TWO STEP EQUATIONS

Remember: Equations are balanced on the equals sign, so what is done to one side of the equation must also be done to the other side of the equation. We will look at equations that require two steps to determine the solution.

INEQUALITIES

An inequality is a math sentence that expresses quantities that are greater than, less than, greater than or equal, or less than or equal to a given number or mathematical expression.

The rules for solving equations are similar to the rules for solving inequalities. To solve addition and subtraction inequalities, keep both sides of the inequality sign in balance by undoing what's been done to the unknown. Isolate the unknown on one side of the inequality.

To solve multiplication and division inequalities, keep both sides of the inequality sign in balance by undoing what's been done to the unknown. Isolate the unknown on one side of the inequality.

Something New

When multiplying or dividing an inequality by a number with a negative sign, "reverse the direction" of the inequality sign.

Two-Step Equations

Rule for equations: Equations are balanced on the equals sign, so what is done to one side of the equation must also be done to the other side of the equation.

Example 1: Solve $7x + 9 = 30$

We need to **isolate** x on the left side of the equals sign. Reverse the order of operations. On the left side subtract 9 (*Step 1*), then divide by 7 (*Step 2*). Keep the equation balanced by doing the same to the right side.

Step 1

$$\begin{array}{rcl} 7x + 9 &=& 30 \\ -\ 9 & & -9 \\ \hline 7x &=& 21 \end{array}$$

Step 2

$$\begin{array}{rcl} 7x \div 7 &=& 21 \div 7 \\ x &=& 3 \end{array}$$

Check:

- Substitute 3 in for x in the original equation.
- Does 7 times 3 plus 9 = 30?
- Yes

Example 2: Solve –32 = 9*p* – 5

We need to **isolate** *p* on the right side of the equals sign. Reverse the order of operations. On the right side add 5 (*Step 1*), then divide by 9 (*Step 2*). Keep the equation balanced by doing the same to the left side.

Step 1

$$\begin{array}{rcl} -32 &=& 9p - 5 \\ +5 & & +5 \\ \hline -27 &=& 9p \end{array}$$

Step 2

$$-27 \div 9 = 9p \div 9$$
$$-3 = p$$

Check:

- Substitute –3 in for p in the original equation.
- Does 9 times –3 minus 5 = –32?
- Yes

Inequalities

An **inequality** is a math sentence that expresses quantities that are **greater than**, **less than**, **greater than or equal**, or **less than or equal** to a given number or mathematical expression.

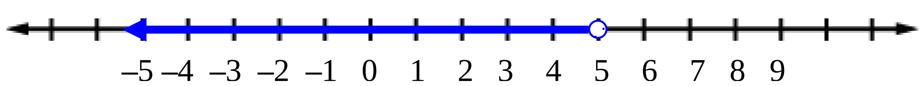
Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Expression	less than	greater than	less than or equal	greater than or equal

Alexia may spend less than $5 for lunch.   How much money may Alexia spend for lunch?

She can spend any amount less than $5 for lunch. To show the amounts she can spend, we'll look at a graph on the number line.

Let *a* stand for the amount she may spend and write an inequality: $a < 5$

Graph the inequality.



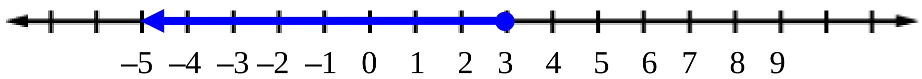
Use an **open dot** at "five" since **5 is not included.**

She may spend any amount less that $5. For example, she could spend $3 or $2.50

More Examples

$x \leq 3$

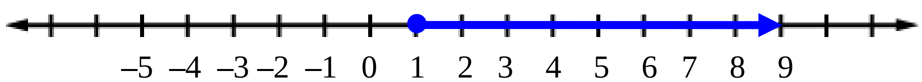
means all numbers less than or equal to three



Use a **closed dot** to include "three" since the expression represents any number **less than or equal to 3**.

$x \geq 1$

means all numbers greater than or equal to one



Use a **closed dot** to include "one" since the expression represents any number **greater than or equal to 1**.

Inequalities – Addition and Subtraction

Inequalities are math sentences that make a statement using inequality signs.

Example 1: Find all numbers that, when three is added, the result is less than eight.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when three is added, the result is less than eight.

all numbers	three is added	result is less than eight
↓	↓	↓
x	$+3$	<8

The inequality is $x+3<8$.

The rules for solving inequalities are similar to the rules for solving equations. Keep both sides of the inequality in balance by **undoing** what's been done to the unknown (*x*).

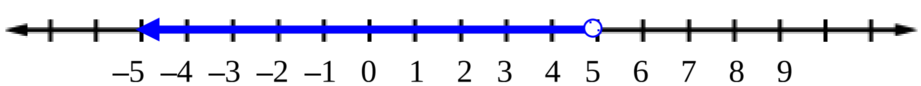
$$x+3<8$$

$$x \cancel{+3} \cancel{-3} < 8-3$$ Undo "+3" by subtracting 3 from both sides.

$$x<5$$

The solution is "all numbers less than 5".

A graph may be used to represent all solutions.



Check: Pick a number less than 5 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for x in the original inequality.

$x+3<8$	Substitute 0 for x
$0+3<8$	Simplify
$3<8$ ✓	True

Example 2: Find all numbers that, when four is subtracted, the result is greater than –6.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when four is subtracted, the result is greater than –6.

all numbers	four is subtracted	result is greater than –6
↓	↓	↓
x	-4	>-6

The inequality is $x-4>-6$.

Keep both sides of the inequality in balance.

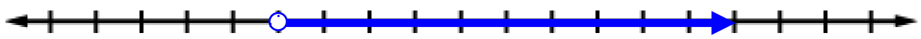
$$x-4>-6$$

$$x\cancel{-4}\cancel{+4}>-6+4$$ Undo "– 4" by adding 4 to both sides.

$$x>-2$$

The solution is "all numbers greater than –2".

A graph may be used to represent all solutions.



Check: Pick a number greater than –2 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for *x* in the original inequality.

$x - 4 > -6$	Substitute 0 for *x*
$0 - 4 > -6$	Simplify
$-4 > -6$ ✓	True

*Remember: the more negative a number is the smaller it is.

Inequalities – Multiplication and Division

Inequalities are math sentences that make a statement using inequality signs.

Example 1: Find all numbers that, when multiplied by four, the result is less than 12.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when multiplied by four, the result is less than 12.

↓	↓	↓
x	$\times 4$	< 12

*The expression $x \times 4$ can be written as $4 \times x$. The expression $4 \times x$ can be simplified to $4x$. In mathematics, it is understood that $4x$ means $4 \times x$ or $x \times 4$.

The inequality is $4x < 12$.

Keep both sides of the inequality in balance.

$$4x < 12$$

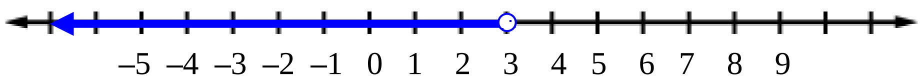
$$\frac{4x}{4} < \frac{12}{4}$$

Undo "$\times 4$" by dividing by 4 on both sides.

$$x < 3$$

The solution is “all numbers less than 3”.

A graph may be used to represent all solutions.



Check: Pick a number less than 3 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for *x* in the original inequality.

$4x < 12$	Substitute 0 for *x*
$4(0) < 12$	Simplify
$0 < 12$ ✓	True

Example 2: Solve $-7x > 21$ for *x*.

Keep both sides of the inequality in balance.

$$-7x > 21$$

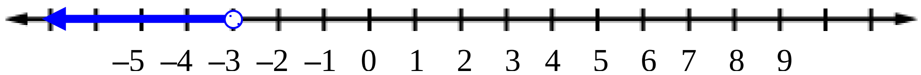
$$\frac{-7x}{-7} > \frac{21}{-7}$$ Undo "$\times -7$" by dividing by -7 on both sides.

$$x < -3$$

*Reverse the inequality symbol when multiplying or dividing by a negative number.

The solution is "all numbers less than –3".

A graph may be used to represent all solutions.



*To demonstrate that the inequality sign must be reversed, we'll test a point that is greater than –3 to show that it tests **false**. We'll test zero since it is greater than –3.

$$-7x > 21$$
$$-7(0) > 21$$
$$0 > 21$$ ✗ False

Now, we'll test a number that is less than –3 to show that it tests **true**, and that the solution is indeed "all numbers less than 3". We'll test –5.

$$-7x > 21$$
$$-7(-5) > 21$$
$$35 > 21$$ ✓ True

Rule: When **multiplying or dividing** an inequality by a **negative number**, **reverse** the direction of the **inequality sign**.

Example 3: Find all numbers that, when divided by eight, the result is less than 4.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when divided by eight, the result is less than 4.

all numbers	divided by eight	result is less than 4
↓	↓	↓
x	$\div 8$	< 4

*The expression $x \div 8$ can be written as $\frac{x}{8}$.

The inequality is $\frac{x}{8} < 4$.

Keep both sides of the inequality in balance.

$$\frac{x}{8} < 4$$

$$(\not{8})\frac{x}{\not{8}} < 4(8)$$ Undo "$\div 8$" by multiplying by 8 on both sides.

$$x < 32$$

The solution is "all numbers less than 32".

Check: Pick a number less than 32 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for *x* in the original inequality.

$\frac{x}{8} < 4$ Substitute 0 for *x*

$\frac{0}{8} < 4$ Simplify

$0 < 4$ ✓ True

Example 4: Solve $15 < \frac{x}{12}$ for *x*.

Keep both sides of the inequality in balance.

$$15 < \frac{x}{12}$$

$(12)15 < \frac{x}{\cancel{12}}(\cancel{12})$ Undo "÷12" by multiplying by 12 on both sides.

$180 < x$ *Move *x* to the left side.

$x > 180$ **Switch the sides of the inequality and reverse the inequality sign.

*The inequality $180 < x$ means 180 is less than all values of *x* OR just simply *x* is any value greater than 180.

**When moving the unknown (*x*) from the right side to the left side, it is necessary to reverse the direction of the inequality sign to keep the meaning of the inequality the same.

The solution is "all numbers greater than 180".

Check: Pick a number greater than 180 to see if that number gives a **true** statement. We'll pick 240 since it is easily divisible by 12. Substitute 240 in for *x* in the original inequality.

$15 < \frac{x}{12}$ Substitute 240 for *x*

$15 < \frac{240}{12}$ Simplify

$15 < 20$ ✓ True