# INEQUALITIES IN CONTEXT

## Unit Overview

In unit 12, you learned to graph a linear inequality. In this unit, we will expand upon that knowledge and put that in context (word problems). You will also learn to identify constraints on the domain and range based on the information in the problem.

## Linear Inequalities in One Variable

*To review inequalities, click here. (Unit 12, Linear Inequalities)*

***Example #1*:** Jarod wants to purchase a new jacket. He can buy it online for 20% off, but he will have to pay shipping which is 7.95. He can purchase the same jacket at a local store at full price but not have to pay shipping. What price makes the jacket less expensive to buy at the store?

Let $c$ = cost of the jacket at full price (local store)

Take the information from the problem, then the online price is the full price ($c$) minus the 20% discount ($0.2c$) plus the cost of shipping (7.95). This gives the online price of $c - 0.2c + 7.95$.

We want to know when the jacket is less expensive at the store, so we want to know when store price < online price. So fill in the 2 expressions.

$c < c - 0.2c + 7.95$

Solve.

1st: Combine like terms on the right side.

$c < (1 - 0.2)c + 7.95$

$c < 0.8c + 7.95$

2nd: Collect the c terms on the left side by subtracting 0.8 from both sides.

$c - 0.8c < 7.95$

$0.2c < 7.95$

3rd: Divide both sides by 0.2

$c < 39.75$

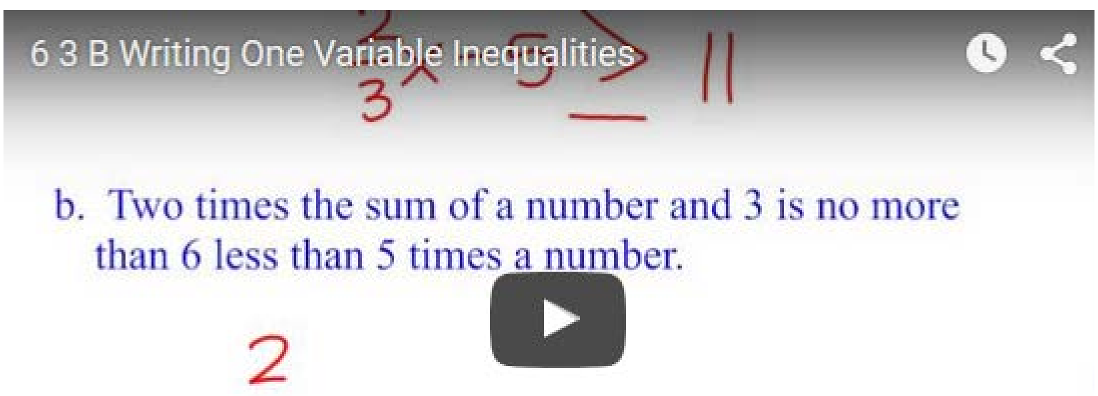
What does this tell us? This tells us that when the jacket is less than $39.75, the jacket is less expensive to buy at the store. However, if the price is more than $39.75, it would be cheaper to buy the jacket online using the discount but pay the shipping.

***Example #2*:** Jayla gets a new job making $*d* per hour (unknown). She wants to earn at least $400 per week. How many hours will she need to work? In this problem, our goal is to learn to set up the problem correctly just using basic information. We will not solve for a final, numerical value.

Let's take a step back and consider this: What if Jayla made $10 per hour? How much would she make per week? Well, that depends on how many hours she works. Let $x$ = number of hours worked. Then she makes $10 × $x$ hours or $10x$. Since she wants to make at least $400, then we could set up an inequality that says $10x \geq 400$. Because she wants to make "at least" $400, this means $400 or more. However, we haven't been told how much Jayla actually makes. We can only use the variable $d$ as given. So, for this problem, we must write $dx \geq 400$.

Why would we need to know how to do such a problem? Especially one with such little information? Well, consider this. You are out of work. You need to find a job. However, before accepting a job, you must know that you will earn enough money to pay your bills. In this example, Jayla needed to earn at least $400 per week. If she knew that $dx \geq 400$ and then learned that a job was available that paid minimum wage at $7.25, she could substitute that in for $d$ and write $7.25x \geq 400$. After dividing both sides by 7.25, she learns that she must work over 55 hours to earn $400. Knowing that is impossible because most jobs don't allow you to work more than 40 hours, she knows that this is not an acceptable job.

***Example #3:*** Click on the link to watch the video "Writing One Variable Inequalities" or click on the video.



After watching this video, we can solve these inequalities to find what the number could be.

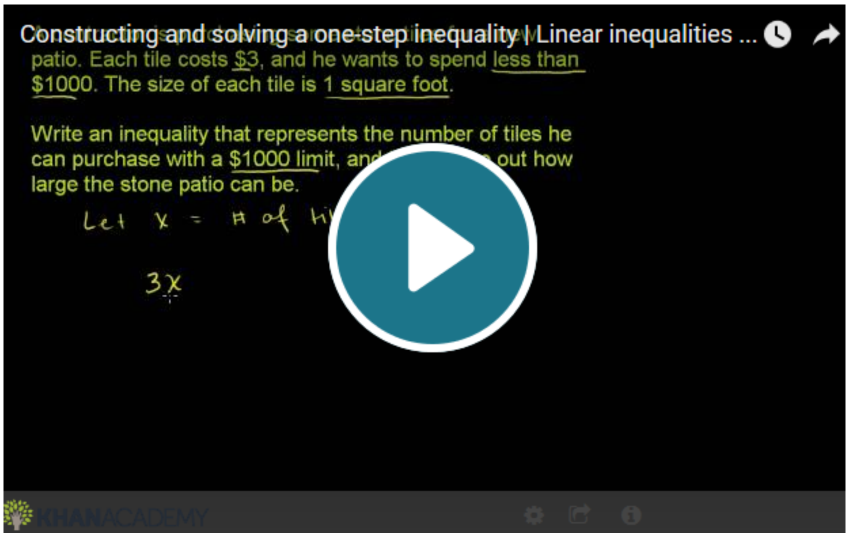
The first example gave $2/3(x) - 5 \geq 11$

Add 5 to both sides gives $2/3(x) \geq 16$

Multiply both sides by 3/2 to get $x \geq 16\ (3/2)$

$x \geq 24$ This means that $x$ must be at least 24 (24 or greater).

***Example #4:*** Click on the link to watch the video "[Constructing and solving a one-step inequality]" or click on the video.




Now, let's take this example further. If $x < 333\ 1/3$ tiles, what is the most number of tiles he can purchase? He cannot buy a partial tile, so 333 is the most he can buy. When graphed, the graph should not exceed past the point where $x = 333$. What is the least number of tiles he could purchase? Well, zero, of course. But then, he wouldn't get a patio!

What is our domain and range for this problem? The domain is represented by the number of tiles. How many tiles can the contractor purchase? He can buy anywhere from 0 to 333 tiles. We write this as

$0 \leq x \leq 333.$

The range is represented by how much the contractor is spending. He will spend anywhere from \$0 to how much 333 tiles cost. Since $333 \times 3 = 999$ tiles, 999 is the upper limit of the range. We write this as

$0 \leq y \leq 999.$

***Stop!*** **Go to Questions #1-10 about this section, then return to continue on to the next section.**

## Linear Inequalities in Two Variables

***Example #1*:** Jasmine works at a bakery. She can make and bake a cake in 2 hours. She can make and bake 2 dozen cookies in 1.5 hours. If she is not allowed to work more than 30 hours per week, how many cakes and cookies will she make?

Let's start by defining our variables.

Let $x$ = # of cakes made

Let $y$ = # of cookies made

Now, write the inequality that fits this problem. Since it takes 2 hours to make a cake, $2x$ represents the time spent making cakes. Likewise, $1.5y$ represents the time spent making cookies. Therefore, the total time is $2x + 1.5y$. Since Jasmine is not allowed to work more than 30 hours, then her time must be 30 hours or less. Since it could be 30 hours, we could use =, but it could also be less than 30 hours so we use $\leq$.

The inequality looks like this: $2x + 1.5y \leq 30$.

To graph this, we will make a table of values. Remember that this is a linear inequality, so we follow the same process as graphing a line. However, first, let's consider our constraints (restrictions) for our variables. What can $x$ be? Are all values possible? What is the least number of cakes that can be made? Zero. She can make no cakes but we cannot make negative cakes. What about cookies? Same, zero. We can make zero cookies, but cannot make negative cookies. So, when making our table, we will not use negative values for $x$ or $y$. Let's start with putting in 0 – 3 for $x$. We will need to solve for $y$.

| $x$ | $y$ |
|---|---|
| **0** | |
| **1** | |
| **2** | |
| **3** | |

To solve for $y$, we will solve the equation $2x + 1.5y = 30$. For now, we will put aside the $\leq$ and find what values give us the full 30 hours.

$x = 0$ $\quad 2(0) + 1.5y = 30$

$1.5y = 30$

$y = 30/1.5 = 20$

$x = 1$ $2(1) + 1.5y = 30$

$2 + 1.5y = 30$

$1.5y = 28$

$y = 28/1.5 = 18.67$

$x = 2$ $2(2) + 1.5y = 30$

$4 + 1.5y = 30$

$1.5y = 26$

$y = 17.3$

$x = 3$ $2(3) + 1.5y = 30$

$6 + 1.5y = 24$

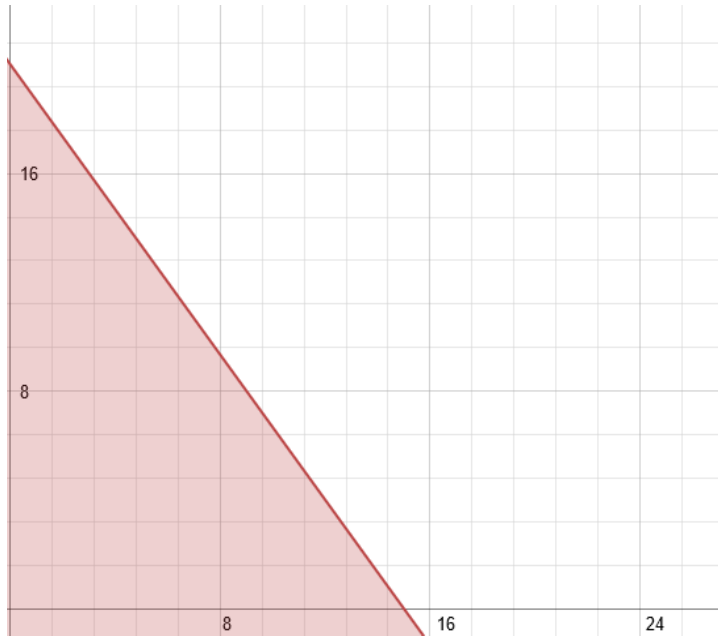
$1.5y = 18$

$y = 18/1.5 = 16$

Now, fill in the table with the values you have found.

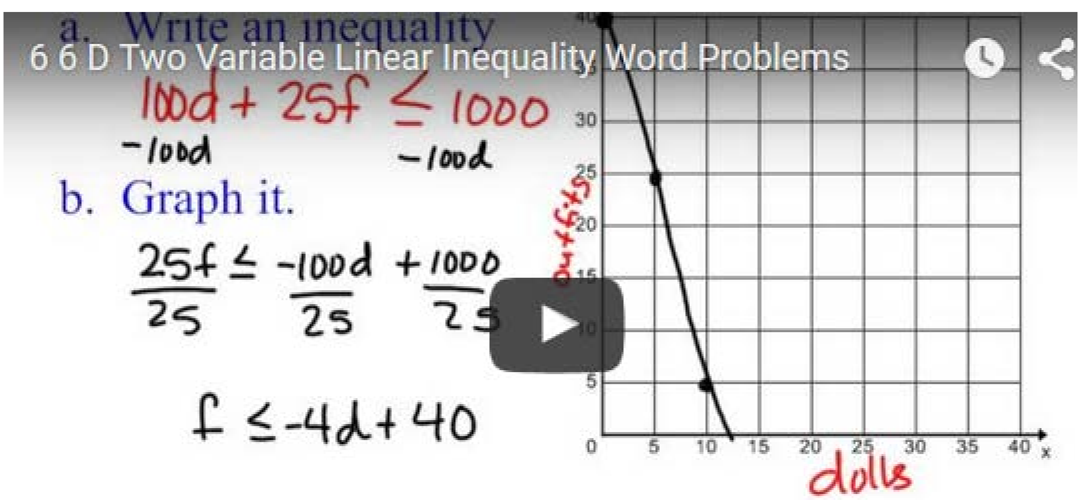
| *x* | *y* |
|---|---|
| **0** | 20 |
| **1** | 18.67 |
| **2** | 17.3 |
| **3** | 16 |

Now, we will graph these points. This will be a solid line because of the = part and we shade below the line due to the < portion. The shading shows all other possible values. Our table gives us the line where the points always give us a total of 30 hours. But the shading represents all of the possibilities when Jasmine works less than 30 hours. Notice that the graph also shows us the *x* value when *y* is zero (# cakes made if no cookies are made.)




Notice that the graph should end at the top at (0, 20) because *x* cannot be less than zero. Likewise, because *y* is not less than zero, the furthest most point on the right is (15, 0).

***Example #2:*** Click on the link to watch the video "[Two Variable Linear Inequality Word Problems]()" or click on the video.



In this example, what is the domain and range? The domain is found on the $x$-axis. This is the number of dolls that can be purchased. Because the most number of dolls that can be purchased is the $x$-intercept, this can be found by substituting 0 in for f (no outfits purchased means the most dolls that can be purchased).

$100d + 25(0) \le 1000$

$100d \le 1000$

$d \le 10$

The domain for this problem is anywhere from 0 to 10 dolls. This is written as $0 \le d \le 10$.

The range represents the number of outfits purchased. Again, the maximum is given on the $y$-axis, so we can find this by substituting in 0 for $d$ (no dolls purchased means the most number of outfits that can be purchased).

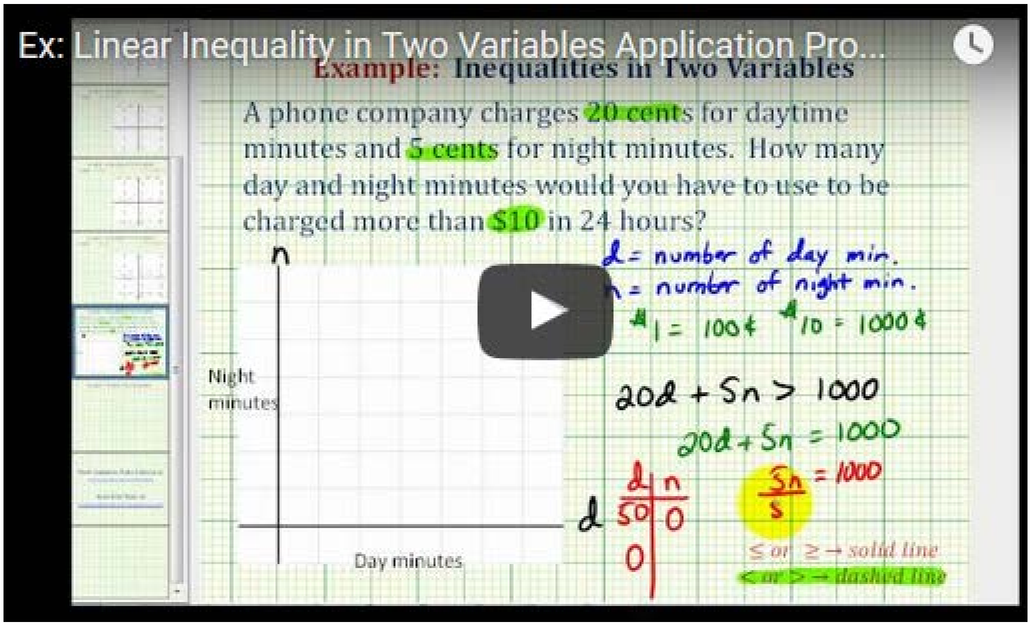
$10(0) + 25f \le 1000$

$25f \le 1000$

$f \le 40$

Of course, this was already found when the intercepts were found to make the graph. The range is therefore, $0 \le f \le 40$.

***Example #3:*** Click on the link to watch the video " Linear Inequality in Two Variables Application Problem (Phone Cost: Day and Night)" or click on the video.



Now that you have watched this example, find the domain and range of this inequality. Since the horizontal access was *d*, this will be our domain. What are the lowest and highest values for this inequality? Notice the graph starts at the *d*-intercept (normally you would think of this as your *x*-intercept but the variable *d* has been used in this problem) of 50, therefore, *x* must be 50 or more. Remember that the problem asked about being charged more than $10, so low values would not make sense. Therefore, our domain is $d \geq 50$. The night minutes are graphed on the *y*-axis but are labeled as n for this problem. This is our range. Notice that the graph starts at 200 night minutes so our range is $n \geq 200$.

***Stop!*** **Go to Questions #11-30 to complete this unit.**