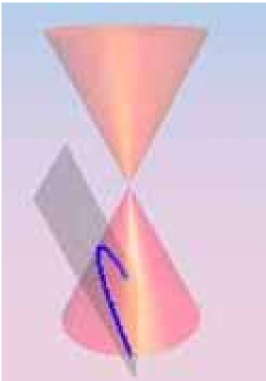
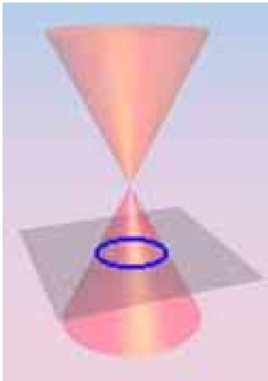
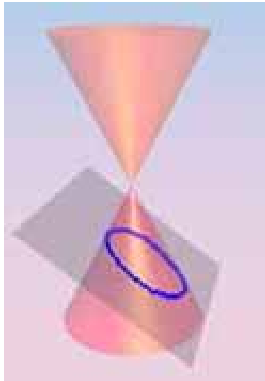
DISTANCE AND MIDPONT FORMULAS; CIRCLES



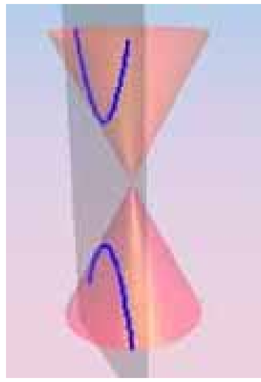
Parabola



Circle



Ellipse



Hyperbola

Unit Overview

In this unit you will begin to investigate the conic sections. The conics are curves formed by the intersection of a plane and a double-napped cone. There are four types of conic sections: a parabola, a circle, an ellipse, and a hyperbola. A diagram of a double-napped cone is given below. A double napped-cone has two cones with the points touching. Notice how slicing the double-napped cone at different angles produces different conic sections. In this unit you will find the distance between two points, the coordinates of the midpoint of a line segment, and the circumference and area of a circle in the coordinate plane. You will also write equations for circles given sufficient information and graph circles.

Distance Formula

The distance between two points, $A(x_{1,}y_1)$ and $B(x_2, y_2)$, on a coordinate plane is as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

*the order of *x* and *y* does not matter.

Example #1: Find the distance between point *S* which is located at (3, 5) and point *T* which is located at (–4, –2).

$$d = \sqrt{(-4-3)^2 + (-2-5)^2}$$

$$d = \sqrt{(-7)^2 + (-7)^2}$$

$$d = \sqrt{49 + 49}$$

$$d = \sqrt{98}$$

$$d = \sqrt{49} \cdot \sqrt{2}$$

$$d = 7\sqrt{2}$$

The distance between *S*(3, 5) and *T*(–4, –2) is $7\sqrt{2}$.

Example #2: Find the distance between point *A* which is located at (5,9) and point *B* which is located at (12, 18).

$$d = \sqrt{(12-5)^2 + (18-9)^2}$$

$$d = \sqrt{(7)^2 + (9)^2}$$

$$d = \sqrt{49 + 81}$$

$$d = \sqrt{130}$$

The prime factorization for 130 is 2 × 5 × 13. Since there are no like factors, $\sqrt{130}$ is in simplest form.

The distance between *A*(5, 9) and *T*(12, 18) is $\sqrt{130}$.

QuickTime Using the Distance Formula to Find Locus (06:40)

Midpoint Formula

The coordinates of the midpoint, M, between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ can be found using the following formula:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example #1: Find the coordinates of the midpoint between point P which is located at (–3, 2) and point Q which is located at (5, –2).

$$M = \left(\frac{-3+5}{2}, \frac{2+(-2)}{2}\right)$$

$$M = \left(\frac{2}{2}, \frac{0}{2}\right)$$

$M(1,\ 0)$ The coordinates of the midpoint between P and Q are $(1, 0)$.

Example #2: Find the coordinates of the midpoint between point P which is located at (–5, 6) and point Q which is located at (1, 7).

$$M = \left(\frac{-5+1}{2}, \frac{6+7}{2}\right)$$

$$M = \left(\frac{-4}{2}, \frac{13}{2}\right)$$

$M(-2,\ \frac{13}{2})$ The coordinates of the midpoint between P and Q are $(-2, \frac{13}{2})$.

QuickTime The Midpoint Formula (04:13)

Stop! **Go to Questions #1-5 about this section, then return to continue on to the next section.**

Center, Circumference, and Area of a Circle in the Coordinate Plane

The midpoint formula can be used to find the center location of a circle on a coordinate plane; and then, by using the distance formula, you can find the area and the circumference of the circle.

Example #1: The diameter of a circle has endpoints located at *P*(8, –3) and *Q*(–2, 1).

a.) The center location of the circle is found by using the midpoint formula.

$$C = \left(\frac{8+(-2)}{2}, \frac{-3+1}{2}\right)$$

$$C = \left(\frac{6}{2}, \frac{-2}{2}\right)$$

$$C\ (3,-1)$$

b.) To find the area of the circle πr^2 we will have to find the distance between the center (3, –1) and either point *P*(8, –3) or *Q*(–2, 1) because this will give us the radius. Let's use point *Q*.

$$r = \sqrt{(-2-3)^2 + (1-(-1))^2}$$

$$r = \sqrt{(-5)^2 + 2^2}$$

$$r = \sqrt{25+4}$$

$r = \sqrt{29}$ This is the radius of our circle.

c.) To find the area of the circle use the formula πr^2.

$$A = \pi(\sqrt{29})^2$$

$$A = 29\pi$$

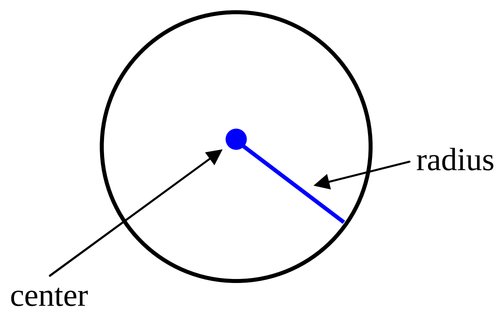
d.) To find the circumference of the circle use the formula $2r\pi$.

$$C = 2\pi\sqrt{29}$$

Stop! **Go to Questions #6-8 about this section, then return to continue on to the next section.**

Equations and Graphs of Circles

Circle: the set of all points in a plane that are a constant distance, called the radius, from a fixed point, called the center.



The standard equation of a circle is:

$(x-h)^2+(y-k)^2=r^2$ with a center at (h,k).

Example #1: Find the center and radius of the circle with the equation $(x-8)^2+(y+3)^2=144$.

$(x-8)^2+(y+3)^2=144$

$(x-8)^2+(y-(-3))^2=12^2$ Rewrite the circle in standard form.

$h=8$ $k=-3$ $r=12$

The center of the circle is (8, –3). The radius is 12.

Example #2: Find the center and radius of the circle with the equation $(x+5)^2+y^2=13$.

$(x+5)^2+y^2=13$

$(x-(-5))^2+(y-0)^2=(\sqrt{13})^2$ Rewrite the circle in standard form.

$h=-5$ $k=0$ $r=\sqrt{13}$

The center of the circle is (–5, 0). The radius is $\sqrt{13}$.

Given the equation of a circle: $(x-2)^2+(y-1)^2=38$

Find the center of the circle. $(x-2)^2+(y-1)^2=38$

"Click here" to check the answer.

The center of the circle is (2, 1).

Find the radius of the circle. $(x-2)^2+(y-1)^2=38$

"Click here" to check the answer.

The radius of the circle is $\sqrt{38}$ **.**

Given the equation of a circle: $x^2+(y+6)^2=64$

Find the center of the circle. $x^2+(y+6)^2=64$

"Click here" to check the answer.

The center of the circle is (0, –6).

Find the radius of the circle. $x^2+(y+6)^2=64$

"Click here" to check the answer.

The radius of the circle is 8.

Example #3: Write the standard equation of a circle whose center is at (–3, 2) and whose radius is 4.

$(x-h)^2+(y-k)^2=r^2$ Use the standard form.

$(x-(-3))^2+(y-2)^2=4^2$ Substitute –3 for *h*, 2 for *k*, and 4 for *r*.

$(x+3)^2+(y-2)^2=16$ Simplify.

The equation of the circle is $(x+3)^2+(y-2)^2=16$.

Example #4: Write the standard equation of a circle whose center is at (0, –2) and whose radius is $\sqrt{11}$.

$(x-h)^2+(y-k)^2=r^2$ Use the standard form.

$(x-0)^2+(y-(-2))^2=(\sqrt{11})^2$ Substitute 0 for *h*, –2 for *k*, and $\sqrt{11}$ for *r*.

$x^2+(y+2)^2=11$ Simplify.

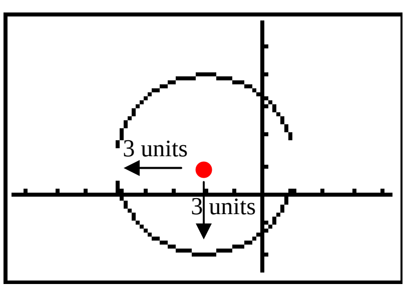
The equation of the circle is $x^2+(y+2)^2=11$.

Example #5: Graph the following circle on a coordinate plane.

$(x+2)^2+(y-1)^2=9$

-locate the center point (–2, 1)

-find the radius, 3, and plot points straight above, below, left and right 3 units from the center and connect them with circular curves.



Example #6: Put the equation of the circle given by $x^2 + 2x + y^2 + 6y = 6$ in standard form.

$x^2 + 2x + \_\_\_ + y^2 + 6y + \_\_\_ = 6 + \_\_\_ + \_\_\_$ Set up two complete the squares.

$x^2 + 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$ Complete the square and balance the equation.

*The red number came from taking half of the linear term 2 and squaring it. Remember you also have to add this to the right side of the equation. The blue term came from taking half of the linear term 6 and squaring it.

$(x+1)^2 + (y+3)^2 = 16$ Write each as perfect square trinomials.

The center of this circle is located at (–1, –3) and it has a radius of 4 units.

Stop! **Go to Questions #9-30 to complete this unit.**