INEQUALITIES AND ABSOLUTE VALUE EQUATIONS

An inequality is a mathematical statement that compares algebraic expressions using greater than (>), less than (<), and other inequality symbols. A compound inequality is a pair of inequalities joined by *and* or *or*. In this unit, properties of inequalities will be used to solve linear inequalities and compound inequalities in one variable. The unit concludes with a study of absolute-value equations and inequalities.

Introduction to Solving Inequalities

Inequality - a mathematical statement that compares algebraic quantities

Inequality Symbols	Meaning	Keyboard Entry
$<$	less than	<
$>$	greater than	>
$\leq$	less than or equal	<=
$\geq$	greater than or equal	>=
$\neq$	not equal	<>

*Solving inequalities is just like solving equations, use opposite operations to isolate the variable.

Example #1: $3(5x - 7) \geq 54$

$$15x - 21 \geq 54$$
$$+21 \quad +21$$
$$15x \geq 75$$
$$x \geq 5$$

*When multiplying or dividing by a negative number, the inequality sign must be reversed.

Let's take a look at why this rule applies.

Let's say that we know 4 > 2. If we multiply both sides of this inequality by a –2, let's see what happens to the inequality.

$4 > 2$ *this is a true inequality

$(-2)4 > (-2)2$ multiply both sides by –2

$-8 > -4$ *the result is not a true statement; however, if we **flip** the inequality sign, the result **will** be a true statement.

↓

$-8 < -4$

Example #2: $2y + 9 < 5y + 15$

$$\begin{aligned} 2y + 9 &< 5y + 15 \\ -5y \quad &\quad -5y \\ -3y + 9 &< 15 \\ -9 \quad &\quad -9 \\ \frac{-3y}{-3} &< \frac{6}{-3} \\ y &> -2 \end{aligned}$$

*Notice that the inequality sign was flipped because of the division by –3.

Example #3: $\frac{3}{4}(x-7) \le x-3$

$(4)\frac{3}{4}(x-7) \le 4(x-3)$ –Multiply both sides by 4.

$3(x-7) \le 4(x-3)$ –Distribute.

$3x - 21 \le 4x - 12$

$-3x \qquad -3x$

$-21 \le x - 12$

$-9 \le x$ –Rewrite with *x* on left side,

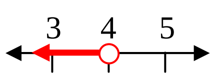
$x \ge -9$ inequality sign is reversed.

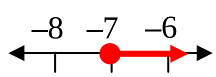
You can represent the solution of an inequality in one variable on a number line.

For < and > an open circle is used to denote that the solution number **is not** included in the solution.

For ≤ and ≥ a closed circle is used to denote that the solution number **is** included in the solution.

Example #4: $x < 4$ $y \ge -7$





compound inequalities: a pair of inequalities joined by "and" or "or".

To solve a compound inequality joined with "**and**", find the values of the variable that satisfy ***both*** inequalities.

*"**and**" means the **intersection** of the solutions

Example #5: $2x + 3 > 1$ and $5x - 9 < 6$

$2x > -2$ and $5x < 15$

$x > -1$ and $x < 3$

The solution is written $\{x / -1 < x < 3\}$ (set notation) "all numbers *x*, such that –1 is less than *x* is less than 3".

To solve a compound inequality joined with "**or**", find the values of the variable that satisfy at **least one** inequality.

"**or**" means the **union** of the solutions

Example #6: $3b + 7 \leq 1$ or $2b - 3 \geq 1$

$3b \leq -6$ or $2b \geq 4$

$b \leq -2$ or $b \geq 2$

The solution is written $\{b / b \leq -2 \text{ or } b \geq 2\}$ (set notation) "all numbers *b* such that *b* is less than or equal to –2 or *b* is greater than or equal to 2".

Solving Absolute Value Equations and Inequalities

absolute value - the distance a number is from zero (always positive).

*Two bars around the number denote absolute value.

$|-5| = 5$ $|6| = 6$

To solve absolute value equations:

1. Rewrite the equation without the absolute value notation.
2. Rewrite a second time using the opposite of what the original equation was equal to, and connect with the word "**or**".
3. Solve both equations and check both answers in the original equation.

Example #1: Solve $|2x-1| = 3$

$2x - 1 = 3$ or $2x - 1 = -3$ the -3 is the opposite of what the original was equal to.

$2x = 4$ or $2x = -2$

$x = 2$ or $x = -1$

Check $|2(2)-1| = 3$ or $|2(-1)-1| = 3$

$|4-1| = 3$ or $|-2-1| = 3$

$|3| = 3$ or $|-3| = 3$

$3 = 3$ or $3 = 3$

Therefore, the solution is 2 or –1.

Example #2: Solve $|2x+1| = x + 5$

$2x + 1 = x + 5$ or $2x + 1 = -x - 5$ *again use the opposite

$x = 4$ or $3x = -6$

$x = -2$

Check: $|2(4)+1| = 4 + 5$ or $|2(-2)+1| = -2 + 5$

$|8+1| = 9$ or $|-4+1| = 3$

$|9| = 9$ or $|-3| = 3$

$9 = 9$ or $3 = 3$

Therefore, the solution is 4 or –2.

absolute value inequalities - an absolute value that contains an inequality.

To solve absolute value inequalities:

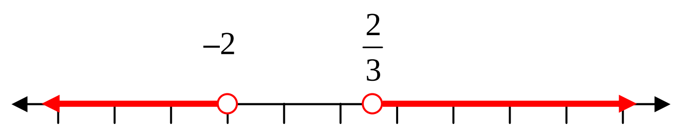
1.) Rewrite the inequality without the absolute value notation.
2.) Rewrite a second time, change the inequality sign, and use opposites.
3.) Solve both inequalities and check both answers in the original inequality.
4.) If the inequality is a $<$ or $\leq$, connect with the word "and".
5.) If the inequality is a $>$ or $\geq$, connect with the word "or".

Example #3: Solve $|3x+2| > 4$

$3x + 2 > 4$ **or** $3x + 2 < -4$ *flip the sign and use the opposite

$3x > 2$ **or** $3x < -6$

$x > \frac{2}{3}$ **or** $x < -2$



Check: $|3x+2| > 4$

To check this problem you will have to choose a number that is less than –2, and then choose a number that is greater than $\frac{2}{3}$.

Check (-3)		Check (1)
$\|3(-3)+2\| > 4$	or	$\|3(1)+2\| > 4$
$\|-9+2\| > 4$	or	$\|3+2\| > 4$
$\|-7\| > 4$	or	$\|5\| > 4$
$7 > 4$ (true)		$5 > 4$ (true)

Therefore, the solution to this absolute value inequality is $\{x/x < -2 \text{ or } x > \frac{2}{3}\}$.

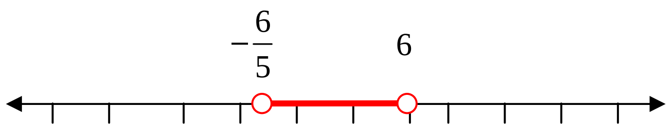
Example #4: Solve $\frac{1}{2}|5x-12|+4\leq 13$

$$\begin{aligned}
\tfrac{1}{2}|5x-12|+4 &\leq 13 \\
-4 \quad & \quad -4 \\
(2)\tfrac{1}{2}|5x-12| &\leq (2)9 \\
|5x-12| &\leq 18
\end{aligned}$$

$5x-12\leq 18$ **and** $5x-12\geq -18$ *flip the sign and use opposite

$5x\leq 30$ $\qquad$ $5x\geq -6$

$x\leq 6$ **and** $x\geq \frac{-6}{5}$



Check: $\frac{1}{2}|5x-12|+4\leq 13$

To check this problem, you will have to choose a number that is greater than $-\frac{6}{5}$ and also less than 6.

Check (0)

$$\begin{aligned}
&\tfrac{1}{2}|5x-12|+4\leq 13 \\
&\tfrac{1}{2}|5(0)-12|+4\leq 13 \\
&\tfrac{1}{2}|-12|+4\leq 13 \\
&\tfrac{1}{2}(12)+4\leq 13 \\
&6+4\leq 13 \\
&10\leq 13 \text{ (true)}
\end{aligned}$$

Therefore, the solution to this absolute value inequality is $\{x/\ x > -\frac{6}{5}$ and $x < 6\}$.