RADICAL EQUATIONS

Unit Overview

In this unit, you will learn how to solve radical equations that are often used to model scientific phenomena. You will also learn about the most familiar theorem in mathematics, The Pythagorean Theorem.

Radical Equations

To solve a radical equation:

1) Isolate the radical.
2) Square both sides of the equation (this will eliminate the radical).
3) Solve the equation.
4) Check your solution.

Example #1: Solve $\sqrt{x+4} = 5$

1) The radical is already isolated.

2) Square both sides of the equation.

$(\sqrt{x+4})^2 = 5^2$

$x + 4 = 25$

3) Solve the equation.

$x = 21$

4) Check your solution by substituting 21 for x in the original equation.

$\sqrt{21+4} = 5$

$\sqrt{25} = 5$ $\quad$ $5 = 5$ true

Example #2: Solve $\sqrt{3x-2}-5=0$

1) Isolate the radical by adding 5 to both sides.

$$\sqrt{3x-2}=5$$

2) Square both sides.

$$(\sqrt{3x-2})^2=5^2$$
$$3x-2=25$$

3) Solve the equation.

$$3x=27$$

$$x=9$$

4) Check your solution.

$$\sqrt{3(9)-2}-5=0$$

$$\sqrt{27-2}-5=0$$

$$\sqrt{25}-5=0$$

$$5-5 \ = \ 0$$

$0=0$ true

Example #3: Solve $6\sqrt{2z-3}=42$

1) Isolate the radical by dividing both sides by 6.

$$\frac{\cancel{6}\sqrt{2z-3}}{\cancel{6}}=\frac{42}{6}$$
$$\sqrt{2z-3}=7$$

2) Square both sides.

$$(\sqrt{2z-3})^2=7^2$$
$$2z-3=49$$

3) Solve the equation.

$$2z=52$$

$$x=26$$

4) Check your solution.

$$6\sqrt{2(26)-3}=42$$

$$6\sqrt{52-3}=42$$

$$6\sqrt{49}=42$$

$$6(7)\ =42$$

$$42=42 \text{ true}$$

Sometimes, it will be necessary to use factoring when solving a radical equation. This will occur when there is a squared term. You will then use the zero product property to solve. **Remember to check both solutions**.

Example #4: Solve $\sqrt{2x+3} = x$

1) The radical is already isolated, so go to step 2 and square both sides.

2) Square both sides.

   $(\sqrt{2x+3})^2 = x^2$

   $2x+3 = x^2$

3) Set the equation equal to zero. (**Keep the squared term positive**, it is easier to factor.)

   $0 = x^2 - 2x - 3$

   $0 = (x-3)(x+1)$

4) Set each factor equal to zero and solve.

   $x - 3 = 0$ $\quad$ $x + 1 = 0$

   $x = 3$ $\quad$ $x = -1$

5) Check both solutions in the original equation.

   $\sqrt{2(3)+3} = 3$ $\quad$ $\sqrt{2(-1)+3} = -1$

   $\sqrt{6+3} = 3$ $\quad$ $\sqrt{-2+3} = -1$

   $\sqrt{9} = 3$ $\quad$ $\sqrt{1} = -1$

   $3 = 3$ $\quad$ $1 \neq -1$

*For $x = -1$, $\sqrt{1} \neq -1$, because $\sqrt{1}$ is defined as a positive square root of 1; therefore, –1 does not satisfy the original equation and is called an **extraneous solution**.

Example #5: Solve $\sqrt{x+12} = x$

$$(\sqrt{x+12})^2 = x^2$$

$$x+12 = x^2$$

$$0 = x^2 - x - 12$$

$$0 = (x-4)(x+3)$$

$$x-4=0 \qquad x+3=0$$

$$x=4 \qquad x=-3$$

Check: $\sqrt{4+12} = 4$ $\qquad$ $\sqrt{-3+12} = -3$

$\sqrt{16} = 4$ $\qquad$ $\sqrt{9} = -3$

4 = 4 *true* $\qquad$ 3 = –3 *false*

Therefore, the only solution to this equation is *x* = 4.

Example #6: Solve $\sqrt{x^2+3x-15}=x$

$$\left(\sqrt{x^2+3x-15}\right)^2=x^2$$

$$x^2+3x-15=x^2$$.

$$\cancel{x^2}+3x-15=\cancel{x^2}$$ *Subtract x^2 from both sides.

$$3x-15=0$$

$$3x=15$$

$$x=5$$

Check: $\sqrt{5^2+3(5)-15}=5$

$$\sqrt{25+15-15}=5$$

$$\sqrt{25}=5$$

5 = 5 *true*

*Be very careful when solving a radical equation that has two terms on one side that need to be squared. Remember that a binomial squared, $(x+2)^2$, means to multiply it by itself, $(x + 2)(x + 2)$, and will require the FOIL process. Follow along with the example below:

Example #7: Solve $\sqrt{x+3}-1=x$

1) Isolate the radical by adding 1 to both sides.

   $\sqrt{x+3}=x+1$

2) Square both sides.

   $(\sqrt{x+3})^2=(x+1)^2$

   $x+3=(x+1)(x+1)$

   $x+3=x^2+2x+1$

3) Set the equation equal to 0.

   $0=x^2+x-2$

4) Factor.

   $0=(x+2)(x-1)$

5) Set each factor equal to 0 and solve.

   $x + 2 = 0$ $\qquad$ $x – 1 = 0$

   $x = –2$ $\qquad$ $x = 1$

6) Check both solutions.

   $\sqrt{-2+3}=-2+1$ $\qquad$ $\sqrt{1+3}=1+1$

   $\sqrt{1}=-1$ $\qquad$ $\sqrt{4}=2$

   $1=-1$ *false* $\qquad$ $2=2$ *true*

   The only solution for this equation is $x = 1$.

To solve an equation that contains a squared term, such as $x^2 = 36$, you will take the square root of both sides. Remember that the square root may have two solutions because the value of *x* can either be positive or negative.

Example #8: Solve $x^2 = 121$

$$\sqrt{x^2} = \sqrt{121}$$

$$x = \pm 11$$

When solving equations, like the example above, you must always simplify the radical solution.

Example #9: Solve $x^2 = 98$

$$\sqrt{x^2} = \sqrt{98}$$

$$x = \pm\sqrt{98}$$

To simplify this radical, ask yourself if 98 contains a perfect square factor. In this example, 98 contains the perfect square factor of 49.

$$x = \pm\sqrt{49} \cdot \sqrt{2}$$

$$x = \pm 7\sqrt{2}$$

Example #10: Solve $7x^2 = 336$

$$\frac{\cancel{7}x^2}{\cancel{7}} = \frac{336}{7}$$ *Isolate x^2.

$$x^2 = 48$$

$$\sqrt{x^2} = \sqrt{48}$$

$$x = \pm\sqrt{48}$$

To simplify this radical, ask yourself if 48 contains a perfect square factor. In this example, 48 contains the perfect square factor of 16.

$$x = \pm\sqrt{16}\cdot\sqrt{3}$$

$$x = \pm 4\sqrt{3}$$

Example #11: Solve $(x-4)^2 = 36$

1) Take the square root of both sides.

$$\sqrt{(x-4)^2} = \sqrt{36}$$

$$x - 4 = \pm 6$$

At this point, you must realize that there are two possible solutions to the equation.

$x - 4 = +6$ and $x - 4 = -6$

$x = 10$ and $x = -2$

Check both solutions:

$(10-4)^2 = 36$	$(-2-4)^2 = 36$
$(6)^2 = 36$	$(-6)^2 = 36$
$36 = 36$	$36 = 36$

Both solutions are true; therefore, $x = 10$ and $x = -2$.

Stop! **Go to Questions #1-13 about this section, then return to continue on to the next section.**

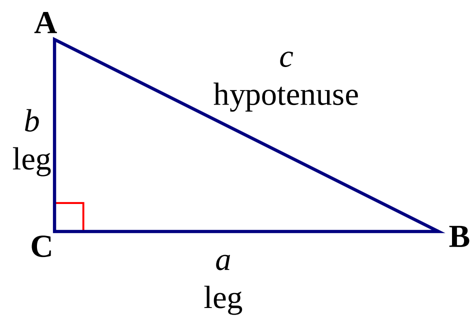
The Pythagorean Theorem

The Pythagorean Theorem is used to find the lengths of the sides of right triangles.

The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

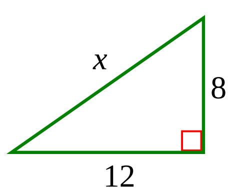
a and *b* are the lengths of the legs and *c* is the length of the hypotenuse (the side opposite the right angle).



To use the Pythagorean Theorem in finding the missing measures, follow the examples below:

Example 1: Find the length of side x (the hypotenuse of the right triangle) .

What is the value of x?



$$a^2 + b^2 = c^2$$

$$12^2 + 8^2 = x^2$$

$$144 + 64 = x^2$$

$$208 = x^2$$

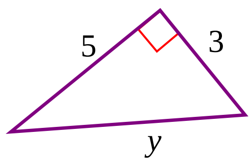
$$\sqrt{208} = \sqrt{x^2}$$

$$\sqrt{16} \cdot \sqrt{13} = x$$

$$4\sqrt{13} = x$$

Example 2: Find the length of side y (the hypotenuse of the right triangle).

What is the value of y?



$$a^2 + b^2 = c^2$$

$$5^2 + 3^2 = y^2$$

$$25 + 9 = y^2$$

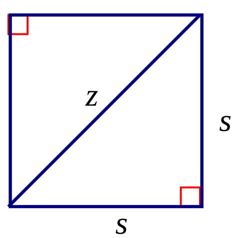
$$34 = y^2$$

$$\sqrt{34} = \sqrt{y^2}$$

$$\sqrt{34} = y$$

Example 3: Find the length of side z.

What is the value of z?



In this example, you should notice that the diagonal of the square is the hypotenuse of the right triangles formed.

$$s^2 + s^2 = z^2$$

$$2s^2 = z^2$$

$$\sqrt{2s^2} = \sqrt{z^2}$$

$$s\sqrt{2} = z$$

The last example derives the formula for the length of a diagonal of a square with sides of length *s*.

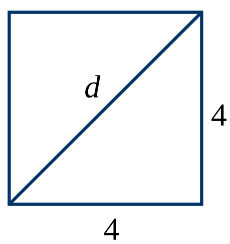
Diagonal of a Square

$$d = s\sqrt{2}$$

s is the length of the side of the square.

Let's take this formula one step further and determine the length of the diagonal of a square whose sides measure 4 inches.

Example 4: Find the length of side *d*.



Substitute 4 for the length of the side to determine the measure of the diagonal.

$s = 4$

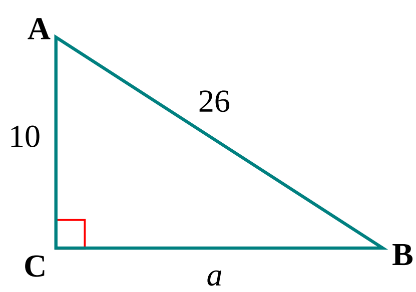
$d = s\sqrt{2}$ *Substitute.

$d = 4\sqrt{2}$

The length of the diagonal in this square is $4\sqrt{2}$.

The Pythagorean Theorem can also be used to find the measure of the **legs** of a right triangle.

Example 5: Find the length of side *a* (a leg of the right triangle).



$a^2 + b^2 = c^2$

$a^2 + 10^2 = 26^2$

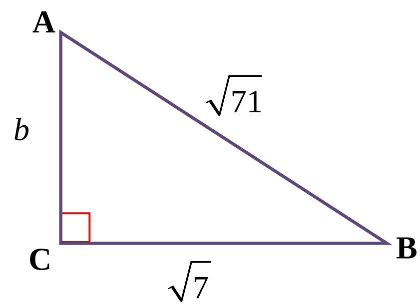
$a^2 + 100 = 676$

$a^2 = 576$

$\sqrt{a^2} = \sqrt{576}$

$a = 24$

Example 6: Find the length of side *b* (a leg of the right triangle).



$$a^2 + b^2 = c^2$$

$$\left(\sqrt{7}\right)^2 + b^2 = \left(\sqrt{71}\right)^2$$

$$7 + b^2 = 71$$

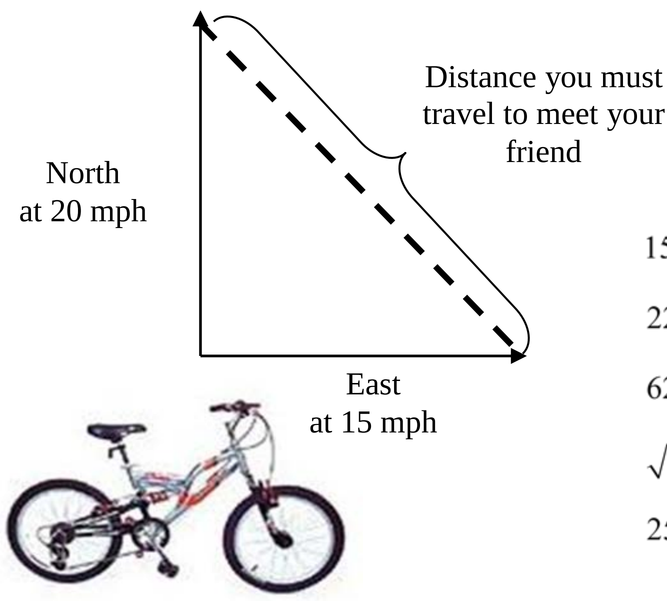
$$b^2 = 64$$

$$\sqrt{b^2} = \sqrt{64}$$

$$b = 8$$

Application

Example 7: You are traveling on your bicycle due north at 20 miles per hour. Your friend, who left at the same time traveling due east at 15 miles per hour, called you an hour later on the cell phone to tell you he had a flat tire. How far must you travel to meet your friend? (Assume you can travel directly to your friend's location.)



$$15^2 + 20^2 = d^2$$

$$225 + 400 = d^2$$

$$625 = d^2$$

$$\sqrt{625} = \sqrt{d^2}$$

$$25 = d$$

The distance you must travel to reach your friend is 25 miles.

Stop! **Go to Questions #14-32 to complete this unit.**