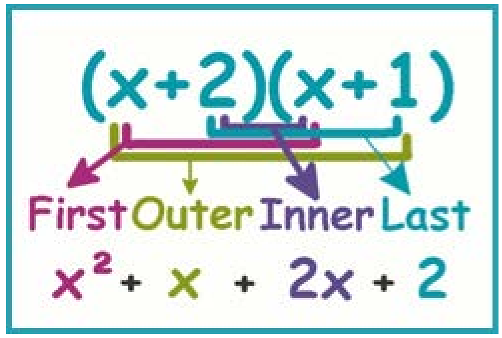
POLYNOMIALS



Unit Overview

In this unit, you will study polynomials in a more detailed manner than you did in previous units. Polynomials are used extensively in geometry and are useful for representing area, volume, gravity, and consumer problems.

Types of Polynomials

Let's review the different types of polynomials that were studied in a previous unit.

Polynomial: an expression that is either a monomial or a sum of monomials

Monomial: an expression that can be written as a number, variable, or product of a number and variable(s)

5 $\quad$ y $\quad$ $-35xy^2$

Binomial: a polynomial that has two terms

$3x+4$ $\quad$ x^2+3y^2 $\quad$ $-5-xy$

Trinomial: a polynomial that has three terms

$7x^2+5x+2$ $\quad$ $xy+yz-zw$

Remember that the degree of a polynomial is the highest degree of any of its terms (this is found by adding all exponents of each term). Finding the degree of a polynomial will determine the number of zeros (places it crosses the *x*-axis) that it has.

Find the degree of each polynomial.

Example #1: $4x^2-3x+7$

First find the degree of each term:

$4x^2$: degree = 2
$-3x$: degree = 1
7: degree = 0 (no variables)

The highest degree is 2 so the degree of the polynomial is 2.

(2) (1) (0) $\quad$ The degree is 2.

Example #2: $x + y + xy^2 + x^2y^3$

First find the degree of each term:

x: degree = 1
y: degree = 1
xy^2: degree = 3 (add the exponents 1 + 2 = 3)
x^2y^3: degree = 5 (add the exponents 2 + 3 = 5)

The highest degree is 5 so the degree of this polynomial is 5.

(1) (1) (3) (5) The degree is 5.

A polynomial is generally written in descending order of the exponents of its terms starting with the term that has the highest exponent. This is called **standard form**.

Example #3: $-2x^5 + 4x^3 + 7x - 9$ is in standard form. (*The numbers –2, 4, and 7 are called the **coefficients** of the polynomial.)

The degree of a polynomial also describes the type of polynomial it is. A polynomial of degree one is a linear polynomial and represents a line. A polynomial of degree two is a quadratic polynomial and represents a parabola which we will discuss in more detail in future units.

Example #4: $3 + x$ is linear because the degree is one.

Example #5: $4x^2 + x - 6$ is quadratic because the degree is two.

Try this: Find the degree of $2x + 7$. Is it linear or quadratic?

"Click here" to check the answer.

Answer: degree = 1, linear

Try this: Write the polynomial, $-2x^5 + 7 + 2x + 3x^2 - 6x^3$, in standard form.

"Click here" to check the answer.

Answer: $-2x^5 - 6x^3 + 3x^2 + 2x + 7$

Stop! **Go to Questions #1-13 about this section, then return to continue on to the next section.**

Multiplying Polynomials

When multiplying polynomials, you will use the distributive property along with the exponent rules you learned in previous units.

Monomial times a Polynomial

Example #1: $2x(-3x + 4)$ — Multiply $2x$ by each term in parenthesis.

$(2x)(-3x) + (2x)(4)$ — Distribute

$-6x^2 + 8x$

Example #2: $-4x^2(2x^2 - 5x + 3)$ — *Careful* Watch your signs.

$(-4x^2 \cdot 2x^2) + [-4x^2 \cdot (-5x)] + (-4x^2 \cdot 3)$ — Distribute

$-8x^4 + 20x^3 - 12x^2$

Example #3: $(3k)(-8k^2) + (-7k^2)(-4k)$

$-24k^3 \quad + \quad 28k^3$ — Product of Powers

$4k^3$ — Collect Like Terms

Example #4: $b(4x^2 + 3x) - x(3b + b^2)$

$4bx^2 + 3bx - 3bx - b^2x$ — Distribute

$4bx^2 \cancel{+3bx} \cancel{-3bx} - b^2x$ — Collect Like Terms

$4bx^2 - b^2x$ — Simplify

Try this: $a(7x - 4) + 2(-3a + x)$

"Click here" to check the answer.

Answer: $7ax - 4a - 6a + 2x$
$7ax - 10a + 2x$

Stop! **Go to Questions #14-17 about this section, then return to continue on to the next section.**

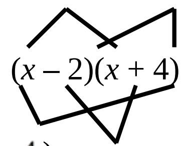
Multiplying Binomials

The distributive property can also be used to multiply two binomials; however, there is a popular mnemonic (memory) method for multiplying binomials called **FOIL**.

- Multiplying the **F**irst terms of each binomial.
- Multiplying the **O**utside terms of the binomials.
- Multiplying the **I**nside terms of the binomials.
- Multiplying the **L**ast terms of each binomial.

First ($x \cdot x$) Last ($-2 \cdot 4$)

Example #5: $(x - 2)(x + 4)$



Outside ($x \cdot 4$) Inside ($-2 \cdot x$)

F		**O**		**I**		**L**
x^2	$+$	$4x$	$-$	$2x$	$-$	8

$x^2 + 2x - 8$

Example #6: $(2x + 3)(6x - 1)$

$(2x \cdot 6x) + (2x \cdot -1) + (3 \cdot 6x) + (3 \cdot -1)$ FOIL

$12x^2 - 2x + 18x - 3$ Multiply

$12x^2 + 16x - 3$ Combine like terms

Try this: $(3x - 2)(-4x + 8)$

"Click here" to check the answer.

Answer: $-12x^2 + 24x + 8x - 16$

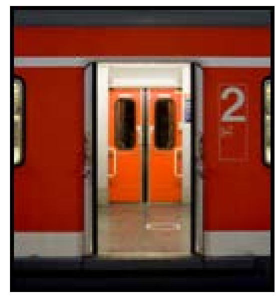
$-12x^2 + 32x - 16$

Example #7: $(5a-2b)(7a-3b)$

$(5a \cdot 7a)+(5a \cdot -3b)+(-2b \cdot 7a)+(-2b \cdot -3b)$	FOIL
$35a^2 - 15ab - 14ab + 6b^2$	Multiply
$35a^2 - 29ab + 6b^2$	Collect like terms

Are Polynomials Closed?

The closure property states that when two elements are combined with a given operaion, the result is another element of that same set. For example, when you multiply 7 × 3 (which are both whole numbers), the result 21 is also a whole number; hence, the whole numbers are closed under multiplication.



So with polynomials, when you multiply two polynomials, will the result still be a polynomial? Before answering, review the examples in the content section titled "Multiplying Polynomials" and "Multiplying Binomials" to see if each answer was also a polynomial.

Are polynomials closed under multiplication?

"Click here" to check the answer.

Answer: yes!

Special Products – Difference of Squares

If two binomials have the same terms and only differ by the signs, the following will result.

Example #8: $(x + 2)(x – 2)$ -both contain x's and both contain 2's
-one (+) and one (–)

$$x^2 - 2x + 2x - 4$$

$$x^2 \cancel{-2x} \cancel{+2x} - 4$$

$$x^2 - 4$$

The result is called a **difference of squares** and will be discussed in the next unit.

Special Products – Squaring a Binomial

When squaring a binomial, (a) square the first term, (b) double the product of the first and last term, and (c) square the last term.

Example #9: $(5x - 3)^2$

square the first term, double the product of first and last term, square the last term

$$(5x)^2 + 2(5x)(-3) + (-3)^2 \quad .$$

$$25x^2 - 30x + 9$$

*Note: This problem may also be completed by using the foil method.

$$\begin{aligned}(5x-3)^2 &= (5x-3)(5x-3)\\ &= 25x^2 - 15x - 15x + 9\\ &= 25x^2 - 30x + 9\end{aligned}$$

Try this: Using difference of squares, find $(x + 6)(x – 6)$

"Click here" to check the answer.

Answer: $x^2 – 36$

Try this: Using squaring a binomial, find $(7x + 4)^2$

"Click here" to check the answer.

Answer: $49x^2 + 56x + 16$

Stop! **Go to Questions #18-21 about this section, then return to continue on to the next section.**

Polynomial Functions

Three examples of polynomial functions used in the real world are volume, surface area, and area. A polynomial function is a function that consists of a monomial, or the sum or difference of two or more monomials.

Example #1: Rachel, the production manager of Krispy Krackers Company wants to change the size of the Krispy Krackers snack box. The box currently has a base with length of 15 centimeters, a width of 8 centimeters, and a height of 25 centimeters.

Find the volume of the box.

$V = lwh$

$V = 15 \cdot 8 \cdot 25$

$V = 3000$ cubic centimeters

**Remember that because you are finding volume, the appropriate unit is "cubic centimeters" also written cu. cm or cm3. Using cm by itself refers to a linear measurement.*

Rachel would like to reduce the volume of the box by 15%. What would the new volume be? (You can use the exponential decay formula for this).

$V = P\,(1 + r)^{n}$

$V = 3000(1 - .15)^{1}$ (Use a –.15 because it is a reduction.)

$V = 3000(.85)$

$V = 2550$ cubic centimeters is the new volume if reduced by 15%

To achieve this 15% reduction, Rachel also decides that she would like to reduce the size of the base of the box; keep the same height and make the length twice the size of the width.

The following polynomial function can be used to represent this situation where x is the width:

$V = lwh$

Height = 25 (kept same)
Width = x (use a variable)
Length = $2x$ (length will be twice the width)

$V = 2x \cdot x \cdot 25$

$V = 2x^2 \cdot 25$

$V = 50x^2$

If Rachel decides to reduce the volume by 15% and change the dimensions of the base, what would the new dimensions be (rounded to the nearest hundredth of a centimeter)?

-reduced volume is 2550

-new dimensions $2x \cdot x \cdot 25$

$V = lwh$

$2550 = 2x \cdot x \cdot 25$

$2550 = 50x^2$ (simplify)

$51 = x^2$ (divide both sides by 50)

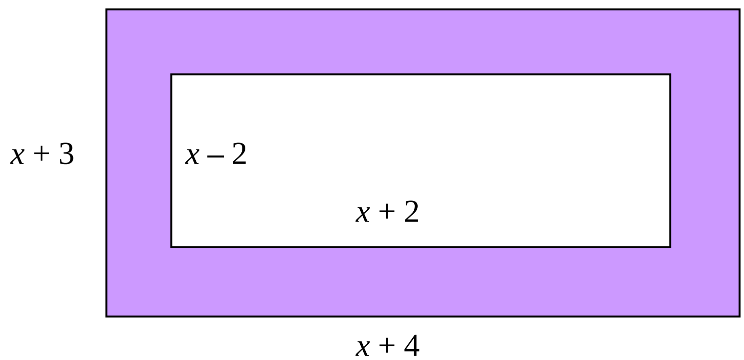
$7.14 = x$ To solve for x, take the square root of both sides. This concept will be discussed in more depth in a later lesson.

The new width of the base (x) will be 7.14 centimeters and the new length ($2x$) will be 14.28 centimeters.

Remember, we were told to round to the nearest one hundredth of a centimeter (hundredths is two decimal places). We are finding length so our unit is cm not cm^3 as when finding volume.

Applications

Example #1: Creating a mat to frame a rectangular picture requires precise cutting. Refer to the diagram below to determine the measurements of the original mat if the area of the mat border (shaded area) is 156 square inches. The original full mat is represented by the full outer rectangle. The picture will go in the white area and the mat border is represented by the purple area.



1) Find the areas of the outer and inner rectangles (remember that area is length *x* width).

   Length of outer rectangle = $(x + 4)$
   Width of outer rectangle = $(x + 3)$
   Area of mat border = area of outer rectangle – area of inner rectangle

area of outer	area of inner
$(x + 4)(x + 3)$	$(x + 2)(x - 2)$

2)

area of outer	–	area of inner	=	156
$(x + 4)(x + 3)$	–	$(x + 2)(x - 2)$	=	156
$x^2 + 4x + 3x + 12$	–	$(x^2 + 2x - 2x - 4)$	=	156
$x^2 + 7x + 12$	–	$(x^2 - 4)$	=	156

*Note the parentheses around this quantity. You need to remember to subtract each term.

$x^2+7x+12-x^2+4=156$ (combine like terms)

$7x+16=156$ (subtract 16 from each side)

$7x=140$ (divide both sides by 7)

$x=20$

3) The length of the original rectangle was $x + 4$. Substitute 20 for x to find the original length.

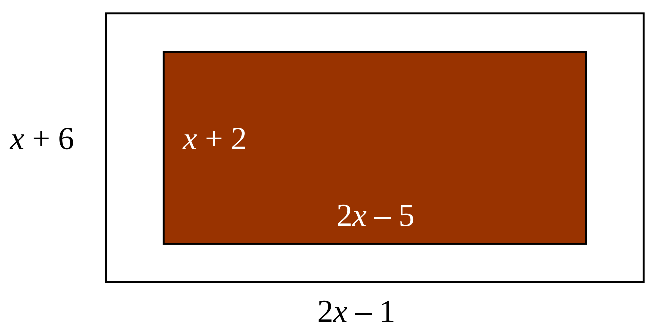
$20 + 4 = 24$

4) The width of the original rectangle was $x + 3$. Substitute 20 for x to find the original width.

$20 + 3 = 23$

5) The original dimensions of the rectangle were 24” × 23”.

Example #2: The McGwires want to put outdoor carpet on their front porch, but they also want to leave 148 square feet of cement showing around the carpeting. Use the diagram below to determine how many square feet of carpet they need.



**Brown represents the carpet while the white surrounding the brown represents the cement.

1) Find the area of the porch and the carpeting.

porch	carpet
$(x + 6)(2x - 1)$	$(2x - 5)(x + 2)$
$2x^2-x+12x-6$	$2x^2+4x-5x-10$
$2x^2+11x-6$	$2x^2-x-10$

2) Subtract the two areas and set the difference equal to 148 because this is the area of the cement showing.

$$2x^2 + 11x - 6 - (2x^2 - x - 10) = 148$$

$$2x^2 + 11x - 6 - 2x^2 + x + 10 = 148$$

$$12x + 4 = 148$$

$$12x = 144$$

$$x = 12$$

Remember, you are subtracting all of the carpeted area.

3) The length of the carpet is $2x - 5$, so substitute 12 for x.

2(12) – 5

24 – 5 = 19

4) The width of the carpet is $x + 2$, so substitute 12 for x.

12 + 2 = 14

5) The dimensions of the carpet are 19 × 14, so the total amount of carpet the McGwires need is 266 square feet.

Remember that feet represents a linear measurement and square feet represents area.

Stop! **Go to Questions #22-34 to complete this unit.**