Course Overview

In this course students will begin by reviewing basic algebra and geometry topics. They demonstrate fluency in operations with real numbers, vectors and matrices; represent and compute with complex numbers; use fractional and negative exponents to find solutions for problem situations; describe and compare the characteristics of the families of quadratics with complex roots, polynomials of any degree, logarithms, and rational functions. Students investigate rates of change, intercepts, zeros and asymptotes of polynomial, rational, and trigonometric functions graphically and with technology; identify families of functions with graphs that have rotation symmetry or reflection symmetry about the y-axis, x-axis, or $y = x$. They solve problems with matrices and vectors, solve equations involving radical expressions and complex roots, solve 3 by 3 systems of linear equations, and solve systems of linear inequalities; solve quadratic expressions, investigate curve fitting, and determine solutions for quadratic inequalities. They investigate exponential growth and decay and use recursive functions to model and solve problems; compute with polynomials and solve polynomial equations using a variety of methods including synthetic division and the rational root theorem; solve inverse, joint, and combined variation problems; solve rational and radical equations and inequalities; and describe the characteristics of the graphs of conic sections. They analyze the behavior of arithmetic and geometric sequences and series. Students use permutations and combinations to calculate the number of possible outcomes recognizing repetition and order; compute the probability of compound events, independent events, and dependent events. They use descriptive statistics to analyze and interpret data, including measures of central tendency and variation.

In some of the units, a graphing calculator will be useful. It is recommended that the graphing calculator be at least a TI-83 model.

LINEAR EQUATIONS

$$y = mx+b \qquad y = b \qquad x = a$$

$$y-y1 = m(x-x1)$$

Unit Overview

An equation that has a graph that is a line is called a linear equation. In this unit you will analyze linear equations in two variables using tables and graphs. By working with rate of change you will understand how the slope of a line can be interpreted in real world situations. Linear equations will be graphed using the slope and y-intercept. You will write a linear equation in two variables given sufficient information. Finally you will write an equation for a line that is parallel or perpendicular to a given line.

Tables and Graphs of Linear Equations

$y = mx + b$ is called a linear equation and the graph is a straight line.

For an equation to be linear, the following must apply:

-There must be at least one variable, and at most 2 variables.
-The exponent on any of the variables must be equal to 1 (no more, no less).

Examples:

$y = 2$ is linear because there is at least one variable.

$3x + 4y = 6$ is linear because there is at most 2 variables.

$2x + y = 3z$ is **not** linear because there are more than 2 variables.

$3x^2 - y^3 = 2$ is **not** linear because the exponent on the x is 2 and the exponent on the y is 3.

$\dfrac{2}{x} = 7$ is **not** linear because the variable is in the denominator which would result in a negative exponent $(x^{-1}) = \dfrac{1}{x}$.

When variables represented in a table of values are linearly related and there is a constant difference in the x-values, there is also a constant difference in the y-values.

Example #1: $y = -2x + 5$

1.) Make a table.
2.) Choose x-values with a constant difference.
3.) Replace the values in the equation for x to find the y values.

$$y = -2(1) + 5 \qquad y = -2(2) + 5 \qquad y = -2(3) + 5 \qquad y = -2(4) + 5$$
$$y = 3 \qquad\qquad\quad y = 1 \qquad\qquad\quad y = -1 \qquad\qquad\quad y = -3$$

x	1	2	3	4
y	3	1	−1	−3

In a linear relationship, a constant difference in consecutive *x*-values results in a constant difference in consecutive *y*-values.
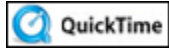
QuickTime Linear Equations and Slope (15:01)

Application (Income)

Example #2: A carpenter charges a fixed fee of $250 for an initial estimate and $150 per hour for all hours worked after that. What is the carpenter's charge for 28 hours of work?
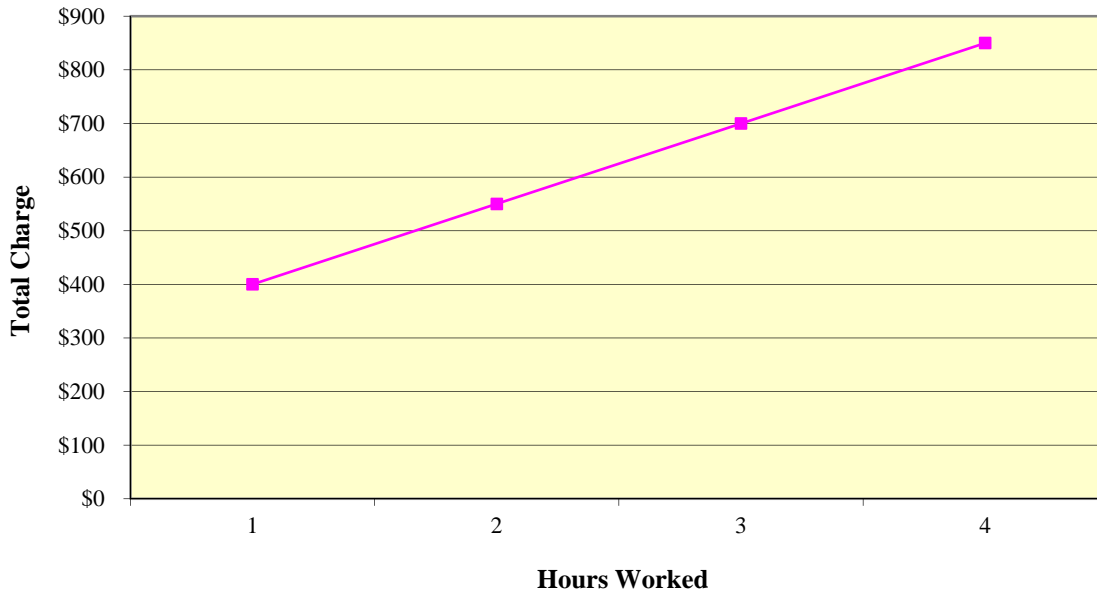
    a.) Make a table of the total charge for 1, 2, 3, and 4 hours worked.
    b.) Graph the points represented by your table on paper and connect them.
    c.) Write a linear equation to model this situation.
    d.) Find the charge for 28 hours of work.

Solution:

a.)

Hours Worked	1	2	3	4
Total Charge	$400	$550	$700	$850

b.)

c.) Translate the verbal description into an equation involving c (charge) and h (hours).

total charge $=$ variable charge $+$ fixed fee

c $=$ $150h$ $+$ 250

Thus, $c = 150h + 250$.

d.) Use the equation. Substitute 28 for h in the equation.

$c = 150h + 250$
$c = 150(28) + 250$
$c = 4450$

The carpenter's charge for 28 hours of work is $4450.

Stop! **Go to Questions #1-14 about this section, then return to continue on to the next section.**

Rates of Change

A rate of change is a rate that describes how one quantity changes in relationship to another quantity.

Let's see how this works.

Find the Rate of Change Using a Table

In the first example, the rate of change in Krista's height is examined.

> *Example #1*: The table shows Krista's height in inches between the ages of 9 and 14. (a) Find the rate of change in her height between the ages of 9 and 12. (b) Find the rate of change in her height between the ages of 12 and 14.

Age (yr)	Height (in)
9	51
12	59
14	63

(a) Find the rate of change in Krista's height between the ages of 9 and 12.

$$\frac{\text{change in height}}{\text{change in years}} = \frac{59-51}{12-9} = \frac{8}{3}$$

The rate of change is $\frac{8}{3}$ or $2\frac{2}{3}$. Krista grew at the rate of $2\frac{2}{3}$ inches each year.

(b) Find the rate of change in her height between the ages of 12 and 14.

$$\frac{\text{change in height}}{\text{change in years}} = \frac{63-59}{14-12} = \frac{4}{2}$$

The rate of change between ages 12 and 14 is 2. Krista grew at the rate of 2 inches each year.

In this example the rate of change of a burning candle is examined.

Example #2: The table shows the height of a candle as it burns at a constant (same) rate. What is the rate of change and explain what "rate of change" means in this situation.

Time (min)	Height (in)
0	12
15	10
30	8

Since the rate of change is constant, you can choose any two pairs of points to calculate the rate of change.

Let's choose (0,12) and (30,8).

$$\frac{\text{change in height}}{\text{change in minutes}} = \frac{8-12}{30-0} = \frac{-4}{30} = \frac{-2}{15} = -\frac{2}{15}$$

The rate of change is $-\frac{2}{15}$. The height of the candle is decreasing by 2/15 of an inch per minute.

Find the Rate of Change Using a Graph

In this example the bicyclist's constant speed is examined as a rate of change.

Example #3: The graph shows the distance a bicyclist travels at a constant speed. Use the graph to find the rate of change in miles per hour.
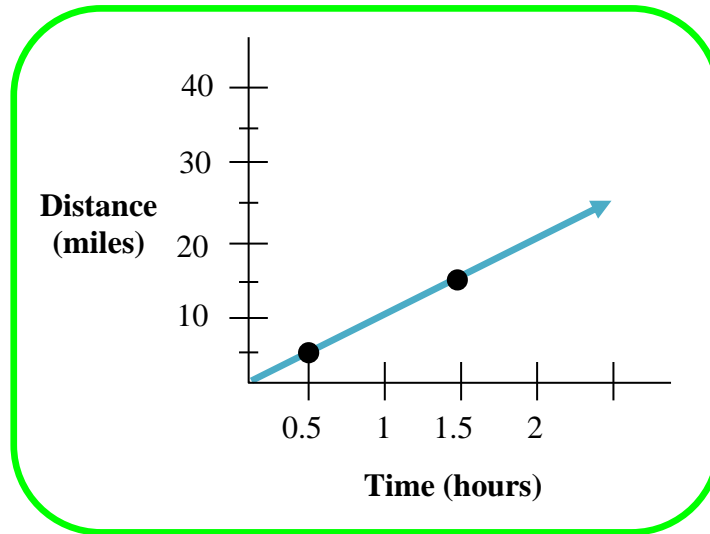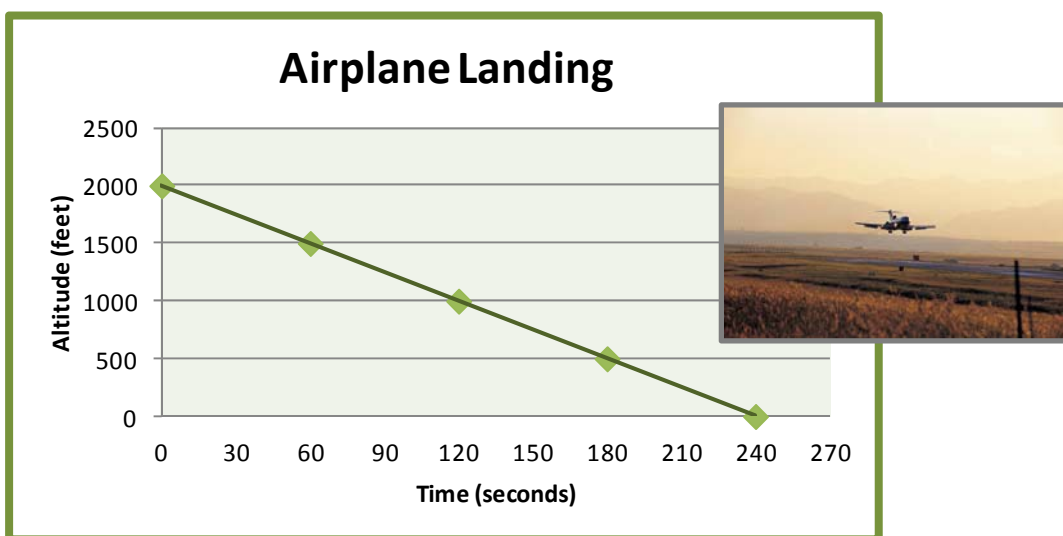

To find the rate of change, pick any two points on the line, such as (0.5,5) and (1.5,15), and then calculate the rate of change.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{15-5}{1.5-0.5} = \frac{10 \text{ miles}}{1 \text{ hour}}$$

The rate of change is 10. The bicyclist travels 10 miles per hour.

In this example rate of change in an airplane's height from the ground is examined as it descends to land.

Example #4: The graph shows the altitude of an airplane as it prepares to land. Find the rate of change and interpret what the rate of change means in this situation.

Airplane Landing

To find the rate of change, pick any two points on the graph, such as (0,2000) and (120, 1000).

$$\frac{\text{change in altitude}}{\text{change in time}} = \frac{2000-1000}{0-120} = \frac{1000}{-120} = \frac{100}{-12} = \frac{25}{-3} = -8\frac{1}{3}$$

The rate of change is $-8\frac{1}{3}$. The airplane is descending at the rate of 8 1/3 feet per second.
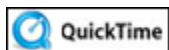
QuickTime Slope-Intercept Form: An Introduction (04:47)

Average Rate of Change

Example #5: Jason opened a savings account on January 1 with a deposit of $150. On April 1, the account balance was $280. What is the average rate of change per month.

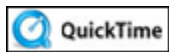$$\frac{\text{change in account balance}}{\text{change in time}} = \frac{280-150}{3} = \frac{130}{3} = 43.33$$

The average rate of change is $43.33 per month.

Stop! **Go to Questions #15-17 about this section, then return to continue on to the next section.**
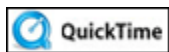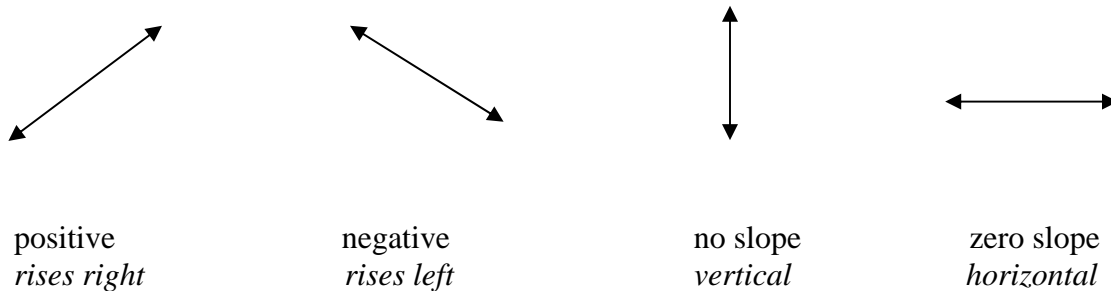
Slopes and Intercepts

The **slope** of a line refers to the steepness and direction of the line and is found by finding the change in vertical units (*y*) divided by the corresponding change in horizontal units (*x*).

The slope of a line is a *rate of change*. Slope $= \dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{\text{rise}}{\text{run}}$.

QuickTime What is Slope? (06:02)

There are four (4) types of slopes.

| positive | negative | no slope | zero slope |
| *rises right* | *rises left* | *vertical* | *horizontal* |

QuickTime Negative, Positive, Zero, and Undefined Slopes (05:33)

The slope of a line can be thought of as "RISE OVER RUN"; that is, "rise" parallel to the *y*-axis (change in *y*) over "run" parallel to the *x*-axis (change in *x*). Let's examine this further.

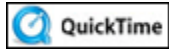**To find the slope of a line given two points:**

$$(x_1, y_1) \text{ and } (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{*ratio for slope is } \frac{rise}{run}.$$

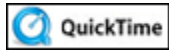*Example #1*: Find the slope of a line going through the points (3, 5) and (–2, –6).

$$m = \frac{-6-5}{-2-3} = \frac{-11}{-5} = \frac{11}{5}$$

Thus the slope (m) is $\frac{11}{5}$.

y-intercept: the point at which the line crosses the *y*-axis (0, *y*)

x-intercept: the point at which the line crosses the *x*-axis (*x*, 0)

slope-intercept form of a line: $y = mx + b$ (m = slope, b = *y*-intercept)

standard form of a linear equation: $Ax + By = C$

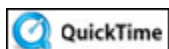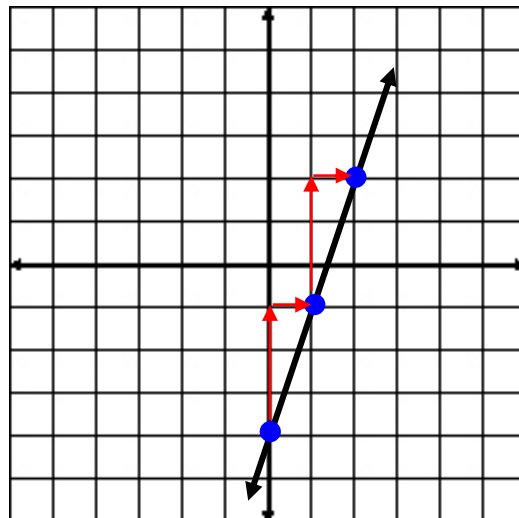**To graph an equation using the slope and y-intercept:**

1.) Solve the equation for *y* if not already done so ($y = mx + b$).

2.) Plot the *y*-intercept which is located at (0, *b*).

3.) Use the ratio of "rise over run" to plot more points.

Example #2: Graph $-3x + y = -4$

$$y = 3x - 4$$

y-intercept is located at (0, –4)

slope is $\frac{3}{1}$, $\left(\frac{rise}{run} \right)$, up 3 to the right 1.

**To graph a line using the *x* and *y*-intercepts:**

1.) Replace *x* with a zero and solve for *y*.
[This will give you the *y*-intercept (0, *y*).]

2.) Replace *y* with a zero and solve for *x*.
[This will give you the *x*-intercept (*x*, 0).]

Example #3: Graph $3x + 6y = -12$ using the *x* and *y*-intercepts.

$$3x + 6y = -12 \qquad\qquad 3x + 6y = -12$$
$$3(0) + 6y = -12 \qquad\qquad 3x + 6(0) = -12$$
$$6y = -12 \qquad\qquad\qquad 3x = -12$$
$$y = -2 \qquad\qquad\qquad\quad x = -4$$

y-intercept (0, –2)   *x*-intercept (–4, 0)

Plot these two points and connect them with a straight line.
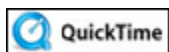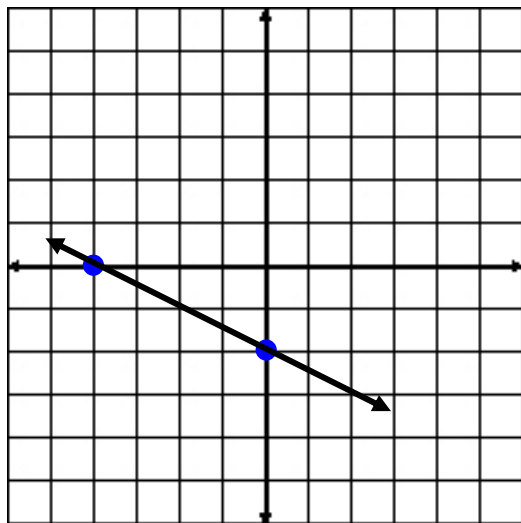
Stop! **Go to Questions #18-27 about this section, then return to continue on to the next section.**

Linear Equations in Two Variables

Point-slope form: $y - y_1 = m(x - x_1)$

   x_1 and y_1 represent the coordinates of the given point.

To find a linear equation using point-slope form:

Given the slope (m) and a point (x_1, y_1)

   1.) Use the point-slope form $y - y_1 = m(x - x_1)$.

   2.) Substitute the given slope for m and the given point for x_1 and y_1.
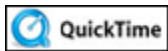
   3.) Solve for y.

Example #1: What is the linear equation of a line that has a slop of $\dfrac{3}{4}$ and passes through the point $(-12, 2)$?

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{4}(x - (-12)) \qquad m = \frac{3}{4}, \ x_1 = -12, \ y_1 = 2$$

$$y - 2 = \frac{3}{4}x + 9$$

$$y = \frac{3}{4}x + 11$$

QuickTime The Point-Slope Equation (01:16)

To find a linear equation using point-slope form:

Given two points

   1.) Find the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. 2.) Choose one of the points given.

   3.) Substitute the slope for m and the chosen point for x_1 and y_1.

4.) Solve for y.

Example #2: Given points $(6, 9)$ and $(5, -6)$

$$m = \frac{-6-9}{5-6} = \frac{-15}{-1} = 15$$ Calculate the slope.

$y - y_1 = m(x - x_1)$ Use the point-slope method to find the equation.

$y - 9 = 15(x - 6)$ Substitute in the values of one point for (x_1, y_1)

$y - 9 = 15x - 90$ Distribute

$y = 15x - 81$ Simplify

Stop! **Go to Questions #28-31 about this section, then return to continue on to the next section.**

Equations of Horizontal and Vertical Lines

Recall that the slopes of horizontal and vertical lines are special cases.

Horizontal lines have a slope of 0.

Vertical lines have an undefined slope or no slope.

In this section we will discuss the equation of both of these lines and how to graph each.

> The equation of a **horizontal line** is $y = b$, where b is the y-intercept.
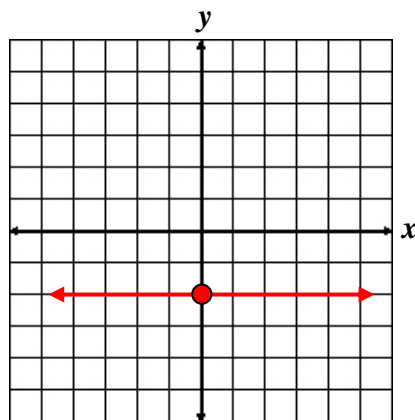>
> The equation of a **vertical line** is $x = a$, where a is the x-intercept.

Equation of a Horizontal Line

Example #1: Graph the equation $y = -2$.

-This is a horizontal line with a y-intercept of $(0, -2)$.

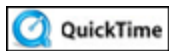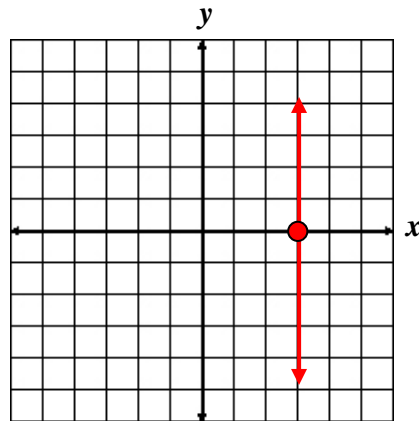-Plot the point $(0, -2)$ and draw a horizontal line through it.

Equation of a Vertical Line

Example #2: Graph the equation $x = 3$.

    -This is a vertical line with an x-intercept of (3, 0).

    -Plot the point (3, 0) and draw a vertical line through it.


QuickTime Horizontal Lines, Vertical Lines, and Lines Through the Origin (01:41)

Stop! **Go to Questions #32-33 about this section, then return to continue on to the next section.**

Parallel and Perpendicular Lines

Lines that never intersect are called parallel lines. There is a special relationship between parallel lines. Study the graph below and see if you can figure out what the special relationship is.
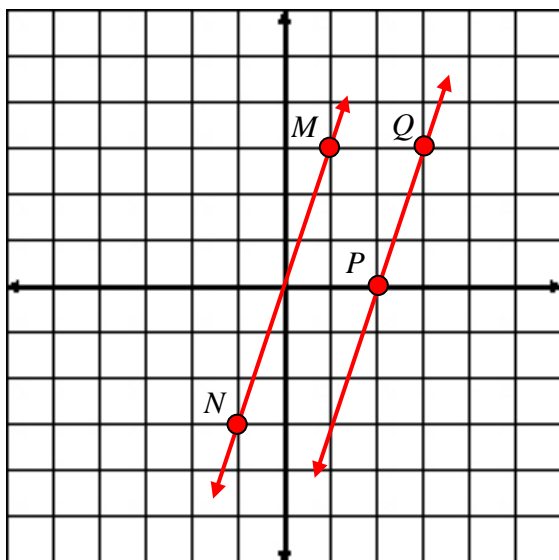
Do the lines have the same slope? Let's see if this conjecture (guess) is true.

1.) Two points on line *MN* are *M*(1, 3) and *N*(−1, −3). Find the slope of line *MN* by using the two points on the line.

$$m = \frac{-3-3}{-1-1} = \frac{-6}{-2} = 3$$ The slope of the line containing *M* and *N* is 3.

2.) Two points on line *PQ* are *P*(2, 0) and *Q*(3, 3). Find the slope of line *PQ* by using the two points on the line.

$$m = \frac{3-0}{3-2} = \frac{3}{1} = 3$$ The slope of the line containing *P* and *Q* is 3.

By finding the slope of both lines, you can see that **parallel lines** have the **same** slope.

QuickTime Making Comparisons with Equations (02:56)
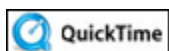
Parallel lines have the same slope.

Example #1: Are the graphs of the equations, $y = -\frac{4}{5}x - 5$ and $y = -\frac{4}{5}x + 2$, parallel lines?

The graphs of the equations are parallel lines because each equation has a slope of $-\frac{4}{5}$.

Perpendicular lines have opposite reciprocal slopes.

Example #2: Are the graphs of the equations, $y = \frac{5}{8}x + 2$ and $y = -\frac{8}{5}x - 3$, perpendicular lines?

The graphs of the equations are perpendicular lines because $\frac{5}{8}$ and $-\frac{8}{5}$ are **opposite reciprocals**.

$$y = \frac{5}{8}x + 2$$

$$y = -\frac{8}{5}x - 3$$
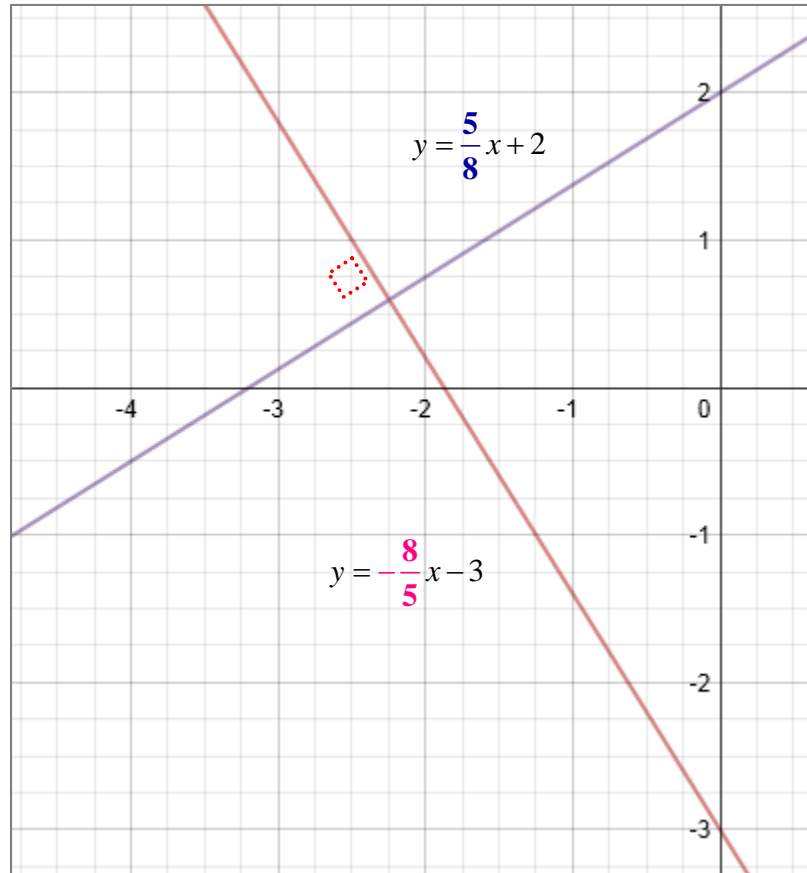
Now, let's practice what we know!

What is the slope of a line that is parallel to $y = -4x + 5$?

"Click here" to check your answer.

The slope of the parallel line is –4.

What is the slope of a line that is perpendicular to $y = \frac{3}{2}x - 7$?

"Click here" to check your answer.

The slope of the perpendicular line is the opposite reciprocal, –2/3.

This knowledge provides us with the ability to write equations of lines that are parallel to or perpendicular to the lines of given equations and containing a certain point.

Example #3: Write an equation, in slope-intercept form, of a line that is parallel to $y = 2x + 3$ and passes through point $(-1, 4)$.

a.) Determine the slope of the given equation.

$y = 2x + 3$ has a slope of 2 so $m = 2$

Since we want a line parallel to this line, we will use the same slope, $m = 2$, because parallel lines have the same slope.

b.) Use the slope-intercept form and substitute the slope you found into the equation.

$y = mx + b$
$y = 2x + b$

c.) Now find the *y*-intercept (b) by substituting the given point's coordinates into the equation in the previous step.

$y = mx + b$
$y = 2x + b$
$4 = 2(-1) + b$
$6 = b$

The equation $y = 2x + 6$ is parallel to $y = 2x + 3$ and passes through the point $(-1, 4)$. When working with linear equations, sometimes they are given in standard form and need to be expressed in slope-intercept form. In this case, solve the equation for *y* and proceed onward. This is shown in the next example.

Example #4: Write the equation of a line perpendicular to $5x + 2y = 10$ and passes through point $(3, -5)$.

a.) Solve the equation $5x + 2y = 10$ for y and determine the slope.

$$5x + 2y = 10$$

$$2y = -5x + 10 \qquad \text{Subtract } 5x \text{ from both sides.}$$

$$y = -\frac{5}{2}x + 5 \qquad \text{Divide all terms by 2.}$$

The slope of this line is $-\frac{5}{2}$. We want to use the opposite reciprocal of this value because we want the equation of a line that is perpendicular to the given equation.

b.) Use $\frac{2}{5}$ as the slope in the slope-intercept form.

$$y = mx + b$$

$$y = \frac{2}{5}x + b$$

c.) Substitute the given point (3, –5) and the slope you determined above, $\frac{2}{5}$, into the slope-intercept form to solve for the *y*-intercept (*b*).

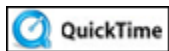$$y = mx + b$$

$$y = \frac{2}{5}x + b$$

$$-5 = \frac{2}{5}(3) + b$$

$$-5 = \frac{6}{5} + b$$

$$\frac{-25}{5} = \frac{6}{5} + b$$

$$\frac{-31}{5} = b$$

The equation $y = \frac{2}{5}x - \frac{31}{5}$ is perpendicular to 5*x* + 2*y* = 10 and passes through point $(3, -5)$.

QuickTime Converting Standard Form to Slope-Intercept Form (02:27)

Let's finish with a quick quiz.

What is the slope of a line that is parallel to $2x - 4y = -3$?

"Click here" to check your answer.

The slope of the parallel line is 1/2.

What is the slope of a line that is perpendicular to $x - 3y = 5$?

"Click here" to check your answer.

The slope of the perpendicular line is –3.

Stop! **Go to Questions #34-40 to complete this unit.**