SYSTEMS OF EQUATIONS

A system of equations is a collection of equations in the same variables. In this unit you will solve systems of equations using three (3) techniques: graphing, substitution, and elimination.

Graphing Systems of Equations

The solution of a system of two linear equations in x and y is any ordered pair, (x, y) that satisfies both equations. The solution (x, y) is also the point of intersection of the graphs.

There are 3 possible solutions.

1 solution	The lines intersect at one point.	The slopes of the lines are different.	Consistent and independent
many solutions	The lines intersect at many points.	The lines are exactly the same.	Consistent and dependent
no solution	The lines do not intersect.	The slopes are the same.	Inconsistent

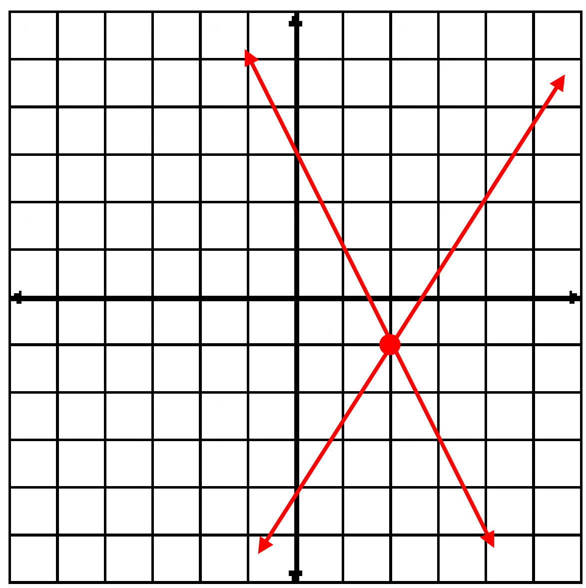
To graph a line, remember to solve for y and use the slope intercept form of $y = mx + b$. Plot the y-intercept, use the slope ratio of $\frac{rise}{run}$ to plot more points, and then connect the points using a straight edge.

Example #1: $\begin{cases} 2x + y = 3 \\ 3x - 2y = 8 \end{cases}$

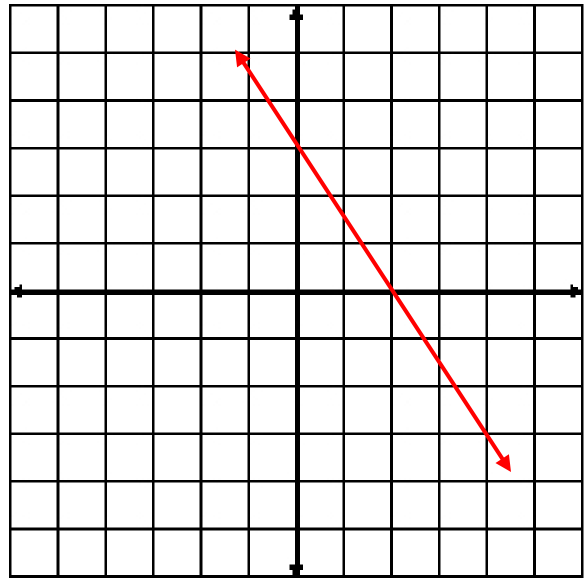
$2x + y = 3$ becomes $y = -2x + 3$

$3x - 2y = 8$ becomes $y = \frac{3}{2}x - 4$

Therefore the solution to this system of equations is **(2, –1)** because that is the point of intersection of the graphs of the two equations. The system would be classified as **consistent and independent**.



Example #2: $\begin{cases} 6x + 4y = 12 \\ 2y = 6 - 3x \end{cases}$

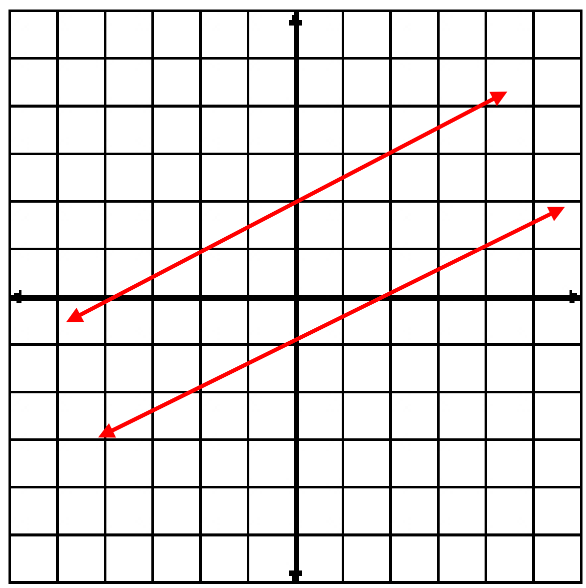


$6x + 4y = 12$ becomes $y = \frac{-3}{2}x + 3$

$2y = 6 - 3x$ becomes $y = \frac{-3}{2}x + 3$

Since these represent the same line they lie on top of each other therefore the solution will be **many solutions** and the system is classified as **consistent and dependent**.

Example #3: $\begin{cases} -2x + 4y = 8 \\ y = \frac{1}{2}x - 1 \end{cases}$



$2x + 4y = 8$ becomes $y = \frac{1}{2}x + 2$

Since these two lines are parallel, there is no intersection; so, the solution is **no solution** and the system is classified as **inconsistent**.

Substitution Method for Solving Systems of Equations

To solve a system by substitution:

1) solve one of the equations for a variable (hint: solve for a variable that has a coefficient of 1)
2) substitute this value into the other equation to find the value of one of the variables
3) substitute this value back into either of the equations to find the second variable

Example #1: $\begin{cases} x - y = 3 \\ 2x + 2y = 2 \end{cases}$

Solve the first equation for either x or y. (Solve for x as this is a positive value)

$x = 3 + y$

Substitute $3 + y$ into the 2nd equation for x.

$$\begin{aligned} 2(3 + y) + 2y &= 2 \\ 6 + 2y + 2y &= 2 \\ -6 \qquad\qquad & -6 \\ 4y &= -4 \\ y &= -1 \end{aligned}$$

Substitute -1 for y in either equation. (Substitute it into the equation already solved for y.

$x = 3 + (-1)$

$x = 2$

Therefore, the solution to this system is $(2, -1)$.

Elimination

To solve a system by elimination:

1.) the coefficients of the same variable must be the same
2.) if the coefficients **are** the same, either subtract the equations or add the equations to eliminate that variable, depending on the signs (same sign – subtract, different signs – add)
3.) substitute this value back into one of the equations to find the other variable

Example #1: $\begin{cases} 2x + y = 8 \\ x - y = 10 \end{cases}$

Since both the y values have the same coefficient, other than the sign, you can add the two equations to eliminate the y.

$$\begin{array}{r} 2x + y = 8 \\ + \quad x - y = 10 \\ \hline 3x = 18 \\ x = 6 \end{array}$$

Solve for x.

Substitute 6 for x in either equation to find y.

$$\begin{aligned} 6 - y &= 10 \\ -y &= 4 \\ y &= -4 \end{aligned}$$

Therefore, the solution to this system is (6, –4)

To solve a system by elimination:

1.) the coefficients of the same variable must be the same
2.) if the coefficients **are not** the same, make them the same by multiplying one or both equations by a factor to make them the same
3.) add or subtract the result to eliminate a variable
4.) substitute that value back into either equation to find the other variable

Example #2: $\begin{cases} x + y = 4 \\ 2x + 3y = 9 \end{cases}$

You have a choice to either eliminate the x or the y.

If you choose to eliminate the x, multiply the top equation by 2 or –2, depending on whether you want to add or subtract.

If you choose to eliminate the y, multiply the top equation by 3 or –3, again depending on whether you want to add or subtract.

a.) eliminating x first

$-2(x + y = 4)$
$2x + 3y = 9$

$$\begin{array}{r} -2x - 2y = -8 \\ +\quad 2x + 3y = 9 \\ \hline y = 1 \end{array}$$

b.) eliminating y first

$-3(x + y = 4)$
$2x + 3y = 9$

$$\begin{array}{r} -3x - 3y = -12 \\ +\ 2x + 3y = 9 \\ \hline -x \quad = \ -3 \\ x = 3 \end{array}$$

In situation "a", substitute 1 for y to find the value of x.

$x + 1 = 4$
$x = 3$

In situation "b", substitute 3 for x to find the value of y.

$3 + y = 4$
$y = 1$

Either way you eliminate, you will receive the same answer, (3, 1).