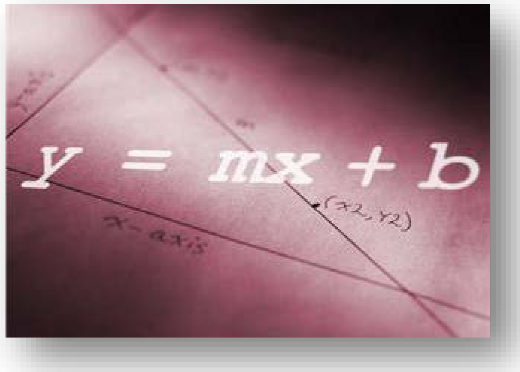
LINEAR FUNCTIONS



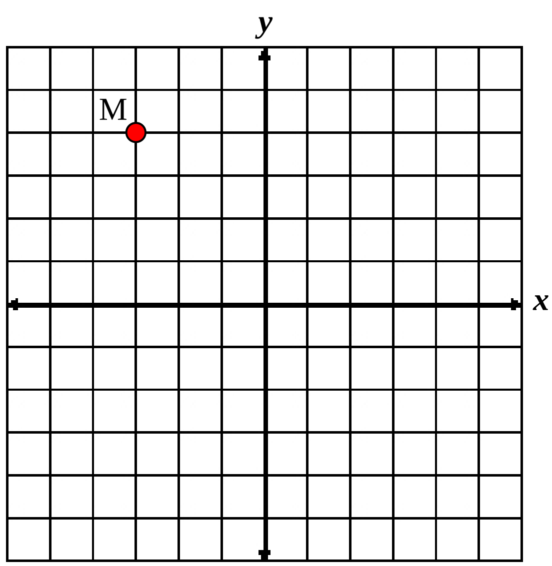
Unit Overview

In this unit, you will study linear functions which are the most basic algebraic functions. You will also learn how to find the slope, or steepness of a line given a graph. The unit will conclude with rate of change which is just slope in terms of real-life problems and direct variation.

Linear Functions and Graphs

Ordered Pair: the *x*- and *y*- coordinates that give the location of a point in a coordinate plane, indicated by two numbers in parentheses (x, y).

Example #1: What is the location of Point M if each line in the grid represents one unit?



Point M is located at (–3, 4).

Relation: a pairing of a set of numbers generally represented as a set of ordered pairs.

Example #2: State the ordered pairs shown in the table (h, w).

Height (*h*) (inches)	**Weight (*w*)**
68	125
64	118
65	112
72	145
64	126
67	130
66	128

The set of ordered pairs (h, w) represented in this table would be:

{(68, 125), (64, 118), (65, 112), (72, 145), (64, 126), (67,130), (66, 128)}

*Notice in this example that there are repeated numbers in the height column. Because of this, the chart and set of ordered pairs only represents a **relation**.

{(68, 125), (64, 118), (65, 112), (72, 145), (64, 126), (67,130), (66, 128)}

Function: a pairing between two sets of numbers in which each element in the first set is paired with **exactly** one element of the second set.

QuickTime Introduction to Functions (01:52)

Example #3: State the ordered pairs shown in the table (x, y).

x	y
11	63
12	64
13	65
14	70
15	72
16	72

The set of ordered pairs (x, y) represented in the table would be:

$$\{(11, 63), (12, 64), (13, 65), (14, 70), (15, 72), (16, 72)\}$$

*Notice in this example that there are no repeated values in the first column. Because of this, the chart and the set of ordered pairs represent a **function**.

In common terms, a relation is a set of ordered pairs.

A function is a set of ordered pairs whose first coordinates (the *x*-coordinates) are all different.

In a relation, the set of first coordinates, (x-values) are called the **domain** of the relation. The set of second coordinates, (y-values) are called the **range**.

From *Example #3* above:

The domain = {11, 12, 13, 14, 15, 16}

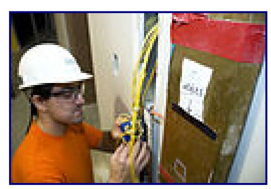
The range = {63, 64, 65, 70, 72}

*Notice that the number 72 is only listed once in the range. If a number is repeated in a relation, it is only listed once in the domain or range.

**Notice that the domain and range are listed in order from least to greatest.

Application of Domain and Range

Example #4: An employee for Elton's Electric Company charges $125 for an estimate and $30 per hour for repairs. This situation can be represented by $y = 125 + 30x$. The employee is permitted to work no more than 38 hours per job and his time is rounded to the nearest half hour.



Fill in the chart with appropriate domain (x) and range (y) values based on the given terms of Elton's Electronic Company for the employee.

Hours worked (x)	**Invoice Total** $y = 125 + 30x$
5 hrs	$275
	$425
8 hrs	
	$185
4.5 hrs	

State the domain values.

"Click here" to check your answer.

Domain: 2, 4.5, 5, 8, 10

State the range values.

"Click here" to check your answer.

Range: 185, 260, 275, 365, 425

*Note: The domain and range values are stated in order from least to greatest.

Would 3.75 hours be a possibility for the domain? Why or why not?

"Click here" to check your answer.

No, because the hours are rounded to the nearest half hour.

Would $1325 be a possibility for the range?

"Click here" to check your answer.

No

*Note: This amount, $1325 would equal 40 hours of work. [125 + 30(40)]. The employee is not permitted to work more than 38 hours in one week.

Graphing a Function -- Swimming (02:12)

Example #5: Determine whether the following relation is a function, and then state the domain and range.

{(2, 4), (4, 5), (–5, 5), (7, 0), (–3, –2)}

a) This relation **is** a function because **all of the *x*-values are different**.

{(2, 4), (4, 5), (–5, 5), (7, 0), (–3, –2)}

b) domain = {2, 4, –5, 7, –3}

range = {4, 5, 0, –2}

An equation in two variables can have an infinite number of solutions which can be represented by ordered pairs.

To determine a solution to an equation in two variables, substitute the given coordinate into the equation and solve for the other coordinate.

Example #6: Complete each ordered pair so that it is a solution to $3x + 2y = 1$.

a) $(1, ?)$ b) $\left(\frac{5}{3}, ?\right)$ c) $(?, 8)$

Substitute 1 for x and solve for y.	Substitute $\frac{5}{3}$ for x and solve for y.	Substitute 8 for y and solve for x.
$3x+2y=1$	$3x+2y=1$	$3x+2y=1$
$3(1)+2y=1$	$3\left(\frac{5}{3}\right)+2y=1$	$3x+2(8)=1$
$3+2y=1$	$5+2y=1$	$3x+16=1$
$2y=-2$	$2y=-4$	$3x=-15$
$y=-1$	$y=-2$	$x=-5$
$(1,-1)$	$\left(\frac{5}{3},-2\right)$	$(-5,8)$

Stop! **Go to Questions #1-17 about this section, then return to continue on to the next section.**

Slope and Rate of Change

The slope of a line describes the steepness of the line. The slope of a line is a rate of change. A rate of change describes how one quantity changes in relation to another. The slope of a line is the ratio of vertical rise to horizontal run.

What is slope? (06:02)

To find the slope of a line graphed on a coordinate plane:

-Identify a point on the line.

-From that point, move up or down until you are directly across from the next point.

-Move left or right to the next point.

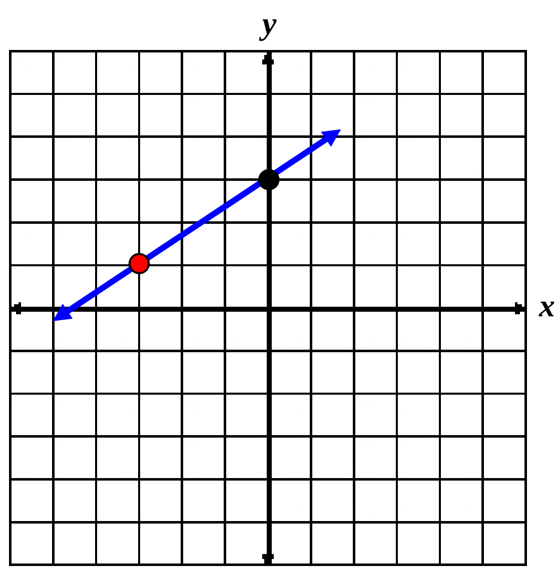
Example #1: From the graph below, determine the slope of the line.

- Put your pencil on the red point.

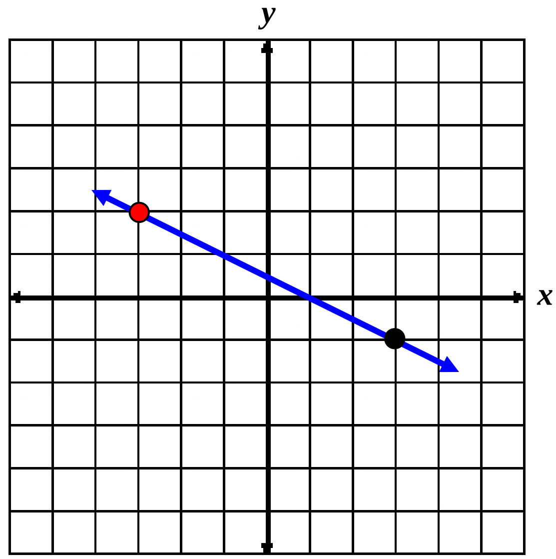
- Move straight up (vertical rise) until your pencil is in the same line as the black point. (2 units)

- Move right (horizontal run) until you reach the black point. (3 units)

You have now determined the slope of the line to be $\frac{2}{3}$.



Example #2: From the graph below, determine the slope of the line.



- Put your pencil on the red point.

- Move straight down (vertical rise) until your pencil is in the same line as the black point. (–3 units)

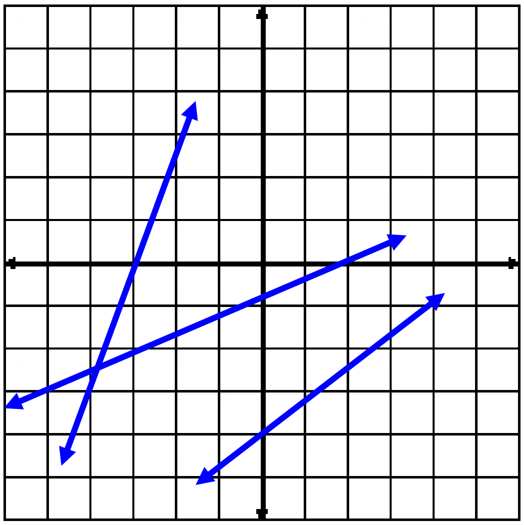
- Move right (horizontal run) until you reach the black point. (6 units)

You have now determined the slope of the line to be $\frac{-3}{6}$. This slope can be reduced to $-\frac{1}{2}$.

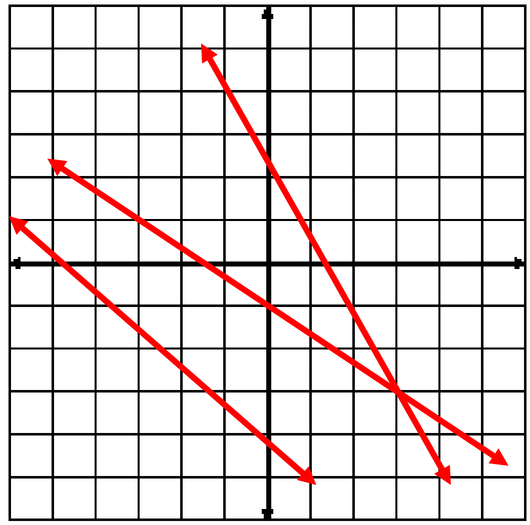
*Notice that if you count down one unit (rise) from the red point and right two units (run), you will be on a point of the line.

Click here to view more examples of slope.

On a coordinate plane, there are lines that have positive slopes and lines that have negative slopes. Below is an illustration of both.



Lines with positive slopes rise to the right.



Lines with negative slopes rise to the left.

At this point, we are going to learn how to find the slope of a line by using two points that lie on the line.

The definition of slope states that given two points, (x_1, y_1) and (x_2, y_2), the formula for finding the slope of a line containing these points is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

*Notice, this is the vertical change over the horizontal change. Slope is a rate of change that describes the change in *y* in relation to the change in *x*.

Example #3: Find the slope of the line containing the point A(–2, –6) and B(3, 5).

Let point B = (x_2, y_2) and point A = (x_1, y_1).

$(x_2, y_2) = (3,5) \quad \rightarrow \quad x_2 = 3,\ y_2 = 5$

$(x_1, y_1) = (-2,-6) \quad \rightarrow \quad x_1 = -2, y_1 = -6$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{5-(-6)}{3-(-2)}$$

$$m=\frac{11}{5}$$

The slope (m) of the line is $\frac{11}{5}$.

Example #4: Find the slope of the line containing the point C(5, –2) and D(8, –2).

Let point D = (x_2, y_2) and point C = (x_1, y_1).

$(x_2, y_2)=(8,-2) \quad \rightarrow \quad x_2=8,\ y_2=-2$

$(x_1, y_1)=(5,-2) \quad \rightarrow \quad x_1=5, y_1=-2$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{-2-(-2)}{8-5}$$

$$m=\frac{-2+2}{8-5}$$

$$m=\frac{0}{3}=0$$

The slope (m) of the line is 0. This line is a horizontal line.

Example #5: Find the slope of the line containing the point E(9, 4) and F(9, 1).

Let point F = (x_2, y_2) and point E = (x_1, y_1).

$(x_2, y_2)=(9,1) \quad \rightarrow \quad x_2=9,\ y_2=1$

$(x_1, y_1)=(9,4) \quad \rightarrow \quad x_1=9, y_1=4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-4}{9-9}$$

$$m = \frac{-3}{0} = \text{undefined}$$ *Division by zero is undefined.

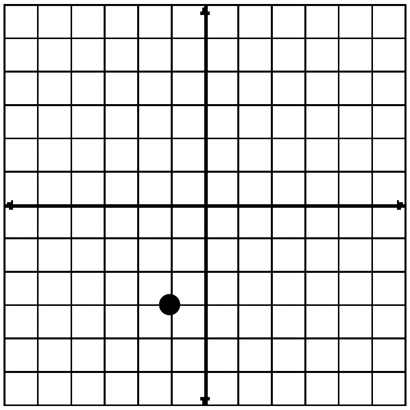
The slope (m) of the line is undefined. This line is a vertical line.

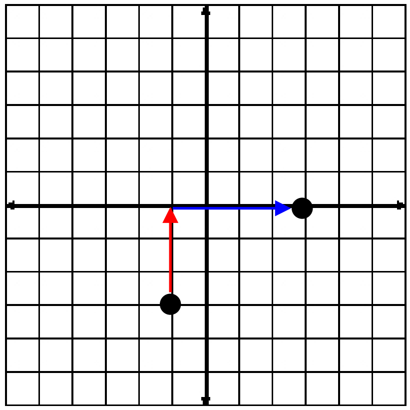
QuickTime Negative, Positive, Zero, and Undefined Slopes (05:33)

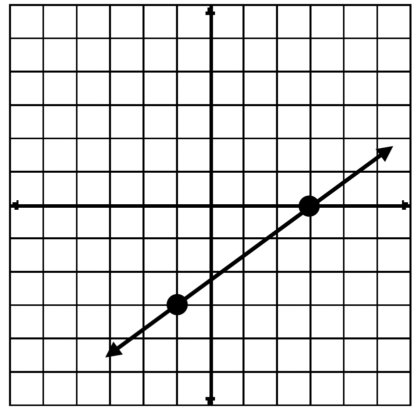
Stop! **Go to Questions #18-29 about this section, then return to continue on to the next section.**

Graphing a Line on a Coordinate Plane Using a Point and the Slope

Example #1: Graph the line containing the point (–1, –3) and having a slope of $m = \frac{3}{4}$.







1. Plot the point (–1, –3).

2. Use the rise (3 units) over run (4 units) ratio for slope to plot a second point.

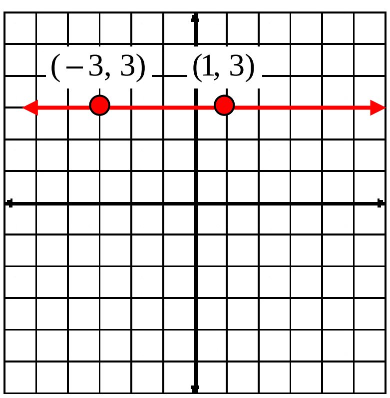
3. Draw a line through the points with a straightedge.

Example: Now you try – Click here to begin the steps in graphing a line through (3,-4) with a slope of 2/3.

How to Draw a Line with a Given Slope Through a Given Point (03:43)

There are two more types of lines that we need to discuss. They are the vertical and horizontal lines that have special types of slopes.

Example #2:



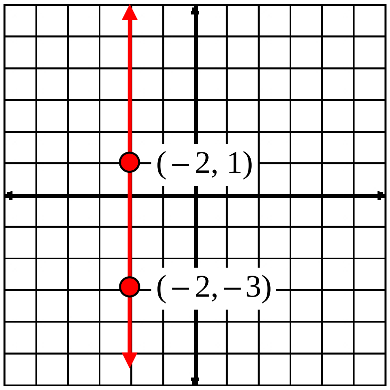
The *y*-coordinate for every point on a horizontal line is the same.

Choose two points on the line and use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the slope.

$$m = \frac{3-3}{1--3} = \frac{0}{4} = 0$$

The slope of every horizontal line is 0.

Example #3:



The *x*-coordinate for every point on a vertical line is the same.

Choose two points on the line and use the slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, to determine the slope.

$$m = \frac{-3-1}{-2-(-2)} = \frac{-4}{0} = \text{ undefined or no slope.}$$

The slope of every vertical line is undefined!

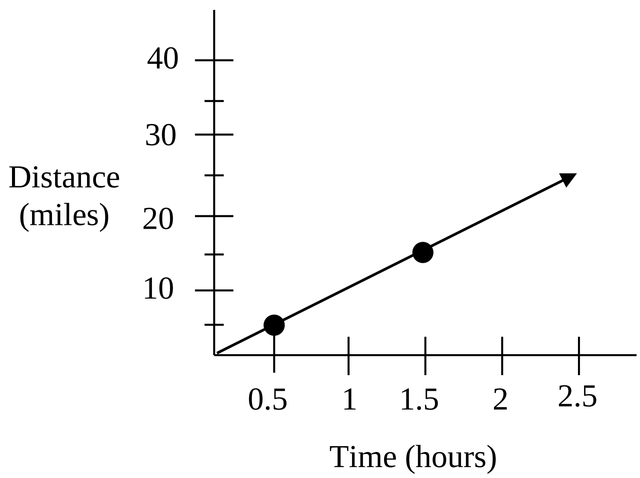
Stop! **Go to Questions #30-33 about this section, then return to continue on to the next section.**

Rate of Change

Rate of change is directly related to slope and can be found using the following formula:

$$\text{rate of change} = \frac{\text{change in distance}}{\text{change in time}}$$

Example 1: The graph shows the distance a bicyclist travels at a constant speed. Find the speed of the bicyclist.



Select two points and use the slope formula.

Let (x_1, y_1) = (0.5, 5)

Let (x_2, y_2) = (1.5, 15)

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{15-5}{1.5-0.5} = \frac{10 \text{ miles}}{1 \text{ hour}}$$

The bicyclist travels 10 miles for every hour he travels.

QuickTime Algebraic Equations Expressing a Function (02:06)

Now, let’s examine a rate of change from a chart and determine what the rate of change means for this problem.

Example 2: Brenton works at a local fast food restaurant and is examining his recent pay checks (amounts do not reflect any type of deductions).

Brenton's time worked and salary is shown in chart below.

Time Worked (in hours)	**Salary Paid (in dollars)**
12 hours	$99
23 hours	$189.75
5 hours	$41.25
35 hours	$288.75

Do these ordered pairs represent a function?

“Click here” to check your answer.

Yes, the domain does not repeat.

Let’s find the rate of change using Brenton's information:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{189.75 - 99}{23 - 12} = \frac{90.75}{11} = 8.25$$

Let’s try another set of data points:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{288.75 - 41.25}{35 - 5} = \frac{247.5}{30} = 8.25$$

This procedure can be done with any of the data points in the chart.

So what does the 8.25 represent in terms of the data?

In this problem, the top number is the change in dollars earned and the bottom number is the change in hours worked.

In mathematics, the symbol Δ (delta) means change.

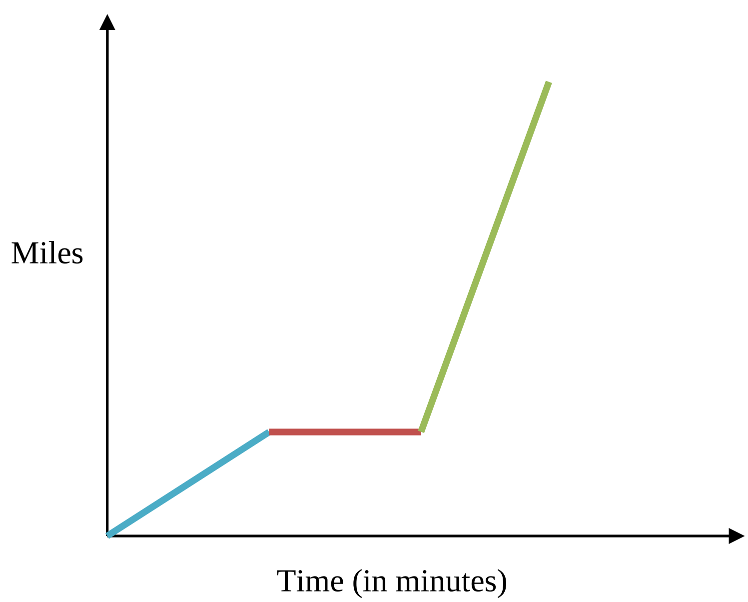
$$\frac{\Delta y}{\Delta x} \quad \text{means} \quad \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in dollars}}{\text{change in hours worked}}$$

Thus, the 8.25 represents the dollars per hour that Brenton is being paid to work at the local fast food restaurant.

Brenton makes $8.25 per hour for his wages. (rate of change is dollars / hour)

Let's examine another rate of change problem.

Example 3: To make the commute to work, Brenton must ride a bus part of the way and walk part of the way. Look at the graph below and describe what the graph represents in terms of Brenton's commute to work and rate of change. Think about what transpires realisticallyon his commute.



Solution:

Notice that the first line in the graph shows a rate of change that is not as great as shown elsewhere in the graph. This line represents Brenton's walk to the bus stop.

Brenton must then wait for the bus at the bus stop. At this point, the line in the graph is flat; time is increasing but the miles are not.

The last line in the graph shows the greatest rate of change. This line represents Brenton's ride on the bus (the bus travels faster – more miles per minute than walking or waiting).

The **rate of change** would be **miles per minutes** in this problem.

Stop! **Go to Questions #34-37 about this section, then return to continue on to the next section.**

Direct Variation

One type of rate of change is known as direct variation. Direct variation can be looked at as a form of $y = mx + b$ where the y-intercept is always 0.

Direct Variation

If y varies directly as x, then $y = kx$, or $\frac{y}{x} = k$, where k is the constant of variation.

*Note: $y = kx$ is the same as $y = mx + b$ where $b = 0$. In this notation, k is used as the coefficent of x instead of m.

Example 1: If y varies directly as x and $y = 6$ when $x = 0.3$, find the constant of variation and write an equation of direct variation.

1. Substitute x and y in to $\frac{y}{x} = k$ and solve for k.

$$\frac{6}{0.3} = k$$

$$20 = k$$

2. Use the constant of variation, k, to write an equation of direct variation.

$$y = kx$$

$$y = 20x$$

Example 2: Bob's salary can be calculated using the equation $y = 7.65x + 35$. Brenton's salary is calculated using the equation $y = 8.25x$. For both persons, x represents the time worked and y represents the amount earned. Which person's salary is an example of direct variation and why?

Solution: Brenton's salary equation is direct variation because it is in the form $y = kx$ with the constant (k) of 8.25. Bob's salary equation is not direct variation because an amount is added (probably tips or other type of incentive).

Example 3: A cell phone company charges \$10 for the first 250 minutes of usage. Given this is an example of direct variation, find the constant of variation and then determine the cost of 400 minutes of use.

Solution:

1. Substitute x and y in to $\frac{y}{x} = k$ and solve for k.

y = number of minutes $\quad\quad$ x = charges in dollars

$$\frac{250}{10} = k$$

$$25 = k$$

The constant of variation (k) is 25.

2. Use the constant of variation (k) to write an equation of direct variation.

$$y = kx$$

$$y = 25x$$

3. Use the direct variation equation to determine the cost of 400 minutes.

y = 400 minutes $\quad\quad$ x = cost in dollars

$y = 25x$	Direct variation equation
$400 = 25x$	Substitute: $y = 400$
$16 = x$	Divide both sides by 25.
$x = 16$	Apply the reflexive property.

The cost for 400 minutes is \$16.00.

Stop! **Go to Questions #38-45 to complete this unit.**