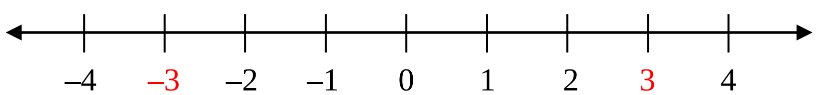
INTEGERS AND EQUATIONS

Unit Overview

In this unit, you will review integers and the rules that apply to adding, subtracting, multiplying and dividing these special numbers. You will also learn about equations and how the Real Number Properties of Equality justify the steps to solve an equation. You will also solve literal equations and formulas.

Comparing Integers

The set of whole numbers consists of 0, 1, 2, 3 … and can be represented on a number line. We can match each whole number with another number that is the same distance from 0 but on the opposite side of 0.



If you take a look at the number line above, 3 or (positive 3) and –3 (negative 3) are on opposite sides of 0 but the same distance from 0. These numbers are called opposites and make up the set of **integers.** Integers are the set of positive and negative whole numbers.

Examples: Name the integer that is suggested by each situation.

a) The temperature is $5°$ below 0.

–5 Below 0 suggests a negative integer.

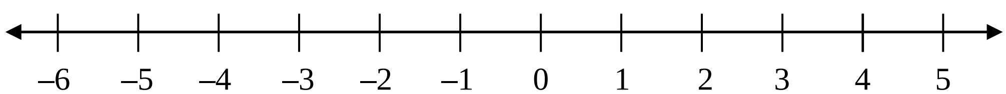
b) Emily’s lemonade stand made a \$24 profit.

24 A profit suggests a positive integer.

Integers (03:50)

To compare integers, we will use the symbol “<” which means less than or the symbol “>” which means greater than. If you remember from the previous unit, these symbols are called **inequality** signs. An inequality can either be true or false. For example, the sentence $12 > 8$ is true and the sentence $6 > 9$ is false.

On a number line, the numbers increase as you move from left to right. For any two numbers, the number that is farther to the right is the larger number and the number farther to the left is the lesser number. Let’s take a look at comparing some integers using the number line below.



Write a true sentence using $<$ or $>$ in place of $\square$.

a) $3 \square 9 \Rightarrow 3 < 9$

Since 3 is to the left of 9 on the number line, 3 is less than 9.

b) $-5 \square 11 \Rightarrow -5 < 11$

Since –5 is to the left of 11 on the number line, –5 is less than 11.

c) $-3 \square -6 \Rightarrow -3 > -6$

Since –3 is to the right of –6 on the number line, –3 is greater than –6.

Hint: An easy way to remember the direction of the inequality sign is that it always points to the smaller number.

If you study the number line above, you will notice that the integers 5 and –5 are the same distance from 0. This brings us to another algebraic concept called **absolute value**. The absolute value of a number is the distance it is from 0, which means that the absolute value will always be **positive**. Absolute value is symbolized by using two straight bars around a number.

Example #1: $|24|$ means the absolute value of 24, which is 24 because 24 is 24 units away from 0.

Example #2: $|-57|$ means the absolute value of –57, which is 57 because –57 is 57 units away from 0.

Stop! **Go to Questions #1-6 about this section, then return to continue on to the next section.**

Adding and Subtracting Integers

When adding and subtracting integers, it will be very helpful to understand the following rules:

Adding Integers

Same sign: If the numbers have the same sign, add the numbers and keep the sign given.

(–5) + (–9) = –14	Both integers are negative, so the answer is negative.
6 + 15 = 21	Both integers are positive, so the answer is positive.

Different signs: If the numbers have different signs, subtract the absolute value of the numbers and use the sign of the larger absolute value.

Example #1:

(–9) + 3 = –6	Subtract 9 and 3 to get 6, and then determine that –9 has the larger absolute value; so, you will use the negative sign.

Example #2:

17 + (–4) = 13	Subtract 17 and 4 to get 13, and then determine that 17 has the larger absolute value; so, you will use the positive sign, or in this case no sign at all represents positive.

QuickTime Addition (06:37)

Subtracting Integers

Change the problem to addition and add the opposite of the second number. Then go to addition rules.

Example #1:	15 – (–8) =	
	15 + (–8) =	change the problem to addition
	15 + (+8) =	change (–8) to a (+8)
	23	Add the numbers because they now have the same sign.

Example #2:	18 – 26 =	
	18 + (26) =	change the problem to addition
	18 + (–26)=	change 26 to (–26)
	–8	Subtract the numbers because they now have different signs and use the sign of the larger absolute value.

QuickTime Subtraction Basics (00:59)

QuickTime Subtracting a Positive Integer from Any Other Integer (00:41)

QuickTime Subtracting a Negative Integer from Any Other Integer (01:03)

Stop! **Go to Questions #7-12 about this section, then return to continue on to the next section.**

Multiplying and Dividing Integers

The rules for multiplying and dividing integers are a little easier to remember because there are only two rules.

Same sign: If the numbers have the same sign, the answer will be **positive**.

$(-4)(-5) = 20$ $\qquad$ $(-35) \div (-7) = 5$

$(6)(3) = 18$ $\qquad$ $(60) \div (10) = 6$

Different signs: If the numbers have different signs, the answer will be **negative**.

$(-4)(8) = -32$ $\qquad$ $(-99) \div (9) = -11$

*When multiplying more than one number, it may be helpful to remember the following rule in determining the sign of your answer:

Even number of negative signs results in a **positive** answer.

Odd number of negative signs results in a **negative** answer.

Example #1: $(-4)(-3)(5)(-2)$
Since there are 3 negative signs, the answer to this will be negative.
Let's check it out by multiplying from left to right.
$(-4)(-3)(5)(-2)$
$(12)(5)(-2)$
$(60)(-2)$
-120

Example #2: $(-2)(-4)(-3)(-2)$
Since there are 4 negative signs, the answer to this will be positive.
Let's check it out by multiplying from left to right.
$(-2)(-4)(-3)(-2)$
$(8)(-3)(-2)$
$(-24)(-2)$
48

QuickTime Multiplication Basics (00:30)

QuickTime Multiplying Positive and Negative Integers (00:58)

Stop! **Go to Questions #13-16 about this section, then return to continue on to the next section.**

Real Number Properties of Equality

In the table below, each of the properties of equality for real numbers are listed with a general explanation of each property. The properties are used to verify steps in solving equations.

Study the Properties of Equality and then answer the questions in the interactive examples below.

Properties of Equality for Real Numbers	
Reflexive Property	For every number a, $a = a$.
Symmetric Property	For all numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For all numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Addition Property of Equality	For all numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For all numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	For all numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	For all numbers a, b, and c, if $a = b$, and if $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property of Equality	For all numbers a and b, if $a = b$, then a may be replaced by b in any equation or expression.
Zero Product Property	For all numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$ or both.

Examples: For each statement below, assume all variables represent real numbers. Identify each property illustrated by the statement.

If $x = y$, then $x + 7 = y + 7$.

Click here to check the property.

Addition Property of Equality

If $x = y$ and $y = 8$, then $x = 8$.

Click here to check the property.

Transitive Property of Equality

If $x = y$, then $x(7) = y(7)$.

Click here to check the property.

Multiplication Property of Equality

$15 = 15$

Click here to check the property.

Reflexive Property

If $x = y$, then $\frac{x}{7} = \frac{y}{7}$.

Click here to check the property.

Division Property of Equality

If $x = 10$, then $10 = x$.

Click here to check the property.

Symmetric Property of Equality

If $x = y + 4$ and $y = 6$, then $x = 6 + 4 = 10$.

Click here to check the property.

Substitution Property of Equality

If $x = y$, then $x - 7 = y - 7$.

Click here to check the property

Subtraction Property of Equality

If $9x = 0$, then $9 = 0$ (not true) or $x = 0$; thus, $x = 0$.

Click here to check your answer.

Zero Product Property

The properties of real numbers will be very useful in solving equations algebraically.

Stop! **Go to Questions #17-18 about this section, then return to continue on to the next section.**

Solving Basic Equations

In the first unit, you learned that there is a specific order in which mathematical operations are performed in simplifying expressions. In this section, you are going to learn that to solve an equation involving more than one operation. You must perform the order of operations IN REVERSE to solve for the unknown variable.

Let's first review solving one-step equations.

Example #1: Solve for $y-6=-21$ for *y*.

$$\begin{array}{l} y\cancel{-6}=-21 \\ \underline{\cancel{+6}\quad +6} \\ y\quad =-15 \end{array}$$

Add 6 to both sides (Addition Property of Equality)

Therefore, $y = -15$ in the equation $y-6=-21$.

Example #2: Solve for $\frac{-m}{3}=5$ for *m*.

$$\frac{-m}{3}=5$$

$$\frac{-1m}{3}=5 \qquad -m=-1m$$

$$-\frac{1}{3}m=5 \qquad \frac{-1m}{3}=-\frac{1}{3}m$$

$$\cancel{-3}\left(\cancel{-\frac{1}{3}}\right)m=-3(5)$$

Multiply both sides by -3

(Multiplication Property of Equality)

$$m=-15$$

Therefore, $m = -15$ in the equation $\frac{-m}{3}=5$.

In the next few examples, we are going to combine the two processes from above in one equation. Remember, that to solve an equation; you must perform the order of operations in **reverse order**, which means:

- add or subtract first
- multiply or divide second

The Golden X (15:06)

Example #3: Solve for $5x+6=31$ for *x*.

$$\begin{aligned} 5x \cancel{+6} &= 31 \\ \underline{\quad \cancel{-6}} &\ \underline{-6} \end{aligned} \qquad \text{Subtract 6 from both sides (Subtraction Property of Equality)}$$

$$\begin{aligned} 5x &= 25 \\ x &= 5 \end{aligned} \qquad \text{Divide both sides by 5 (Division Property of Equality)}$$

Check the answer by replacing *x* with 5 in the original equation.

$$5x+6=31$$
$$5(5)+6=31$$
$$25+6=31$$
$$31=31 \checkmark \text{(True)}$$

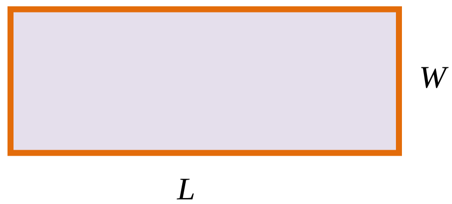
Therefore, $x = 5$ in the equation $5x+6=31$.

Example #4: Solve for $-6z-18=-132$ for *z*.

Stop! **Go to Questions #19-22 about this section, then return to continue on to the next section.**

Literal Equations / Formulas

As we have seen in the previous unit, sometimes in math and science it is necessary to rewrite an equation or a formula to highlight another quantity of interest. We will now investigate when the literal equation/formula has several unknown variables and more than one property of equality is needed to isolate the quantity of interest.



The formula for finding the perimeter of a rectangle is $P = 2L + 2W$.

Let's examine how to solve for L if it is the quantity of interest.

Example #1: Solve the perimeter formula for L.

Think: What inverse operations can be used to isolate L.

$P = 2L + 2W$

$P = 2L + 2W$ — Highlight the quantity of interest.

$$\begin{array}{rcl} P & = & 2L + 2W \\ -2W & & -2W \\ \hline P - 2W & = & 2L \end{array}$$

First subtract $2W$ from both sides, then simplify.

Left side, remember: P and $-2W$ are not like terms and cannot be combined

Right side: $2L \cancel{+2W} \cancel{-2W} = 2L$

$$\frac{P-2W}{2} = \frac{2L}{2}$$

Divide both sides by 2 (Division Property of Equality)

$$\frac{P-2W}{2} = L$$

Simplify the right side. $\left(\frac{\cancel{2}^{1}L}{\cancel{2}^{1}} = 1L = L\right)$

$$L = \frac{P-2W}{2}$$

Apply the Symmetric Property of Equality to rewrite the formula so that L is on the left side of the equals sign.

The final equation is $L = \dfrac{P-2W}{2}$.

Now, let's solve the perimeter formula of a rectange for *W*.

$$P = 2L + 2W$$

What is the quantity of interest?

Click here to check the answer.

"*W* "

What inverse operation should be done first?

Click here to check the answer.

Subtract 2*L* from both sides.

What is the simplifed equation after performing the first inverse operation?

Click here to check the answer.

$$P - 2L = 2W$$

What inverse operation should be done next?

Click here to check the answer.

Divide both sides by 2.

What is the final equation of the perimeter formula solved for *W* ?

Click here to check the answer.

$$W = (P - 2L) / 2$$

This algebraic technique can be very useful when evaluating literal equations or formulas for a specific variable.

Example #2: If the perimeter of a rectangle is 48 centimeters and the length of is 15 centimeters, what is the width of the rectangle?

In the previous scenario, we solve the perimeter formula for *W*. We will use this formula to make our work easier!

$$W = \frac{P - 2l}{2} \qquad P = 48 \qquad l = 15$$

Now, the problem is just a matter of substituting, then evaluating to find the solution.

$$W = \frac{48 - 2(15)}{2}$$

$$W = \frac{48 - 30}{2}$$

$$W = \frac{18}{2} = 9$$

The width of the rectangle is 9 centimeters.

Let's check our answer using the original formula.

$$P = 2L + 2W$$

$$48 = 2(15) + 2(9)$$

$$48 = 30 + 18$$

$$48 = 48 \checkmark \text{(True)}$$

Stop! **Go to Questions #23-24 about this section, then return to continue on to the next section.**

Solving Equations with Variables on Both Sides

Example #1: Solve for $4x-7=8-x$ for x.

$4x-7=8\cancel{-x}$	
$\underline{+x \qquad\qquad \cancel{+x}}$	Add x to both sides (Addition Property of Equality)
$5x\cancel{-7}=8$	Simplify
$\underline{\cancel{+7} \quad +7}$	Add 7 to both sides (Addition Property of Equality)
$5x=15$	
$\dfrac{5x}{5}=\dfrac{15}{5}$	Divide both sides by 5 (Division Property of Equality)
$x=3$	

Check the answer by replacing x with 3 in the original equation.

$$4x-7=8-x$$
$$4(3)-7=8-3$$
$$12-7=5$$
$$5=5\ \checkmark\text{(True)}$$

Therefore, x = 3 in the equation $4x-7=8-x$.

QuickTime A Real-Life Example: Who's Faster - Men or Women? (03:31)

Example #2: Carley and Nathan are shopping for new guitar strings at the local music store. Nathan buys 2 packs of strings. Carley buys 2 packs of strings and a music book. The book costs \$16. Their total cost is \$72. How much is one pack of strings?

Let x = the cost of a pack of strings

Both teens bought 2 packs of strings: $2x$

Nathan buys 2 packs of strings.	Carley buys 2 packs of strings and a music book.
$2x$	$2x + 16$

Total purchase of both teens = *Total cost*

$2x + 2x + 16 \quad = \quad \$72$

Equation: $2x + 2x + 16 = 72$

Solution:

$2x + 2x + 16 = 72$

$4x + 16 = 72$ Collect the x's.

$4x = 56$ Subtraction Property of Equality (Subtract 16 from both sides)

$x = 14$ Division Property of Equality (Divide both sides by 4)

The cost of a pack of guitar strings is \$14.00.

Example #3: Solve for $13+4x=-2x-5x+79$ for x.

$13+4x=-2x-5x+79$	Collect like terms on the right side of the equation.
$13+4x=\cancel{-7x}+79$	$[-2x-5x=-2x+(-5x)=-7x]$
$\quad +7x \quad \cancel{+7x}$	Add $7x$ to both sides (Addition Property of Equality)
$\cancel{13}+11x=79$	
$\cancel{-13} \qquad -13$	Subtract 13 from both sides (Subtraction Property of Equality)
$11x=66$	Divide both sides by 6 (Division Property of Equality)
$x=6$	

Check the answer by replacing x with 6 in the original equation.

$$13+4x=-2x-5x+79$$
$$13+4(6)=-2(6)-5(6)+79$$
$$13+24=-12-30+79$$
$$37=-42+79$$
$$37=37 \checkmark \text{(True)}$$

Therefore, x = 6 in the equation $13+4x=-2x-5x+79$.

QuickTime Solving Equations with Variables on Both Sides (03:19)

To extend the process of solving equations, there may be times when you will have to use the distributive property first to eliminate any parentheses. You will then combine any like terms and solve. Take a look at the example below.

Example #4: Solve for $5(d+4)=7(d-2)$ for d.

Step #1: Eliminate the parentheses by using the distributive property on each of the quantities.

$5d+20=7d-14$	$5(d+4)=5(d)+5(4)=5d+20$
	$7(d-2)=7(d)-7(2)=7d-14$

Step #2: Move the variables to one side of the equation and the numbers to the other side of the equation so that you can solve for the unknown.

$$\cancel{5d} + 20 = 7d - 14$$
$$\underline{\cancel{-5d} \qquad -5d} \qquad \text{Subtract } 5d \text{ from both sides (Subtraction Property of Equality)}$$
$$20 = 2d \cancel{-14}$$
$$+14 \qquad \cancel{+14} \qquad \text{Add 15 to both sides (Addition Property of Equality)}$$
$$34 = 2d$$

Step #3: Divide both sides by 2 to solve for the unknown.

$$\frac{34}{2} = \frac{2d}{2} \qquad \text{Divide both sides by 2 (Division Property of Equality)}$$
$$17 = d \qquad \text{Apply the Symmetric Property of Equality (If } a = b\text{, then } b = a.\text{)}$$
$$d = 17$$

Check the answer by replacing *d* with 17 in the original equation.

$$5(d + 4) = 7(d - 2)$$
$$5(17 + 4) = 7(17 - 2)$$
$$5(21) = 7(15)$$
$$105 = 105 \checkmark \text{(True)}$$

Therefore, $d = 17$ in the equation $5(d + 4) = 7(d - 2)$.

Another Way to Solve Equations with Variables on Both Sides (01:56)

Example #5: Solve for $3(x - 6) + 2 = 4(x + 2) - 21$ for *x*.

Step #1: Eliminate the parentheses by using the distributive property on each of the quantities.

$$3x - 18 + 2 = 4x + 8 - 21 \qquad 3(x - 6) = 3(x) - 3(6) = 3x - 18$$
$$4(x + 2) = 4(x) + 4(2) = 4x + 8$$

Step #2: Combine any like terms on either side of the equals sign. In this case, combine (–18 + 2) on the left and (+8 – 21) on the right.

$$3x - 18 + 2 = 4x + 8 - 21$$
$$3x - 16 = 4x - 13$$

Step #3: Move the variables to one side of the equation and the numbers to the other side of the equation so that you can solve for the unknown.

$$\begin{array}{rcll} \cancel{3x} - 16 & = & 4x - 13 & \\ \cancel{-3x} & & -3x & \text{Subtraction Property of Equality} \\ \hline -16 & = & x \cancel{-13} & \\ +13 & & \cancel{+13} & \text{Addition Property of Equality} \\ -3 & = & x & \\ x & = & -3 & \text{Symmetric Property of Equality} \end{array}$$

Check the answer by replacing *x* with –3 in the original equation.

$$3(x-6)+2=4(x+2)-21$$
$$3[(-3)-6]+2=4(-3+2)-21$$
$$3(-9)+2=4(-1)-21$$
$$-27+2=-4-21$$
$$-25=-25\ \checkmark\ \text{(True)}$$

Therefore, $x = -3$ in the equation $3(x-6)+2=4(x+2)-21$.

QuickTime Collecting Variables: Which Side of the Equation? (03:36)

Example #6: Solve for $\frac{2}{3}(6k+9)-4=\frac{1}{3}(3k+12)+4$ for *k*.

Step #1: Eliminate the parentheses by using the distributive property on each of the quantities.

$$4k+6-4=k+4+4$$

$$\frac{2}{3}(6k+9)=\frac{2}{3}(6k)+\frac{2}{3}(9)$$
$$=\frac{2}{\cancel{3}^{1}}\cdot\frac{\cancel{6}^{2}k}{1}+\frac{2}{\cancel{3}^{1}}\cdot\frac{\cancel{9}^{3}}{1}$$
$$=4x+6$$

$$\frac{1}{3}(3k+12)=\frac{1}{3}(3k)+\frac{1}{3}(12)$$
$$=\frac{1}{\cancel{3}^{1}}\cdot\frac{\cancel{3}^{1}k}{1}+\frac{1}{\cancel{3}^{1}}\cdot\frac{\cancel{12}^{4}}{1}$$
$$=k+4$$

Step #2: Combine any like terms on either side of the equals sign. In this case, combine (+6 – 4) on the left and (+4 + 4) on the right.

$$4k+6-4=k+4+4$$
$$4k+2=k+8$$

Step #3: Move the variables to one side of the equation and the numbers to the other side of the equation so that you can solve for the unknown.

$$\cancel{4k}+2=k+8$$
$$\cancel{-k}\qquad -k \qquad \text{Subtraction Property of Equality}$$
$$3k+2=\cancel{8}$$
$$-2\quad \cancel{-2} \qquad \text{Subtraction Property of Equality}$$
$$3k=6 \qquad \text{Division Property of Equality}$$
$$k=2$$

Check the answer by replacing *k* with 2 in the original equation.

$$\frac{2}{3}(6k+9)-4=\frac{1}{3}(3k+12)+4$$
$$\frac{2}{3}(6(2)+9)-4=\frac{1}{3}(3(2)+12)+4$$
$$\frac{2}{3}(12+9)-4=\frac{1}{3}(6+12)+4$$
$$\frac{2}{3}(21)-4=\frac{1}{3}(18)+4$$
$$\frac{2}{\cancel{3}^{1}}\left(\frac{\cancel{21}^{7}}{1}\right)-4=\frac{1}{\cancel{3}^{1}}\left(\frac{\cancel{18}^{6}}{1}\right)+4$$
$$14-4=6+4$$
$$10=10\ \checkmark\text{(True)}$$

Therefore, $k = 2$ in the equation $\frac{2}{3}(6k+9)-4=\frac{1}{3}(3k+12)+4$.

QuickTime The Battle of the Genders Continues: Solving for Speed (03:33)

Stop! **Go to Questions #25-33 to complete this unit.**