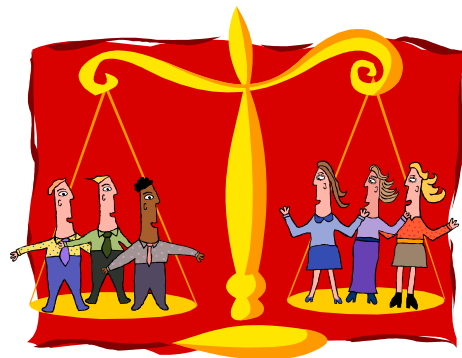


SOLVING EQUATIONS AND APPLICATIONS



Unit Overview

In this unit, you will solve equations and justify the steps using properties of equality. You will apply linear equations to solve a variety of application problems.

Introduction to Solving Equations

Properties of Equality

For real numbers a , b , and c :

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, where $c \neq 0$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Zero Product Property	If $ab = 0$, then $a = 0$ or $b = 0$ or both.

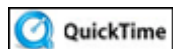
One of the most important properties in algebra is the **distributive property**. This property ties addition or subtraction together with multiplication. The distributive property allows you to write expressions in different forms and is given with the following definition.

Distributive Property

The sum or difference of two numbers multiplied by a number is the sum or difference of the product of each number and the number used to multiply.

For any number x , y , and z , $x(y + z) = xy + xz$

The expression $x(y + z)$ is read “ x times the quantity of $y + z$ ”



The Distributive Properties of Multiplication over Addition (01:26)

An equation is like a balance scale. To keep both sides equal, any operation **must** be performed on **each** side.

equation: a statement that has two expressions are equal

$3x + 9 = 15$ is an equation.

variable: a symbol that represents an unknown value (generally a letter)

The x in the example above is a variable.

solution: any value of a variable that makes the equation true

2 is the solution to the example $3x + 9 = 15$

terms: the components of an expression that are added or subtracted

In “ $5 + 3x - x - 1$ ”, each of these represents a term (5, $3x$, x , 1).

like terms: terms that contain the same variable with the same exponent

$3x$ and x are considered like terms as well as 5 and 1.

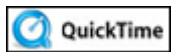
simplified: when all like terms have been combined and all parentheses have been removed

In the expression $5 + 3x - x - 1$, the simplified version is $4 + 2x$.

isolate the variable: rearrange and simplify an algebraic equation so that the variable that is being isolated is on one side of the equation and everything else is on the other side.

To solve equations:

- Combine all like terms on each side (eliminate all parentheses by using the distributive property).
- Use opposite operations to isolate the variable.
- Justify the steps using the Properties of Equality.



Inverse Operations for Isolating Variables--Bowling (01:48)

Examples of Solving Equations:

Use opposite operations to isolate the variable and justify the steps using the Properties of Equality.

Example #1: $3x - 8 = 5x - 20$
 $\quad -5x \quad -5x$

*Subtraction Property (Subtract $5x$ from both sides of the equation.)

$$\begin{array}{r} -2x - 8 = -20 \\ \quad +8 \quad +8 \end{array}$$

*Addition Property (Add 8 to both sides of the equation.)

$$\begin{array}{r} \frac{-2x}{-2} = \frac{-12}{-2} \end{array}$$

*Division Property (Divide both sides of the equation by -2 , to undo -2 times x .)

$$x = 6$$

The solution to the equation is 6 .

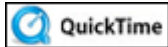
To *check* your work, replace x with 6 and verify the equal results.

$$\begin{array}{r} 3x - 8 = 5x - 20 \\ 3(6) - 8 = 5(6) - 20 \end{array}$$

*Substitution Property (Substitute 6 in for x .)

$$\begin{array}{r} 18 - 8 = 30 - 20 \\ 10 = 10 \end{array}$$

*Reflexive Property ($a = a$)



Solving Equations with Variables on Both Sides (03:19)

Example #2: $-\frac{3}{5}x + 12 = 4$

$$-12 \quad -12$$

*Subtraction Property (Subtract 12 from both sides of the equation.)

$$-\frac{3}{5}x = -8$$

$$-\frac{5}{3}\left(-\frac{3}{5}x\right) = -8\left(-\frac{5}{3}\right)$$

*Multiplication Property

(multiply both sides of the equation by $(-5/3)$, the reciprocal of $-3/5$.)

$$1x = \frac{40}{3}$$

$$x = \frac{40}{3}$$

Example #3: $-4(x - 9) = 32$

*Distribute $-4(x) + -4(-9)$

$$-4x + 36 = 32$$

$$-36 \quad -36$$

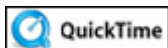
*Subtraction Property (Subtract 36 from both sides of the equation.)

$$-4x = -4$$

$$\frac{-4x}{-4} = \frac{-4}{-4}$$

*Division Property (Divide both sides by -4 to undo -4 times x.)

$$x = 1$$



Solving Two-Step Equations and Word Problems (04:25)

To solve equations with fractions:

Example #4: $\frac{x}{9} + \frac{x}{6} + \frac{x}{3} = 1$

*Use LCD = 18

$$18\left(\frac{x}{9} + \frac{x}{6} + \frac{x}{3}\right) = 18(1)$$

*Multiplication Property,
multiply both sides by 18.

$$18\left(\frac{x}{9}\right) + 18\left(\frac{x}{6}\right) + 18\left(\frac{x}{3}\right) = 18(1)$$

*Distribute

$$\frac{\cancel{2}18}{1} \left(\frac{x}{\cancel{9}^1}\right) + \frac{\cancel{3}18}{1} \left(\frac{x}{\cancel{6}^1}\right) + \frac{\cancel{6}18}{1} \left(\frac{x}{\cancel{3}^1}\right) = 18(1)$$

*Cancel Fractions.

$$2x + 3x + 6x = 18$$

*Simplify

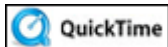
$$11x = 18$$

*Collect like terms.

$$x = \frac{18}{11}$$

*Division Property,

divide both sides by 11 and
leave the answer as an
improper fraction.



Solving Linear Equations with Rational Expressions (04:24)

proportion: an equation that states two ratios are equal

To solve proportions:

- Cross multiply.
- Solve by using opposite operations.
- Justify the steps using the Properties of Equality.

Example #5: $\frac{2x-1}{5} = \frac{3x}{7}$

*Cross multiply $\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$

$$7(2x-1) = 5(3x)$$

*Distribute and simplify

$$14x - 7 = 15x$$

$$-14x \quad -14x$$

*Subtraction Property (Subtract 14x from both sides.)

$$-7 = 1x$$

$$-7 = x$$

* $1x = x$ (The 1 is understood to be there.)

$$x = -7$$

Symmetric Property (If $a = b$, then $b = a$.)

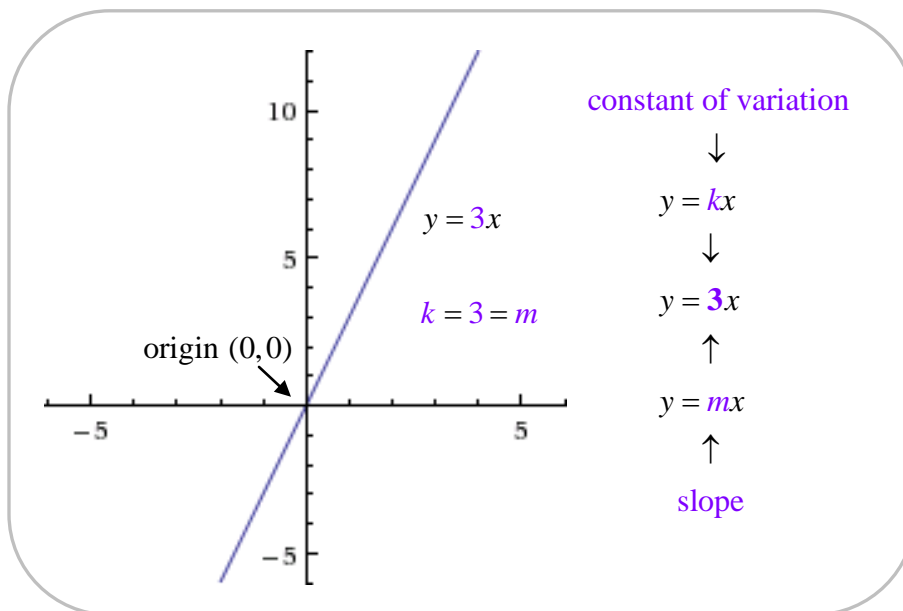
Stop! Go to Questions #1-13 about this section, then return to continue on to the next section.

Direct Variation - A Special Case of a Linear Equation

The variable y varies directly as x if there is a nonzero constant k such that $y = kx$. The equation, $y = kx$, is called a direct variation equation and the number k is called the constant of variation. The graph of a direct variation is a line that goes through the origin $(0,0)$. The slope of the line is the constant of variation.

Graph of a Direct Variation

This line is the graph of a direct variation because it passes through the origin.



There are many situations in which one quantity varies directly as another:

- An employee's wages vary directly as the number of hours worked.
- The amount of sales tax varies directly as the total price of the merchandise.

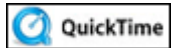
To find the constant of variation, k , and the direct-variation equation, use the following steps:

- 1.) In $y = kx$, replace the x and y with the given values.
- 2.) Solve for k .
- 3.) Replace k in the direct variation equation, $y = kx$.

Example #1: Find the constant of variation, k , and the direct-variation equation, if y varies directly as x and $y = -72$ when $x = -18$.

$$\begin{aligned}y &= kx \\-72 &= k(-18) \\ \frac{-72}{-18} &= \frac{k(-18)}{-18} \\ 4 &= k \\ y &= 4x\end{aligned}$$

The constant of variation (k) is 4 and the direct-variation equation is $y = 4x$.



Shortcuts for Solving Proportion Problems: Writing a Formula (03:30)

Example #2: Each day Michael roller blades for exercise. When traveling at a constant rate, he travels 4 miles in about 20 minutes. At this rate, how long will it take Michael to travel 7 miles?

- 1.) Write a direct variation equation, $d = rt$, that models Michael's distance as it varies with time.
- 2.) Find the constant of variation, r .

$$r = \frac{4 \text{ mi}}{20 \text{ min}} = \frac{1 \text{ mi}}{5 \text{ min}} \text{ or } \frac{1}{5} \text{ mile per minute}$$

- 3.) Write the direct variation equation.

$$d = \frac{1}{5}t$$

- 4.) Use the direct variation equation to solve the problem.

$$d = \frac{1}{5}t$$

$$7 = \frac{1}{5}t \quad \text{*Substitution (} d = 7 \text{)}$$

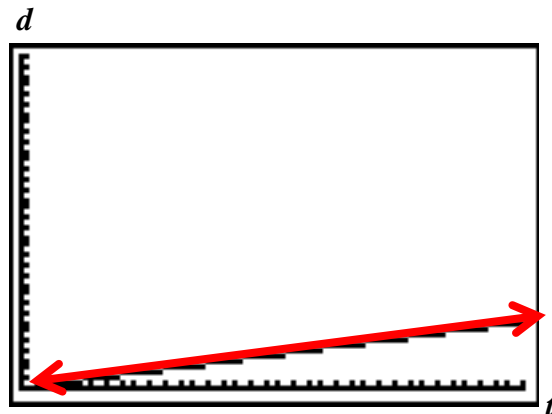
$$7(5) = (5)\frac{1}{5}t \quad \text{*Multiply both sides by 5}$$

$$35 = 1t$$

$$35 = t \quad \text{The 1 is "understood".}$$

Thus, at this rate, it will take Michael 35 minutes to travel 7 miles.

The function above is graphed below. Study the graph carefully. As t **increases**, d **increases**. For example, when t is 5, d is 1. When t increases to 10, d increases to 2. When t increases to 35, d increases to 7. Note: the graph is **linear** and passes through the origin.



*This graph was created on a graphing calculator, causing pixelation of the straight line. Thus, a **straight red line** was added to show the true appearance of the linear graph.

Using Proportions to Solve Direct-Variation Problems

The general formula for direct variation is $y = kx$, where k represents the constant of variation.

The general form can be solved for k .

$$y = kx \quad \rightarrow \quad \frac{y}{x} = \frac{k \cancel{x}}{\cancel{x}} \quad \rightarrow \quad \frac{y}{x} = k$$

$\frac{y}{x} = k$ is read "y varies directly as x" and k is the constant of variation.

Example #3: If y varies directly as x , and $y = 25$ when $x = 15$, find y when $x = 6$.

$$\frac{25}{15} = \frac{y}{6} \quad \text{Use two forms of } \frac{y}{x} \text{ in a proportion.}$$

$$(25)(6) = 15(y) \quad \text{Write the cross products.}$$

$$150 = 15y \quad \text{Simplify}$$

$$10 = y \quad \text{Divide each side by 15.}$$

$$y = 10$$

Example #4: If y varies directly as x and $y = 20$ when $x = 36$, find x when $y = 8$.

$$\frac{20}{36} = \frac{8}{x} \quad \text{Use two forms of } \frac{y}{x} \text{ in a proportion.}$$

$$20x = (36)(8) \quad \text{Write the cross products.}$$

$$20x = 288 \quad \text{Simplify}$$

$$x = 14.4 \quad \text{Divide each side by 20.}$$

Example #5: Mark's wages vary directly as the number of hours he works. For 6 hours he earns \$43.80. How much money will he make in 35 hours?

$$\frac{\text{wages}}{\text{\# of hours}} \quad \text{Set up a guide in words for } \frac{y}{x}.$$

$$\frac{43.80}{6} = \frac{y}{35} \quad \text{Use two forms of } \frac{y}{x} \text{ in a proportion.}$$

$$6y = (43.80)(35) \quad \text{Write the cross products.}$$

$$6y = 1533 \quad \text{Simplify}$$

$$y = \$255.50 \quad \text{Divide each side by 6.}$$

Example #6: In a certain state in the United States of America, the base sales tax is 6%. Thus, the amount of tax owed is directly proportional to the amount of money spent. A person that spends \$1.00 will pay a tax of \$0.06.

a) What amount of tax is assessed to the purchase of a vehicle that costs \$13,500

$$\frac{\text{amount of tax owed}}{\text{amount spent}} \quad \text{Set up a guide in words for } \frac{y}{x}.$$

$$\frac{0.06}{1.00} = \frac{y}{13,500} \quad \text{Use two forms of } \frac{y}{x} \text{ in a proportion.}$$

$$1y = (0.06)(13,500) \quad \text{Write the cross products.}$$

$$y = 810 \quad \text{Simplify}$$

The amount of tax assessed on the vehicle is \$810.

b) If the tax for the purchase of a diamond pendant is \$375, what is the cost of the item?

$$\frac{\text{amount of tax owed}}{\text{amount spent}} \quad \text{Set up a guide in words for } \frac{y}{x}.$$

$$\frac{0.06}{1.00} = \frac{375}{x} \quad \text{Use two forms of } \frac{y}{x} \text{ in a proportion.}$$

$$1(375) = (0.06)(x) \quad \text{Write the cross products.}$$

$$375 = 0.06x \quad \text{Simplify}$$

$$6250 = x \quad \text{Divide each side by 0.06.}$$

The cost of the diamond pendant is \$6250.

Stop! Go to Questions #14-22 about this section, then return to continue on to the next section.

Using Equations to Solve Problems

Linear equations can be used to solve a variety of application problems.

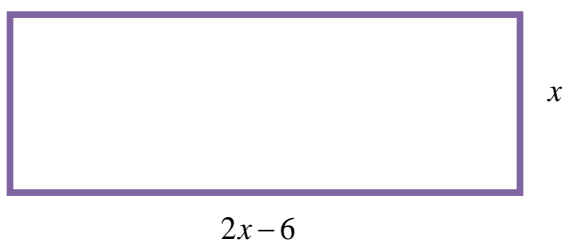
Geometry

perimeter: distance around the outside of a figure.

Example #1: The length of a rectangle is 6 feet less than twice the width. The perimeter of the rectangle is 96 feet. Find the width and the length of the rectangle.

Let $x =$ width

Then $2x - 6 =$ length (The length of a rectangle is 6 feet less than twice the width (x .)



Write an equation based on the idea that perimeter is the total length around the rectangle.

$$\text{width} + \text{length} + \text{width} + \text{length} = 96$$

$$x + (2x - 6) + x + (2x - 6) = 96$$

$$x + 2x - 6 + x + 2x - 6 = 96$$

$$6x - 12 = 96 \quad \text{*Simplify}$$

$$6x = 108 \quad \text{*Addition Property (Add 12 to both sides.)}$$

$$x = 18 \quad \text{*Division Property (Divide both sides by 6.)}$$

The width of the rectangle (x) is 18 feet.

For the length of the rectangle, substitute the value of x into the expression for the length ($2x - 6$).

$$2x - 6 = 2(18) - 6 = 36 - 6 = 30$$

The length of the rectangle is 30 feet.

Check: Add up the four sides of the rectangle.

$$18 + 30 + 18 + 30 = 96 \checkmark$$

Find the Average

average: total of all numbers, divided by how many numbers there are.

Example #2: Ami has test scores of 88, 90 and 84. What score does Ami need on the next test to have an average of 90? Write an equation and solve.



Let x = score on the Test 4.

To find the average, add up all four tests scores and divide by 4.

$$\frac{\text{sum of the four test scores}}{4} = 90 \quad \text{Guide in words}$$

$$\frac{88 + 90 + 84 + x}{4} = 90$$

Set up the equation letting x be Test 4.

$$88 + 90 + 84 + x = 360$$

Multiply each side by 4.

$$262 + x = 360$$

Simplify the left side of the equation.

$$x = 98$$

Subtract 262 from each side

Ami needs to score a 98 on Test 4.

Check:
$$\frac{88 + 90 + 84 + x}{4} = 90$$

$$\frac{88 + 90 + 84 + 98}{4} = \frac{360}{4} = 90 \checkmark$$

Business

revenue: the total amount of money received for goods

cost: the amount of money associated with purchasing the goods to sell

profit: the positive gain calculated by subtracting the cost from the revenue

Example #3: It costs a restaurant owner 13 cents per glass for soda, which is sold for \$1.79 per glass. How many glasses of soda must the restaurant owner sell to make a profit of \$1245?

Let x = the number of glasses of soda to be sold

Then $1.79x$ = revenue and $0.13x$ = cost

$$\text{Revenue} - \text{Cost} = \text{Profit} \quad \text{*Guide in words}$$

$$1.79x - 0.13x = 1245 \quad \text{*Simplify}$$

$$1.66x = 1245 \quad \text{*Divide both sides by 1.66.}$$

$$x = 750$$

The restaurant owner must sell 750 glasses of soda to make a profit of \$1245.

Check: $1.79x - 0.13x = 1245$

$$\begin{aligned} 1.79(750) - .13(750) &= 1245 \\ 1342.50 - 97.50 &= 1245 \checkmark \end{aligned}$$

Uniform Motion

An object that moves at a constant rate is said to be in **uniform motion**.

Many uniform motion problems can be solved by using the formula $d = rt$, where d is the distance traveled, r is the rate of speed and t is the time.

Example #4: Two trains depart from a station traveling in opposite directions. The eastbound train travels 55 miles per hour. The westbound train travels at a rate of 65 miles per hour. In how many hours will the trains be 432 miles apart? Write an equation and solve.



Let's draw the problem as we set it up to help visualize the scenario.

Let t = time that each train travels.

$55t$ = distance (d) traveled by the eastbound train ($d = rt$)

$65t$ = distance (d) traveled by the westbound train. ($d = rt$)



Model the equation in words:

distance west + distance east = total distance

$$55t + 65t = 432$$

$$120t = 432 \quad \text{*Simplify the left side}$$

$$t = 3.6 \quad \text{*Divide both sides by 120.}$$

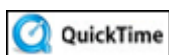
It will take 3.6 hours for the trains to travel 432 miles apart.

Check:

Distance for the westbound train ($d = 65t$) is $65(3.6) = 234$ miles

Distance for the eastbound train ($d = 55t$) is $55(3.6) = 198$ miles

Total miles traveled $198 + 234 = 432$ miles ✓



Practice Problem: Distance (04:23)

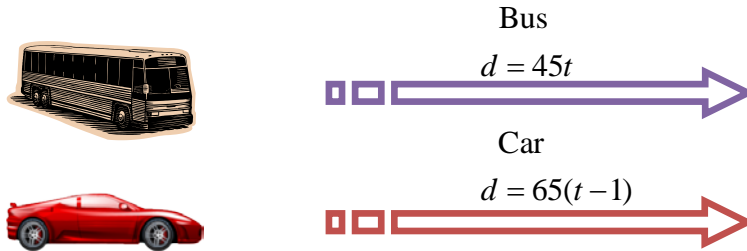
Example #5: A bus traveling at an average rate of 45 mph left the city at noon. An hour later a car traveling in the same direction left from the same place traveling at an average rate of 65 miles per hour. How long will it take the car to catch up with the bus?

Let t = time traveled by the bus

Then, $t - 1$ = time traveled by the car (The car left an hour later.)

$45t$ = distance (d) traveled by the bus ($d = rt$)

$65(t - 1)$ = distance (d) traveled by the car. ($d = rt$)



Since the bus and the car left from the same place, the distance covered by the bus and the car are the same.

Model the equation in words:

distance bus traveled = distance car traveled

$$45t = 65(t - 1) \quad \text{Set the distances equal to each other.}$$

$$45t = 65t - 65(1) \quad \text{Distribute}$$

$$45t = 65t - 65 \quad \text{Simplify}$$

$$-65t \quad -65t$$

$$-20t = -65 \quad \text{Subtract } 65t \text{ from both sides.}$$

$$t = 3.25 \quad \text{Divide both sides by } -20.$$

It will take the car 3.25 or 3 1/4 hours to catch up to the bus.

Check:

Distance traveled by the bus ($d = 45t$) is $45(3.25) = 146.25$ miles

Distance traveled by the car [$d = 65(t - 1)$] is $65(3.25 - 1) = 65(2.25) = 146.25$ miles

146.25 miles = 146.25 miles ✓



Mixture Problem

Example #6: One fruit drink A contains 4% sugar. Fruit drink B contains 8% sugar. You want to create 24 quarts of a fruit drink that contain 5% sugar by combining fruit drink A and fruit drink B. How much of each type of fruit drink should you use? Write an equation and solve.



Let x = number of quarts of fruit drink A
Then $24 - x$ = number of quarts of fruit drink B

$$0.04x \quad = \text{amount of sugar in fruit drink A}$$
$$0.08(24 - x) = \text{amount of sugar in fruit drink B}$$

$$0.05(24) \quad = \text{amount of fruit drink in the mixture}$$

Model the equation in words:

sugar in fruit drink A + sugar in fruit drink B = sugar in the mixture

$$0.04x + 0.08(24 - x) = 0.05(24)$$

$$0.04x + \mathbf{0.08(24)} - \mathbf{0.08x} = 0.05(24) \quad \text{*Distribute}$$

$$-0.04x + 1.92 = 1.2 \quad \text{*Simplify}$$

$$-0.04x = -0.72 \quad \text{*Subtract 1.92 from each side.}$$

$$x = 18 \quad \text{*Divide each side by } -0.04.$$

The mixture will contain 18 quarts of fruit drink A.

For fruit drink B, substitute 18 in the expression $24 - x$.

$$24 - x = 24 - \mathbf{18} = 6$$

The mixture will contain 6 quarts of fruit drink B.

Check:

18 quarts of fruit drink A contains 4% sugar, $18(0.04) = 0.72$ quarts of sugar

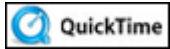
6 quarts of fruit drink B contains 8% sugar, $6(0.08) = 0.48$ quarts of sugar

The mixture, 24 quarts, contains 5% sugar, $24(0.05) = 1.2$ quarts of sugar

Sugar in A + Sugar in B = Total amount of sugar in mixture

$$0.72 + 0.48 = 1.2$$

$$1.2 = 1.2 \checkmark$$



Practical Problem #2: Price & Pounds (05:53)

Investment

Simple interest problems can be solved by using the formula $i = prt$, where i is the interest, p is the principal, r is the interest rate, and t is the time.

Example #7: Henry invested \$3000 at 6% annual simple interest. How much money must he invest at 8% annual simple interest so that his investments will earn \$356 in one year? Write an equation and solve.



Let x = the principal at 8%

Interest at 6% for 1 year = $\$3000(.06)(1) = \180

Interest at 8% for 1 year = $x(.08)(1) = 0.08x$

Model the equation in words:

interest at 6% + interest at 8% = total interest earned

$$180 + 0.08x = 356$$

$$0.08x = 176$$

*Subtract 180 from each side.

$$x = 2200$$

*Divide each side by 0.08.

Henry must invest \$2200 at 8% for one year.

Check : interest at 6% + interest at 8% = total interest earned

$$180 + 0.08x = 356$$

$$180 + 0.08(2200) = 356$$

$$180 + 176 = 356$$

$$356 = 356 \checkmark$$

Work Problem



Example #8: Jack can mow the lawn in 3 hours. His sister Jill can mow the same lawn in 2 hours. How long would it take Jack and Jill working together to mow the lawn? Write an equation and solve.

Let t = time to mow the lawn working together

Jack's work: $\frac{1}{3}$ = amount of the lawn mowed by Jack in one hour.

Then, $\frac{1}{3}t$ = amount of the lawn mowed by Jack in t hours

Jill's Work: $\frac{1}{2}$ = amount of the lawn mowed by Jill in one hour.

Then, $\frac{1}{2}t$ = amount of the lawn mowed by Jill in t hours

Model the equation in words:

amt of lawn mowed by Jack + amt of lawn mowed by Jill = 1 lawn mowed

$$\frac{1}{3}t + \frac{1}{2}t = 1$$

$$6\left(\frac{1}{3}t + \frac{1}{2}t\right) = 1(6)$$

*Multiply both sides by 6.

The LCD (least common denominator) of 3 and 2 is 6.

$$6\left(\frac{1}{3}t\right) + 6\left(\frac{1}{2}t\right) = 6$$

*Distribute and Simplify

$$2t + 3t = 6$$

*Simplify $\frac{\cancel{6}^2}{1} \cdot \frac{1}{\cancel{3}_1}t = 2t$, $\frac{\cancel{6}^3}{1} \cdot \frac{1}{\cancel{2}_1}t = 3t$

$$5t = 6$$

*Simplify

$$t = \frac{6}{5} = 1\frac{1}{5} = 1.2$$

Working together Jack and Jill will mow the lawn in 1.2 hours.

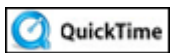
Check: work done by Jack + work done by Jill = 1 lawn mowed

$$\frac{1}{3}t + \frac{1}{2}t = 1$$

$$\frac{1}{3}(1.2) + \frac{1}{2}(1.2) = 1$$

$$0.4 + 0.6 = 1$$

$$1.0 = 1 \checkmark$$



Problem Solving: A Work Problem with Rate (05:31)

Formulas

formula: A formula is an equation that expresses relationship between two or more variables.

Sometimes it is necessary to solve a formula for a specific variable. Solving for a variable in a formula involves similar steps to solving for a variable in an equation. To solve a formula for a specific variable, you must *isolate* the variable on one side of the equation.

For example, let's look at Ohm's Law:

$$V = IR$$

where V represents **volts**, I represents **amperes**, and R represents **resistance**.

Example #9: Solve $V = IR$ for I

This means we must isolate I by itself on one side of the equation.

$$V = IR$$

$$\frac{V}{R} = \frac{IR}{R}$$

*Since I is multiplied by R , divide both sides of the equation by R .

$$\frac{V}{R} = \frac{I\cancel{R}^1}{\cancel{R}^1}$$

*Cancel the R's.

$$\frac{V}{R} = I$$

Example #10: In the perimeter formula, $P = 2l + 2w$, P represents perimeter, l represents length, and w represents width. Solve the formula for w .

This means we must isolate w on one side of the equation.

$$P = 2l + 2w$$

$$-2l \quad -2l$$

$$P - 2l = 2w$$

*Subtract $2l$ from each side of the equation.

$$\frac{P - 2l}{2} = \frac{2w}{2}$$

*Divide both sides by 2.

$$\frac{P - 2l}{2} = w$$

*Simplify the right side. $\frac{\cancel{1}w}{\cancel{1}} = 1w = w$

$$w = \frac{P - 2l}{2}$$

We can simplify this formula further.

$$w = \frac{P - 2l}{2}$$

$$w = \frac{P}{2} - \frac{2l}{2}$$

*Write the right side as two fractions.

$$w = \frac{P}{2} - \frac{\cancel{1}2l}{\cancel{1}2}$$

*Cancel

$$w = \frac{P}{2} - l$$

*Simplify

Example #11: In the surface area formula for a rectangular prism, $S = 2(wh + lw + hl)$, S represents surface area, l represents length, and w represents width, and h represents height. Solve the formula for l .

$$S = 2(wh + lw + hl)$$

$$S = 2wh + 2lw + 2hl$$

*Distributive Property

$$S = 2wh + 2lw + 2hl$$

$$-2wh \quad -2wh$$

$$S - 2wh = 2lw + 2hl$$

*Isolate the terms that have the variable l by subtracting $2wh$ from each side.

$$S - 2wh = 2l(w + h)$$

*Factor $2l$ from the right side.

$$\frac{S - 2wh}{2(w + h)} = \frac{2l(w + h)}{2(w + h)}$$

*Divide each side by $2(w + h)$.

$$\frac{S - 2wh}{2(w + h)} = \frac{\cancel{2}^1 l \cancel{(w + h)}^1}{\cancel{2}^1 \cancel{(w + h)}^1}$$

*Cancel.

$$\frac{S - 2wh}{2(w + h)} = l$$

*Simplify

$$l = \frac{S - 2wh}{2(w + h)}$$

Example #12: The equation $C = \frac{5}{9}(F - 32)$ converts temperatures in degrees Fahrenheit F to degrees Celsius C . Solve the formula for F , and then use the formula to convert 15° Celsius to Fahrenheit.

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9} (F - 32) \quad \text{*Multiply both sides by } \frac{9}{5}, \text{ the reciprocal of } \frac{5}{9}$$

$$\frac{9}{5}C = \frac{\overset{1}{\cancel{9}} \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{9}}} (F - 32) \quad \text{*Cancel}$$

$$\frac{9}{5}C = 1(F - 32) \quad \text{*Simplify}$$

$$\frac{9}{5}C = F - 32 \quad \text{*Distribute the 1.}$$

$$\frac{9}{5}C = F - 32 \quad \text{*Add 32 to both sides}$$

$$+32 \quad +32$$

$$\frac{9}{5}C + 32 = F \quad \text{*Simplify}$$

$$F = \frac{9}{5}C + 32 \quad \text{*Symmetric Property (If } a = b, \text{ then } b = a)$$

To convert 20° Celsius to Fahrenheit, substitute $C = 20$ in the formula.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(20) + 32$$

$$F = 36 + 32 \quad \text{*} \frac{\overset{9}{\cancel{9}} \cdot \overset{20^4}{\cancel{20}}}{\underset{1}{\cancel{5}} \cdot 1} = \frac{36}{1} = 36$$

$$F = 68$$

The Fahrenheit temperature for 20° C is 68° F.

Stop! Go to Questions #23-32 to complete this unit.