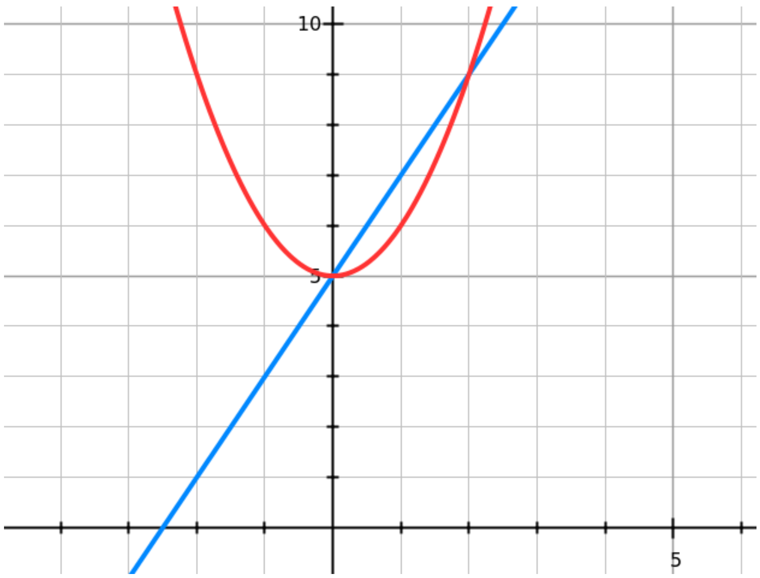
FUNCTIONS – INTERSECTIONS, INTERPRETATIONS, AND DEEPER UNDERSTANDING

Unit Overview

In this unit, you will study graphs and function behavior including linear, quadratic, exponential, absolute value and trigonometric functions. You will look at intersection points, intercepts, intervals where the function is positive or negative, intervals where the function increases or decreases, and end behavior. You will learn to find the midline, amplitude, and period for trigonometric graphs. You will finish by comparing functions through word problems.

Intersection Points

First, we are going to look at intersecting graphs. This is where two graphs cross, or intersect each other. For example, look at the graph below:



Notice that there is one line and one parabola. These two functions cross at two points. Reading the graph, you see that they intersect at (0,5) and (2,9).

In this section, you will find intersections of two functions by looking at graphs, tables, and equations.

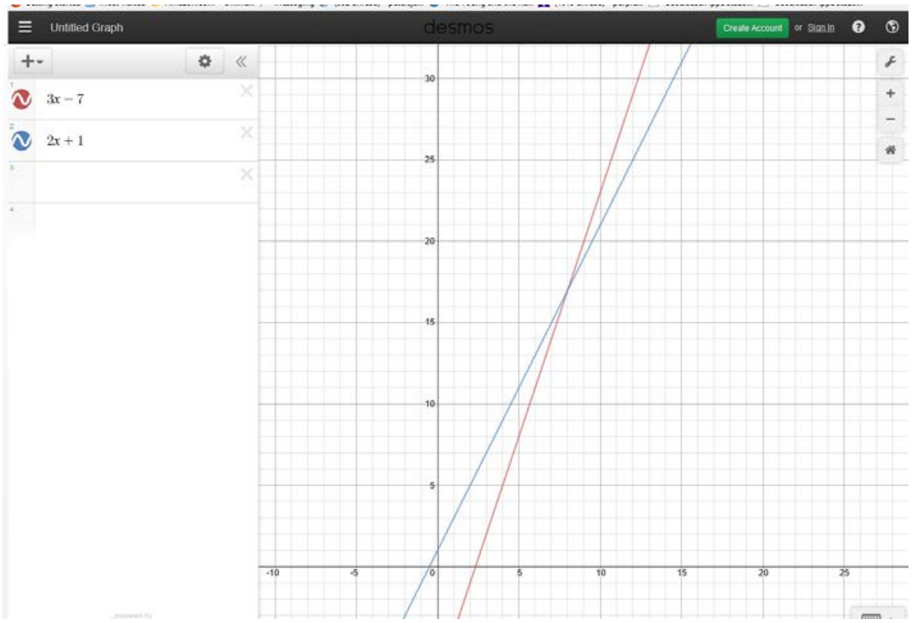
Recall: What do the points on a graph represent?

Answer: All the points along the graph represent solutions to the equation (or function). Recalling this, what do the intersection points of two graphs represent?

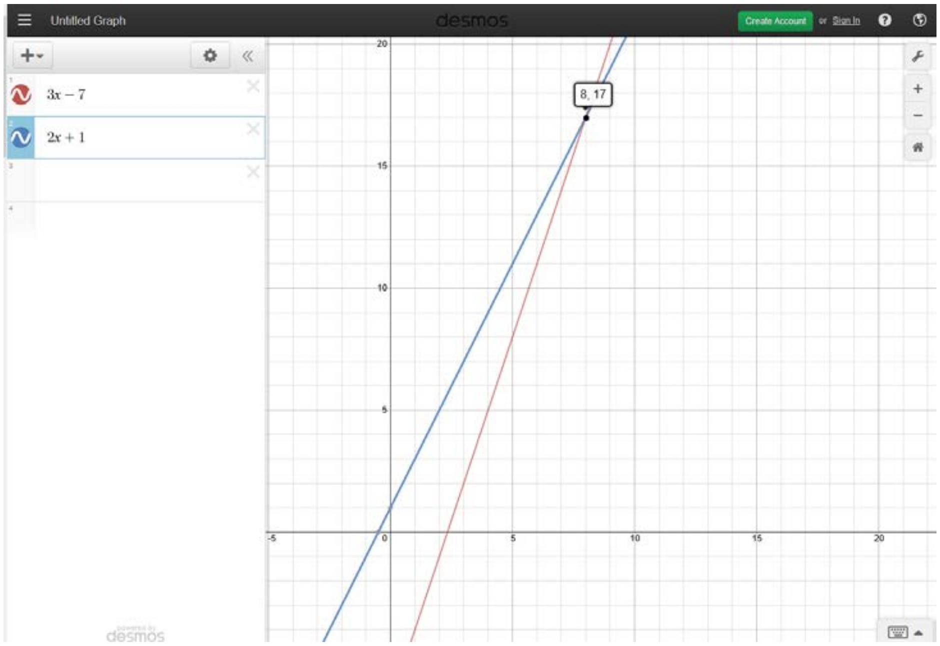
Answer: The intersection points represent solutions to both equations. In other words, an intersection point is a solution to both equations. If more than one intersection point exists, then there is more than one solution that fits both equations.

Example #1: Find the intersection points for the following functions, $f(x)=3x-7$ and $f(x)=2x+1$.

Remember from an earlier unit that you can solve a system of equations by graphing, by substitution, or by elimination. We are going to use the graphing method in this unit. However, we are going to expand that by looking at equations that are non-linear as well. Let's start with this linear example as a review. You will need to either graph by hand, with a graphing calculator, or use an online graphing utility. I am going to demonstrate using https://www.desmos.com/calculator. Go to this site and enter the two equations. When you enter the equations, it will look like this:

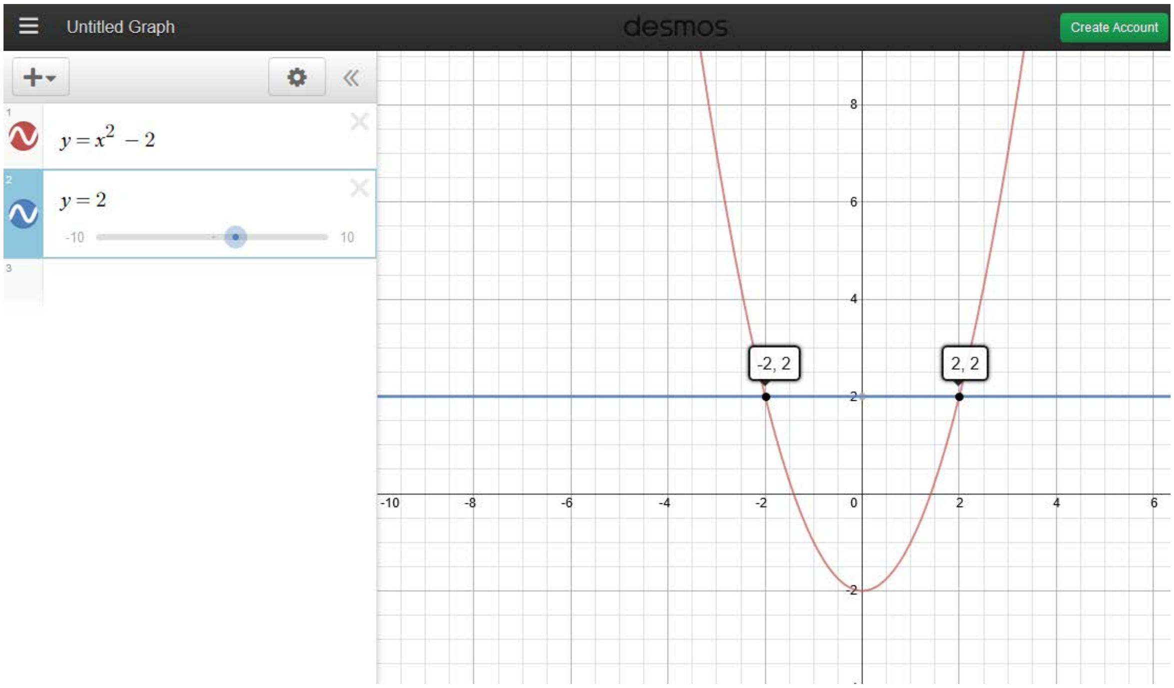


Enter the equations on the left. You do not have to type the $f(x)$ = portion. Just type "$3x – 7$" in the first rectangle and "$2x + 1$" in the second rectangle. You may need to click and "pull" the graph down by moving your mouse in order to see the intersection point. You can also zoom in and out by clicking on the + and – on the right hand side of the screen. In this example, you may be able to easily read the intersection point from the graph. Find (8, 17) and you should see the lines intersect. However, should a point not cross at a whole number or be difficult to tell for sure, you can click on the intersection point and the point will then show for you. Like this:



The intersection point for $y = 3x - 7$ and $y = 2x + 1$ is (8, 17).

Example #2: Find the intersection point(s) for the equations $y = x^2 - 2$ and $y = 2$. First, clear the equations you already used. Just use the backspace key. Now, type the new equations into https://www.desmos.com/calculator to get the graphs. To type an exponent, use the ^ symbol. This is above the 6 key. You will type $y = x$^2 – 2 for the first equation. For the second, type $y = 2$. Notice that to the left of the equation, you see a color. This will indicate which graph is which when you see it. It is likely that you will be able to easily identify the points of intersection, but if not, click on them and they will show. It should look like this:



The intersection points are (–2, 2) and (2, 2).

You should also know how to find the intersection points algebraically. This is the same as finding the solutions to a system of equations in units 10-11 as this is a system of equations. Only this time, one is linear and one is quadratic. This system is pretty simple as you already know $y = 2$. Simply substitute $y = 2$ into the first equation like this: $2 = x^2 - 2$. Now, solve for x.

$$2 = x^2 - 2$$

$$4 = x^2 \qquad \text{Add 2 to both sides.}$$

$$\sqrt{4} = \sqrt{x^2} \qquad \text{Take squre root of both sides.}$$

$$x = \pm 2$$

Therefore, the points of intersection are (–2, 2) and (2, 2).

If given a more complicated system such as $x^2 + y^2 = 25$ and $y = x - 5$, you use the same steps you've learned for solving a system of equations. You just may have additional steps. It would look like this:

1st: Substitute $x - 5$ in for y into the first equation.

$$x^2 + (x-5)^2 = 25$$

2nd: Solve for x.

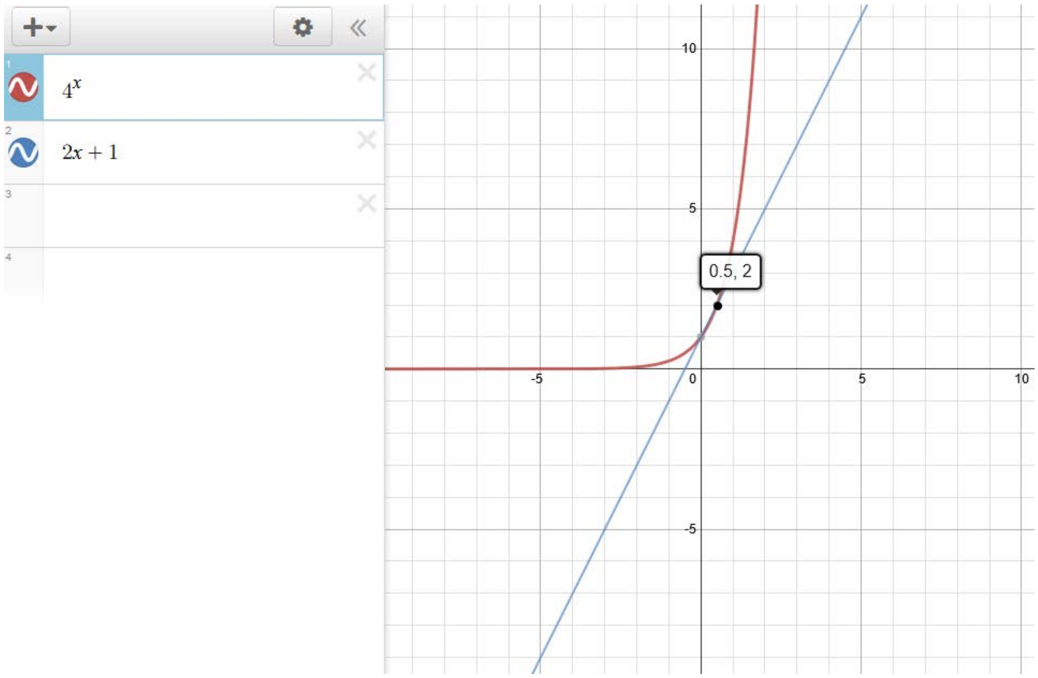
$x^2 + x^2 - 10x + 25 = 25$	Multiply out the $(x-5)^2$.
$2x^2 - 10x = 0$	Combine like terms and subtract 25 from both sides.
$x(2x-10) = 0$	Factor.
$x = 0 \quad \textit{and} \quad 2x - 10 = 0$	Set each factor to zero and solve.
$2x = 10$	
$x = 5$	

3rd: Substitute the x-values in and solve for y.

$y = x - 5$	
$y = 0 - 5 = -5$	Giving the ordered pair $(0, -5)$.
$y = 5 - 5 = 0$	Giving the ordered pair $(5, 0)$.

You can check your solutions by graphing.

Example #3: Find the intersection points of $y = 4^x$ and $y = 2x + 1$. (Remember to clear the old equations first!) Again, you can do this by hand by making a table of values, or for linear equations, use slope to help you graph. You can use a graphing calculator or continue to use the graphing utility at https://www.desmos.com/calculator. Remember to use ^ when entering exponents. So, for this problem, you will enter y = 4^x. Click on the intersection point. It should look like this:



The intersection point for these equations is (0.5, 2).

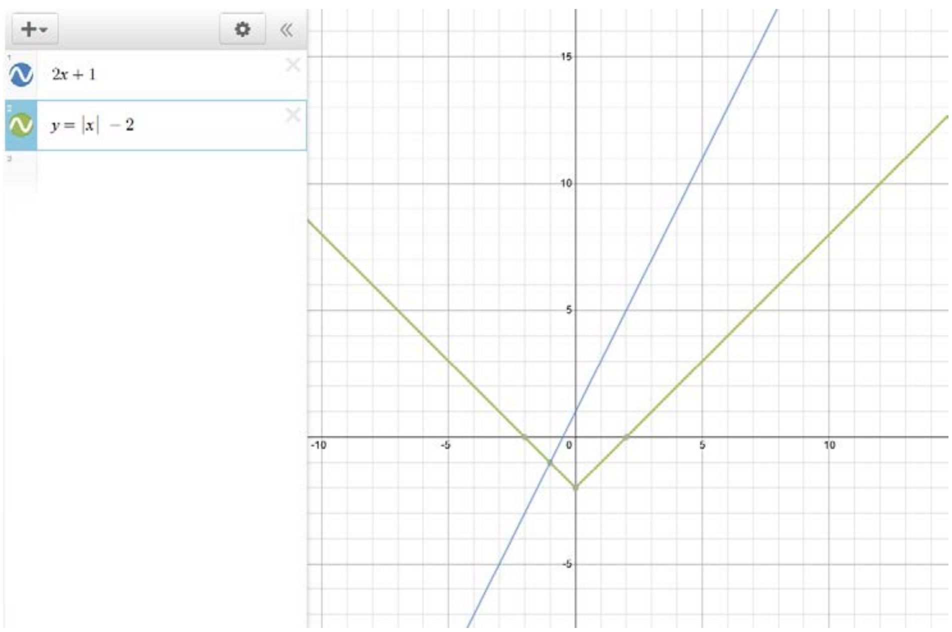
Example #4: Find the intersection point for $y = |x| - 2$ and $y = 2x + 1$. The first equation uses the absolute value symbol. To type this, hold the shift key while pushing the | key (underneath the backspace key and also with the \ key). Type these equations into https://www.desmos.com/calculator. You could also try making a table of values to see where the graphs intersect. Let's try this. Start with a simple table with x-values and 1 column for each equation's y-value:

x	$y = \|x\| - 2$	$y = 2x + 1$
–2		
–1		
0		
1		
2		

Now, fill it in:

x	$y = \|x\| - 2$	$y = 2x + 1$
–2	$y = \|-2\| - 2 = 2 - 2 = 0$	$y = 2 \times -2 + 1 = -4 + 1 = -3$
–1	$y = \|-1\| - 2 = 1 - 2 = -1$	$y = 2 \times -1 + 1 = -2 + 1 = -1$
0	$y = \|0\| - 2 = 0 - 2 = -2$	$y = 2 \times 0 + 1 = 0 + 1 = 1$
1	$y = \|1\| - 2 = 1 - 2 = -1$	$y = 2 \times 1 + 1 = 2 + 1 = 3$
2	$y = \|2\| - 2 = 2 - 2 = 0$	$y = 2 \times 2 + 1 = 4 + 1 = 5$

Notice that you get the same y-value for $x = -1$. This means you have an intersection point at $x = -1$ and $y = -1$ or $(-1, -1)$. To check this, we will also try the graphing method. It should look like this:

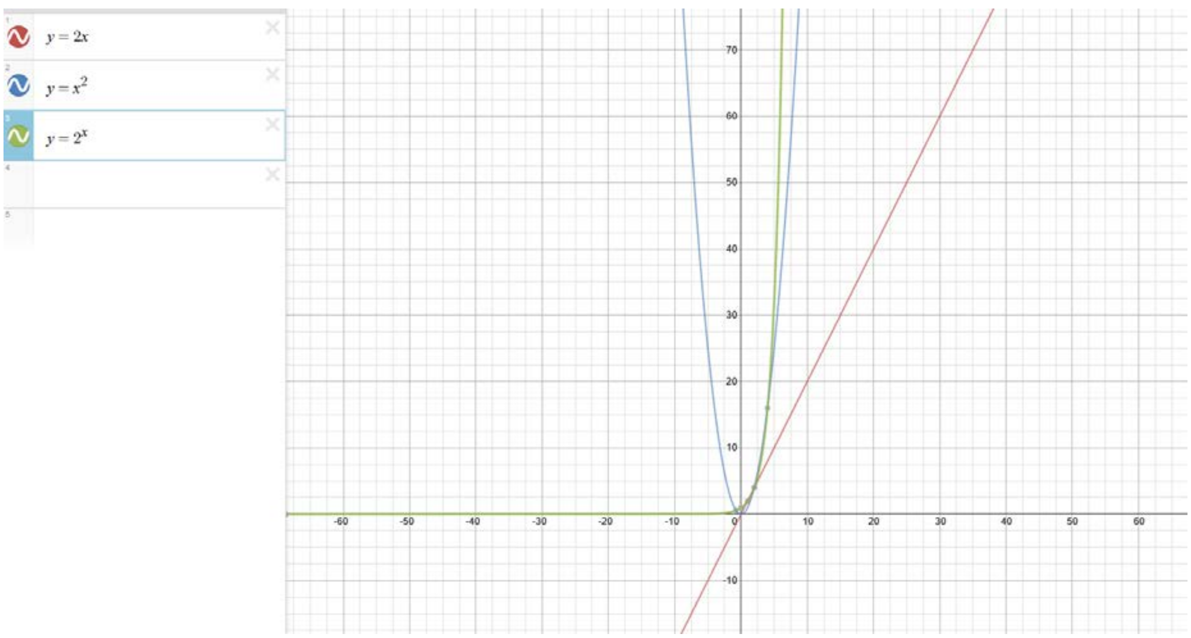


You can see from the graph that $(-1, -1)$ is an intersection point. Remember that you can also click the intersection point and the graphing utility will list it for you.

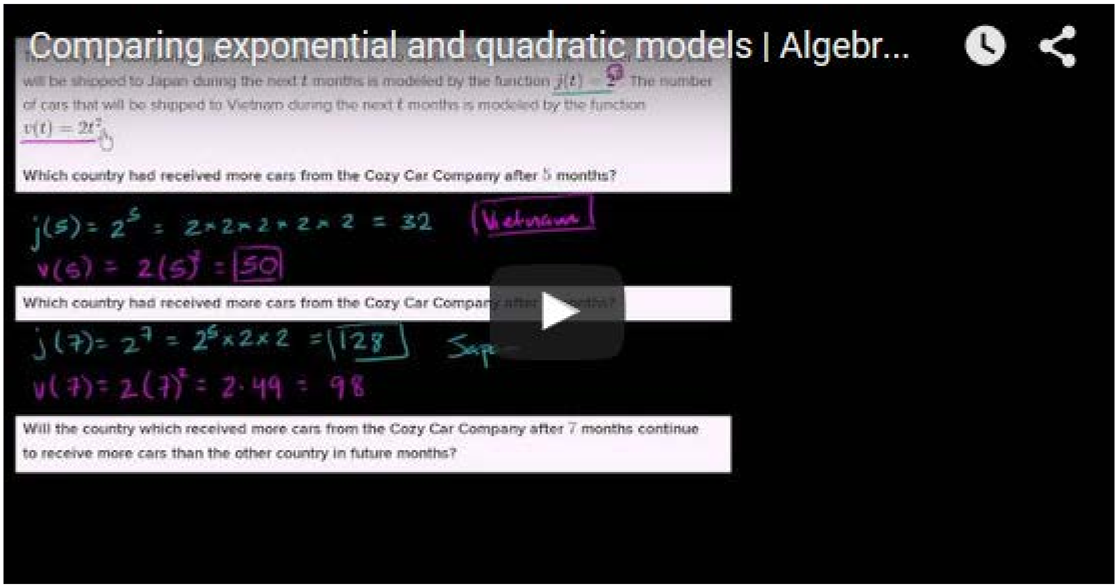
Stop! **Go to Questions #1-7 about this section, then return to continue on to the next section.**

Linear vs. Quadratic vs. Exponential Functions

Now that we've graphed many equations and found intersection points, let's look at the behavior of these graphs. Let's put 3 of our functions together. Using https://www.desmos.com/calculator, first clear any equations you have previously typed. For this problem, graph $y = 2x$, $y = 2^x$, and $y = x^2$. We are not looking for intersection points, but at graph behavior. The first thing to notice is this: which graph increases the fastest? Look at the linear graph (the straight line). How fast does it increase? Now, look at the quadratic graph (x^2). Now, look at the exponential graph (2^x). Notice that the exponential graph begins increasing much faster than the others. The quadratic and exponential graph quickly overtake the linear graph, but the exponential graph begins increasing even faster than the quadratic graph. In other words, for each x-value, the y-value is getting much larger for the exponential graph than for the linear or quadratic graph.



Click on the link to watch the video "[Comparing exponential and quadratic models]" or click on the video.



Do practice here: [Compare the growth rates of exponentials and polynomials]

Stop! **Go to Questions #8-14 about this section, then return to continue on to the next section.**

Function Features

In this section, you will identify key features of a function including intercepts, intervals where the function is increasing or decreasing, intervals where it is positive or negative, maximums and minimums, and end behavior. You will also look at features of trigonometric functions including amplitude, midline, and periodicity.

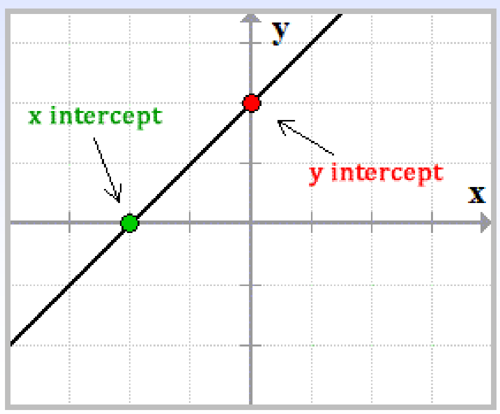
Finding x-intercepts and y-intercepts

You may recall from an earlier unit, finding *x*- and *y*- intercepts for a graph. Remember that if you have an equation in slope-intercept form ($y = mx + b$), you can easily read the *y*-intercept (b) from the equation. However, if given the graph, you should also be able to read the intercepts from the graph.

Intercepts

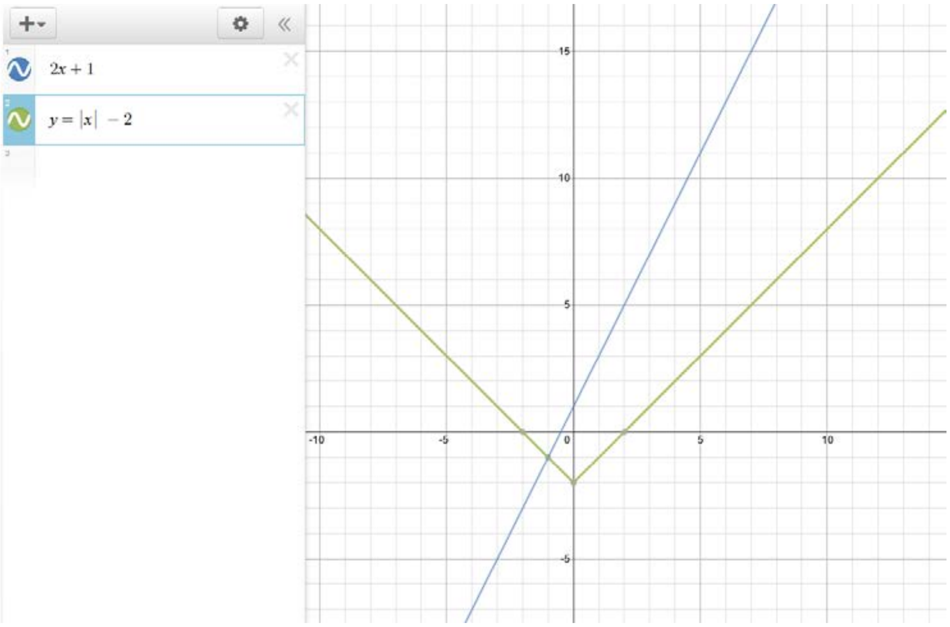
***x*-intercept(s)**: the place(s) where the graph crosses the *x*-axis

***y*-intercept(s)**: the place(s) where the graph crosses the *y*-axis

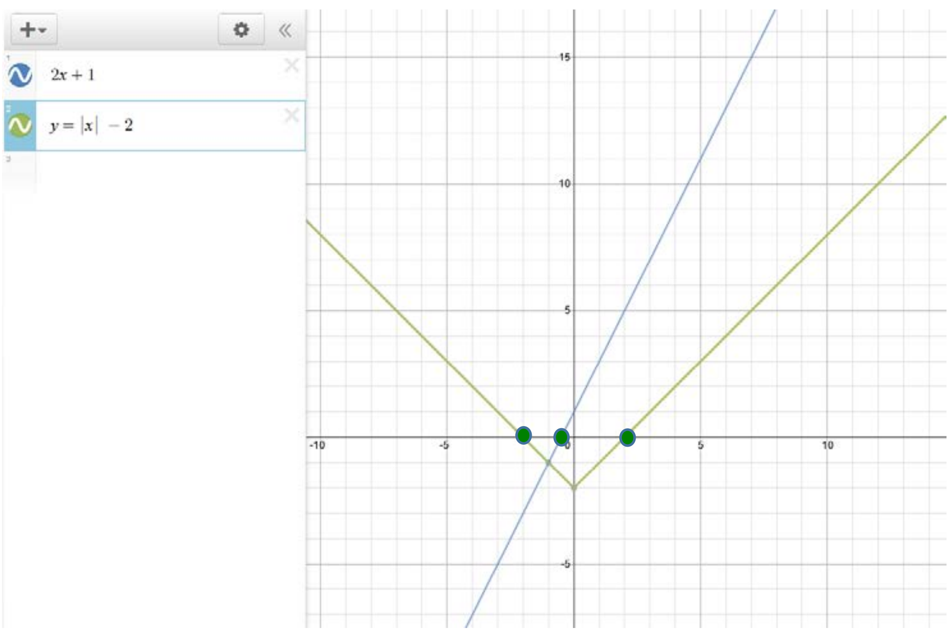


Example #1: Recall the equations and graphs from earlier in the lesson, $y = 2x + 1$ and $y = |x| - 2$.

Their graphs are below.



The blue graph is $y = 2x + 1$ and the green graph is $y = |x| – 2$. What are the *x*-intercepts for each function? Remember the *x*-intercepts are where the graph cross the *x*-axis. Because the *x*-axis is where $y = 0$, you could solve algebraically by substituting 0 for *y* and then solving for *x*. Since we are given the graph, if it crosses at nice integer values, we can read these directly from the graphs.



The *x*-intercepts have been marked with green dots. Notice the *x*-intercepts for the absolute value graph does cross at integer values and is easy to read from the graph. The *x*-intercepts are (–2, 0) and (2, 0). The linear equation (in blue) does cross between 0 and –1. To solve this, we can substitute 0 for *y* and solve algebraically.

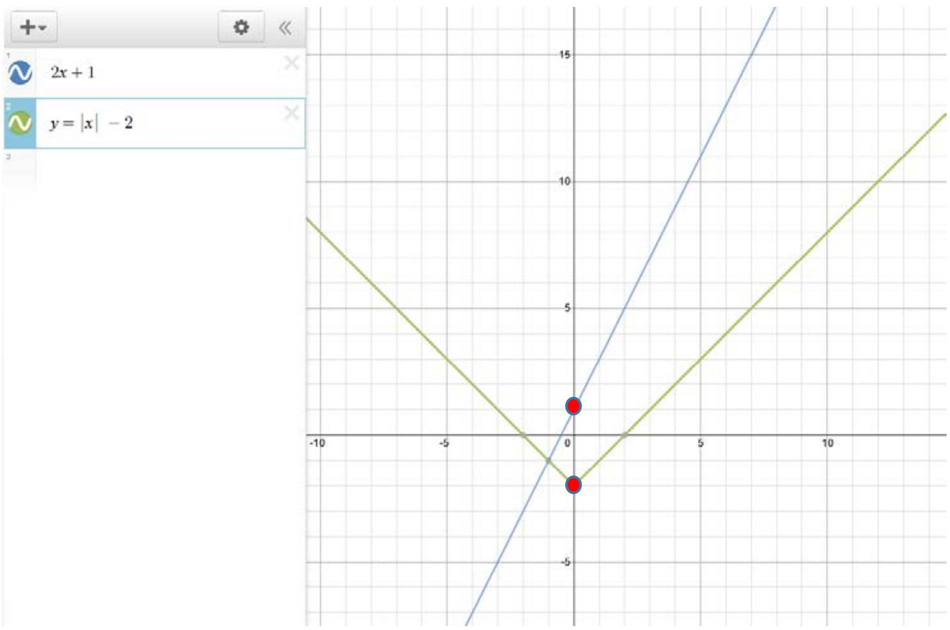
$$0 = 2x + 1$$

$-1 = 2x$ Subtract 1 from both sides.

$-\frac{1}{2} = x$ Divide both sides by 2.

$(-\frac{1}{2}, 0)$ x-intercept.

The *y*-intercepts are found by looking at where the graphs cross the *y*-axis.



The red dots above show where the two graphs intersect (or cross) the *y*-axis. You can read these points from the graph. For $y = 2x + 1$, the *y*-intercept is (0, 1). For $y = |x| - 2$, the y-intercept is (0, –2). Remember the *y*-axis is where $x = 0$, so if the point cannot be easily read from the graph, you can solve algebraically by substituting $x = 0$ into the equation and solving for *y*.

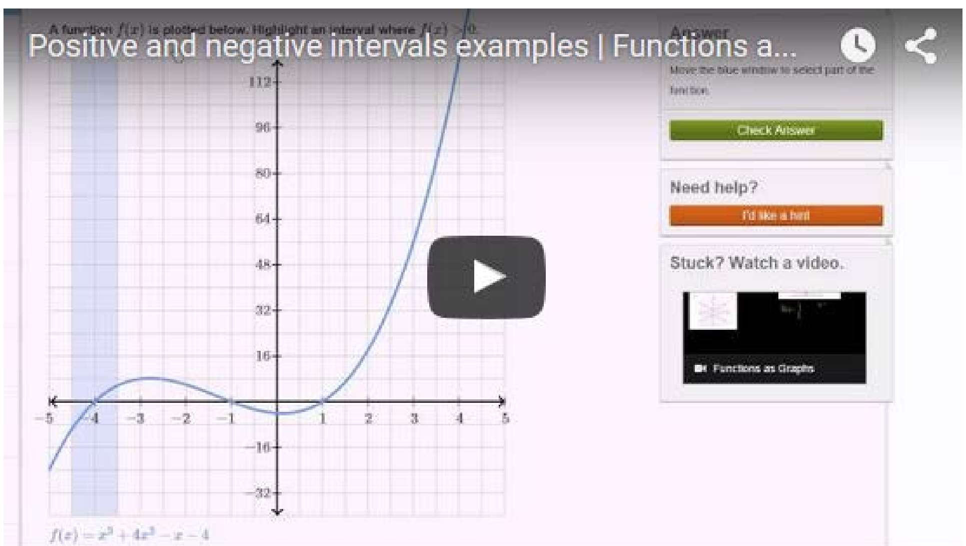
Stop! **Go to Questions #15-17 about this section, then return to continue on to the next section.**

Increasing / Decreasing and Positive / Negative Intervals

Watch this video that explains when a function is increasing or decreasing as well as intervals where the function is positive and where the function is negative.

Click on the link to watch the video "Introduction to increasing, decreasing, positive or negative intervals " or click on the video.

Click on the link to watch the video "Positive and negative intervals examples | Functions and their graphs" or click on the video.



Stop! **Go to Questions #18-22 about this section, then return to continue on to the next section.**

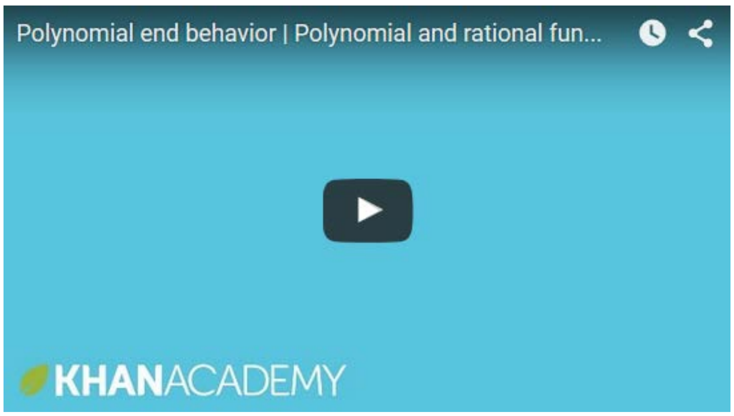
End Behavior

End behavior is how we describe what a graph does at the "ends" or how it continues on in both directions. We describe what a graph does as *x* continues to positive infinity and what the graph does as *x* continues to negative infinity.

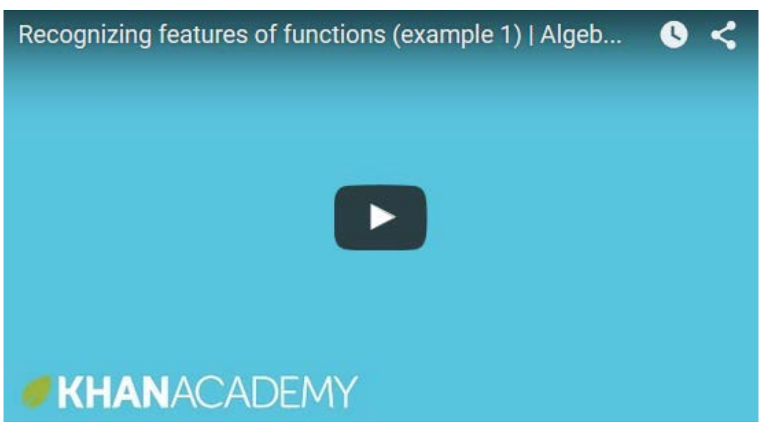
Watch these videos to see examples of end behavior.

Video 1:

Click on the link to watch the video "Polynomial end behavior | Polynomial and rational functions " or click on the video.

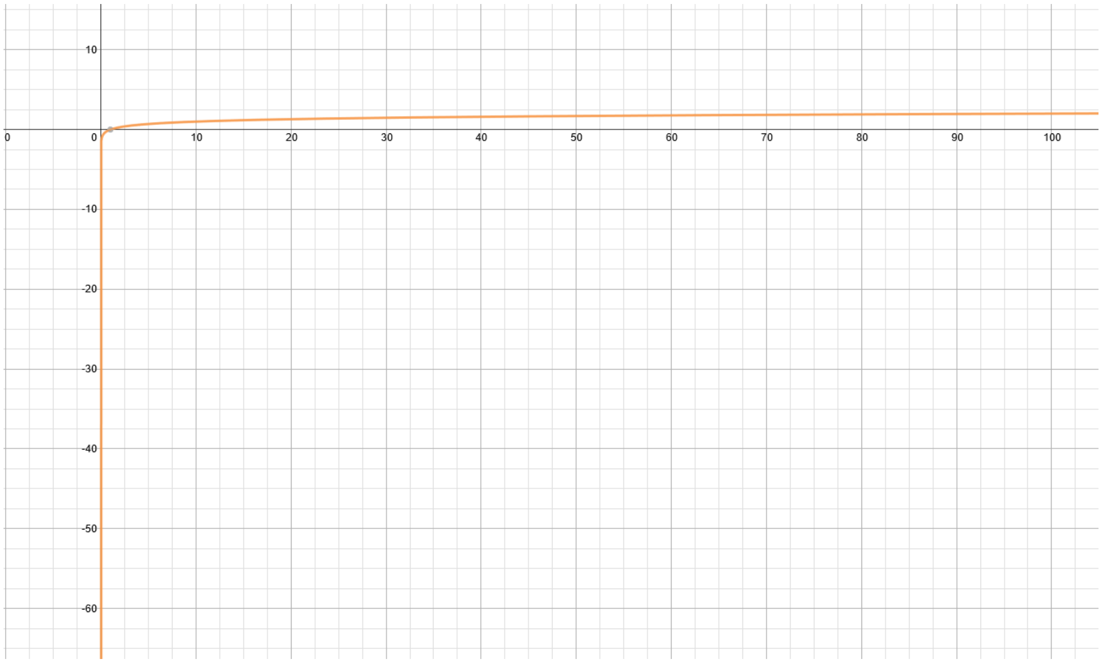


Video 2:
Click on the link to watch the video "Recognizing features of functions" or click on the video.



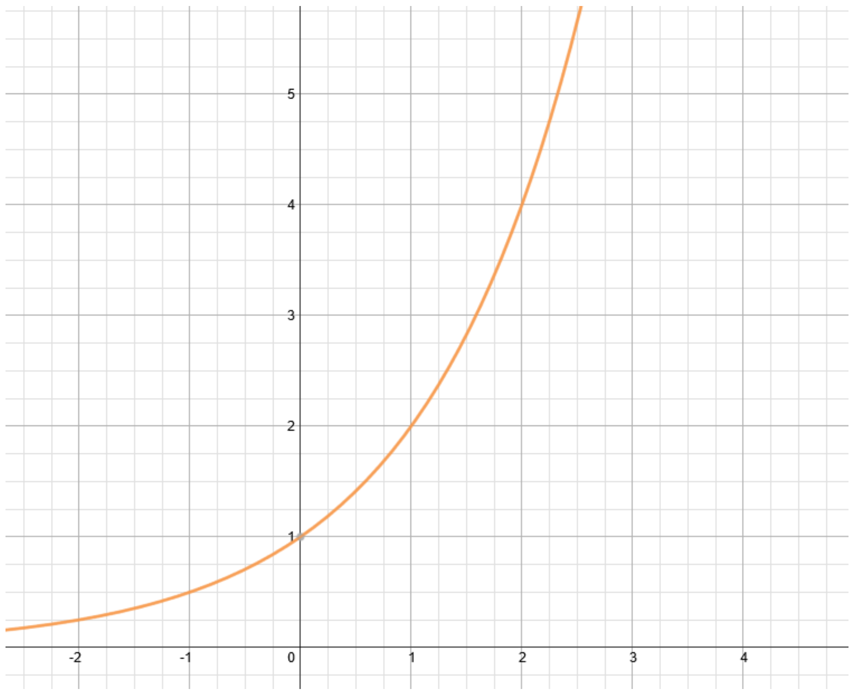
You will not need to understand how to graph the equations of higher degree for this section. You will need to be able to look at a graph and tell the end behavior by seeing the graph. You are expected to know how to graph linear and quadratic equations.

Example #1: Consider the logarithmic function $y = \log x$. The graph looks like this:



The end behavior as x approaches 0, y approaches negative infinity. As x approaches positive infinity, y approaches positive infinity. If asked, you could also identify where the function is positive as $x > 0$.

Try this! Look at the graph below and state the end behavior as x becomes more positive and as x becomes more negative. As review, also state the y-intercept by looking at the graph as well as the interval where the function is positive. This is an exponential function, $f(x) = 2^x$.



"Click here" to check your answer

1. End Behavior

As x goes to negative infinity (or becomes more negative), the y-value approaches zero.

As x goes to positive infinity (or becomes more positive), the y-value approaches positive infinity.

2. y-intercept

The y-intercept is (0, 1) and is read from the graph. Remember the y-intercept is the point where the graph crosses the y-axis.

3. Interval where the function is positive

The function is positive (meaning y is positive) from negative infinity to positive infinity. The function is always positive for all values of x.

Stop! **Go to Questions #23-30 to complete this unit.**