EQUATIONS IN TWO VARIABLES; PARALLEL AND PERPENDICULAR LINES

In this unit you will learn how to write the equation of a line given specific information such as the slope and a point on the line or two points that lie on the line. You will also investigate the graphs and equations of parallel and perpendicular lines.

Linear Equations in Two Variables

Parallel and Perpendicular Lines

Linear Equations in Two Variables

Point-slope form: $y - y_1 = m(x - x_1)$

x_1 and y_1 represent the coordinates of the given point.

To write a linear equation:

Given the slope (*m*) and a point (x_1, y_1)

1.) Use the point-slope form: $y - y_1 = m(x - x_1)$.

2.) Substitute the given slope for *m* and the given point for x_1 and y_1.

3.) Solve for *y*.

Example #1: Find the equation of a line whose slope is 4 and goes through the point (2, 3).

1.) Use the point slope form: $y - y_1 = m(x - x_1)$.

2.) Replace *m* with the slope 4, y_1 with 3, and x_1 with 2.

3.) Solve for *y*.

$y - 3 = 4(x - 2)$

$y - 3 = 4x - 8$

$y = 4x - 5$

The equation of the line that has a slope of 4 and passes through the point (2, 3) is $y = 4x - 5$.

Example #2: Find the equation of a line with slope = $\frac{3}{4}$ and passes through the point (–12, 2).

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{4}(x - (-12))$

$y - 2 = \frac{3}{4}x + 9$

$y = \frac{3}{4}x + 11$

Given two points

1.) Find the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$.

2.) Choose one of the points given.

3.) Substitute the slope for *m* and the chosen point for x_1 and y_1.

4.) Solve for *y*.

Example #3: Given points (6, 9) and (5, –6)

$$m = \frac{-6-9}{5-6} = \frac{-15}{-1} = 15$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 15(x - 6)$$

$$y - 9 = 15x - 90$$

$$y = 15x - 81$$

Example #4: Find the equation of a line that passes through the points (6, 1) and (0, –3)

$$m = \frac{-3-1}{0-6} = \frac{-4}{-6} = \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{2}{3}(x - 6)$$

$$y - 1 = \frac{2}{3}x - 4$$

$$y = \frac{2}{3}x - 3$$

The equation of a line that passes through the points (6, 1) and (0, –3) is $y = \frac{2}{3}x - 3$.

Parallel and Perpendicular Lines

Parallel lines have the same slope.

Perpendicular lines have opposite reciprocal slopes.

Example #1: $\frac{2}{3}$ and $\frac{-3}{2}$ would be opposite reciprocals.

To Write Equations of Lines Parallel to Given Equations

Parallel to a given equation through a given point:

1. Find the slope of the given equation ($y = mx + b$).
2. Use the point-slope form $y - y_1 = m(x - x_1)$ to replace the m (slope) and the (x_1, y_1) with the given point.
3. Solve for y.

Example #2: Write the equation that is parallel to $2x + y = 4$ going through the point (–1, 3).

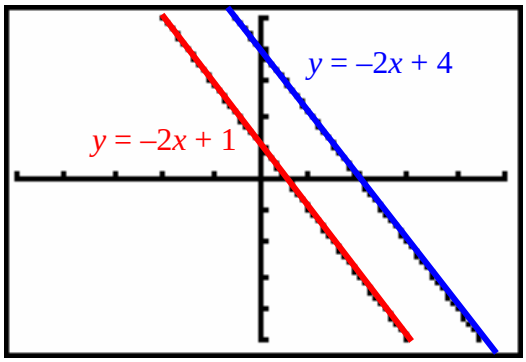
Write $2x + y = 4$ in slope-intercept form ($y = mx + b$)

$y = -2x + 4$ point (–1, 3)
$m = -2$ $x_1 = -1, \quad y_1 = 3$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -2(x - (-1))$$
$$y - 3 = -2(x + 1)$$
$$y - 3 = -2x - 2$$
$$y = -2x + 1$$

*Thus $y = -2x + 1$ is the equation of a line that is parallel to $y = -2x + 4$ going through the point (–1, 3).

Let's take a look at the graphs of the equations and make sure they are parallel.



The graphs of the two equations confirm that the two are parallel.

Example #3: Write the equation of a line that is parallel to $y = -3x + 4$ going through the point $(0, -1)$.

$y - y_1 = m(x - x_1)$

$m = -3$ $\qquad$ $y = -3x + 4$

$y - (-1) = -3(x - 0)$ $\qquad$ point $(0, -1)$

$y + 1 = -3x - 0$

$y = -3x - 1$

To Write Equations Perpendicular to Given Equations

Perpendicular to a given equation going through a given point:

1.) Find the slope of the given equation $y = mx + b$ and determine the opposite reciprocal of that number.

2.) Use the point slope form $y - y_1 = m(x - x_1)$ to replace m (slope) and the (x_1, y_1) with the given point.

3.) Solve for y.

Example #4: Write an equation of a line that is perpendicular to $y = 7x + 3$ going through the point $(8, -1)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{-1}{7}(x - 8) \qquad * \ \frac{-1}{7} \text{ is the opposite reciprocal of } 7$$

$$y + 1 = \frac{-1}{7}x + \frac{8}{7}$$

$$y = \frac{-1}{7}x + \frac{8}{7} - 1$$

$$y = \frac{-1}{7}x + \frac{8}{7} - \frac{7}{7}$$

$$y = \frac{-1}{7}x + \frac{1}{7}$$

Thus $y = \frac{-1}{7}x + \frac{1}{7}$ is the equation of a line that is perpendicular to $y = 7x + 3$ going through the point $(8, -1)$.

Example #5: Write the equation of a line that is perpendicular to $2x + y = 6$ and passes through the point $(4, 5)$.

$2x + y = 6$	-solve the given equation for y to determine the slope, which is -2.
$y = -2x + 6$	-we need to use the opposite reciprocal of -2, to replace in our equation since we are looking for the equation of a line that is **perpendicular to** the given equation.

$y - y_1 = m(x - x_1)$ $\qquad m = \frac{1}{2} \quad (x_1, y_1) = (4, 5)$

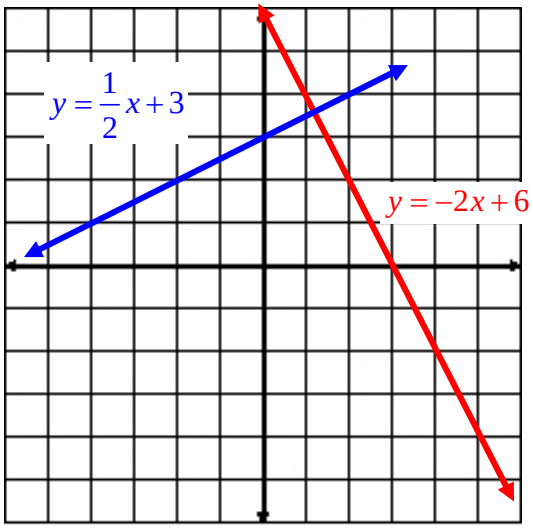
$y - 5 = \frac{1}{2}(x - 4)$ -replace m, x_1, and y_1 with the given values

$y - 5 = \frac{1}{2}x - 2$ -distribute

$$y = \frac{1}{2}x + 3$$

The equation of the line that is perpendicular to $2x + y = 6$ passing through the point (4, 5) is $y = \frac{1}{2}x + 3$.

Let's take a look at the graphs of the two equations to confirm they are perpendicular. We will look at a graph that has not been made on the calculator because the pixels on the calculator distort the graphs.



The graphs of the equations confirm that the two are perpendicular.