solution.

Example #5: Find the **number** and **type** of solutions for: $x^2 + 1 = 0$

$a = 1$, $b = 0$, $c = 1$ Notice that there is no linear term (or x term) so $b = 0$.

$b^2 - 4ac = 0^2 - 4(1)(1) = 0 - 4 = -4$

Because our answer is negative, this equation has 2 imaginary solutions.

Example #6: Find the solutions for $x^2 + 8x + 20 = 0$. For this problem, we need to use the full quadratic formula to find the solutions.

$a = 1$, $b = 8$, $c = 20$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(20)}}{2(1)} = \frac{-8 \pm \sqrt{64-80}}{2} = \frac{-8 \pm \sqrt{-16}}{2} = \frac{-8 \pm \sqrt{16}\sqrt{-1}}{2} = \frac{-8 \pm 4i}{2} = -4 \pm 2i$$

Stop! **Go to Questions #1-30 for this unit.**