LINEAR AND NONLINEAR PROGRESSIONS

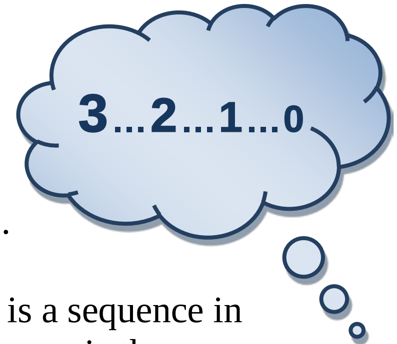
Linear progressions are sequences that are a list of numbers whose change is found by adding or subtracting the same constant each time. The graph of a linear progression makes a "straight line" as the change is the same from term to term. Nonlinear progressions are sequences that are lists of numbers that are found by multiplying or dividing by the same constant each time *or* by adding or subtracting different constants. The graph of a nonlinear progression can be a *curved line* or a *crooked line* as the change varies from term to term.

FUNCTIONS, GRAPHING FUNCTIONS, AND LINEAR EQUATIONS

A function is the relationship between two variables where the input variable yields exactly one output value determined through the function rule. We will examine input values, function rules, and output values organized in a table.

If we write ordered pairs from the input and output values of functions, we can graph functions in the coordinate plane. The graph of a linear function, also called a linear equation, is a straight line. We will plot ordered pairs to find the straight line on which the points of a linear equation are located.

Sequences



sequence – A sequence is a list of numbers in a certain order that connect through a pattern.

term – A term is any of the numbers in a sequence.

arithmetic sequence – An arithmetic sequence is a sequence in which the difference between any two consecutive terms is the same.

{5, 10, 15, 20 ...}

Notice that each number in the above sequence is five more than the number before it.

+5 +5 +5

{5, 10, 15, 20 ...}

To find the next three numbers in the sequence, **follow the pattern**, adding 5 each time.

+5 +5 +5

{5, 10, 15, 20, 25, 30, 35 ...}

Example 1: Find the next three numbers in the given arithmetic sequence.

{9, 10.5, 12, 13.5 ...}

Since this sequence is an arithmetic sequence, there will be a common difference between any two successive (side by side) terms.

To find the common difference, subtract two successive terms; then, check by subtracting two more successive terms.

$$\begin{array}{r} 12.0 \\ -\underline{10.5} \\ 1.5 \end{array} \qquad \begin{array}{r} 13.5 \\ -\underline{12.0} \\ 1.5 \end{array}$$

The common difference is 1.5.

To find the next three terms, add 1.5.

$$13.5+1.5=15 \qquad 15+1.5=16.5 \qquad 16.5+1.5=18$$

The next three terms in the sequence are 15, 16.5, and 18.

{9, 10.5, 12, 13.5, 15, 16.5, 18 …}

geometric sequence – A geometric sequence is a sequence in which the ratio between any two successive terms is the same.

{6, 12, 24, 48 ...}

Notice that each number in the above sequence is twice as much as the number before it. This sequence is a geometric sequence. Each successive term has a ratio of 2.

x2 x2 x2

{6, 12, 24, 48 ...}

To find the next three numbers in the sequence, **follow the pattern**, multiplying by two each time.

x2 x2 x2

{6, 12, 24, 48, 96, 192, 384 ...}

The next three terms in the sequence are 96, 192, and 384.

Example 2: Find the next three numbers in the geometric sequence.

{1.5, 1.8, 2.16, 2.592 ...}

Since this sequence is a geometric sequence, there will be a common difference between any two successive (side by side) terms.

To find the common ratio, divide two successive terms; then, check by dividing two more successive terms.

$$\begin{array}{r} 1.2 \\ 1.8_{\wedge}\overline{)2.1_{\wedge}6} \\ \underline{18} \\ 36 \\ \underline{36} \end{array} \qquad \begin{array}{r} 1.2 \\ 2.16_{\wedge}\overline{)2.59_{\wedge}2} \\ \underline{216} \\ 432 \\ \underline{432} \end{array}$$

Notice that each number in the sequence is 1.2 times as much as the number before it. Each successive term has a ratio of 1.2. In other words, multiply by 1.2 to get the next term in the sequence.

x 1.2   x 1.2   x 1.2

{1.5, 1.8, 2.16, 2.592 ...}

To find the next three numbers in the sequence, **follow the pattern**, multiplying by 1.2 each time.

x 1.2   x 1.2   x 1.2

{1.5, 1.8, 2.16, 2.592, 3.1104, 3.73248, 4.478976 …}

The next three terms in the sequence are 3.1104, 3.73248, and 4.478976.

Other Types of Sequences

Example 3: Look for a pattern and find the next three numbers in the sequence.

{1, 2, 4, 7, 11, 16 ...}

Notice: Each number in the sequence can be found by adding one more than what was added to the previous number.

+1 +2 +3 +4 +5

{1, 2, 4, 7, 11, 16 ...}

To find the next three numbers in the sequence, **follow the pattern**, adding one more each time.

$$\{1, 2, 4, 7, 11, 16, \overset{+6}{22}, \overset{+7}{29}, \overset{+8}{37} \ldots\}$$

The next three terms in the sequence are 22, 29, and 37.

Example 4: Look for a pattern and find the next three numbers in the sequence.

$$\{1, 8, 27, 64 \ldots\}$$

Notice: Each number in the sequence is the "cube" of the position of the term.

Sequence:	1, 8, 27, 64 ...
	↓ ↓ ↓ ↓
Number of Term:	1 2 3 4
Cube:	$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$

To find the next three numbers in the sequence, **follow the pattern**, cubing the next three "term numbers".

Sequence:	1, 8, 27, 64 ...
	↓ ↓ ↓ ↓ ↓ ↓ ↓
Number of Term:	1 2 3 4 5 6 7
Cube:	$5^3 = 125$ $6^3 = 216$ $7^3 = 343$
Sequence:	1, 8, 27, 64, 125, 216, 343

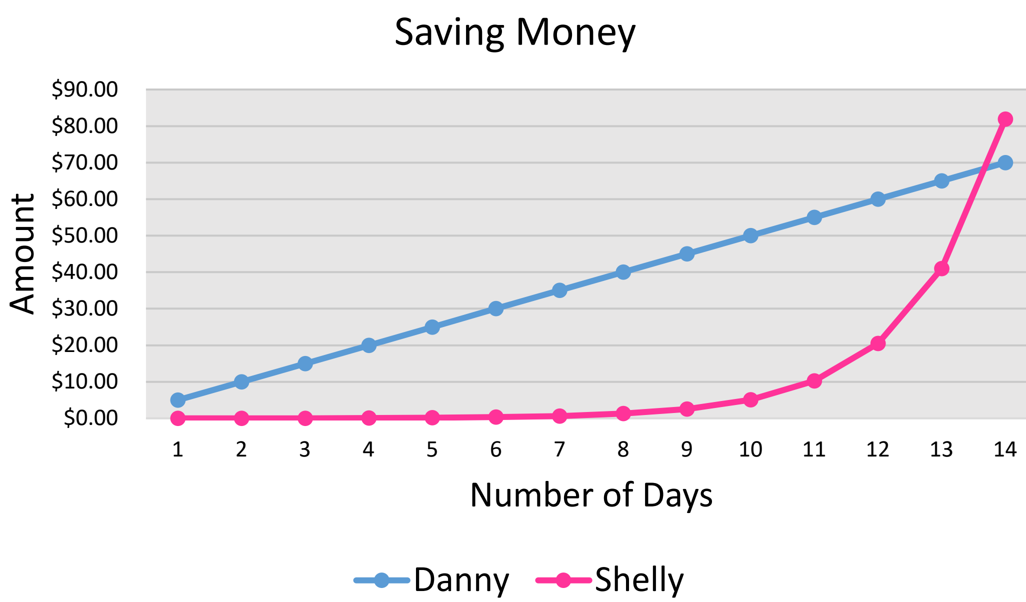
The next three terms in the sequence are 125, 216, and 343.

Linear and Nonlinear Relationships

Danny and Shelly plan to save money for 14 days. Danny is going to add $5.00 to his savings account every day for 14 days. Shelly is going to start with a penny, but every day is going to double the amount that is in the bank. Who will have more money after 14 days?

Create a table and record the data for 14 days.

Day	Danny	Shelly
1	$5.00	$0.01
2	$10.00	$0.02
3	$15.00	$0.04
4	$20.00	$0.08
5	$25.00	$0.16
6	$30.00	$0.32
7	$35.00	$0.64
8	$40.00	$1.28
9	$45.00	$2.56
10	$50.00	$5.12
11	$55.00	$10.24
12	$60.00	$20.48
13	$65.00	$40.96
14	$70.00	$81.92



Note: Shelly has more money after day 14.

Danny’s amount changes equally and the graph is a straight line. This graph shows a linear growth. Danny’s graph is a **linear graph**.

Shelly’s amount of money starts out slowly, but then grows quickly starting at about the 11th day. The graph is a curved line rather than a straight line. Shelly’s growth is said to be nonlinear. Shelly’s graph is a **nonlinear graph**.

Functions

Look at the function $y = 5x$.

Input a number (x) through the function rule ($5x$) to get the output (y).

Below is a chart for several input values, the application of the function rule, and the output values.

$y = 5x$		
y = 5 times x		
Input (x)	Function rule ($5x$)	Output (y)
1	5 times 1	*5*
2	5 times 2	*10*
3	5 times 3	*15*
4	5 times 4	*20*
5	5 times 5	*25*

Look at the function $y = x + 4$.

Input a number (x) through the function rule ($x + 4$) to get the output (y).

Below is a chart for several input values, the application of the function rule, and the output values.

$y = x + 4$		
$y = x$ plus 4		
Input (x)	Function rule ($x + 4$)	Output (y)
1	12 + 4	*16*
2	13 + 4	*17*
3	14 + 4	*18*
4	15 + 4	*19*
5	16 + 4	*20*

Functions as Graphs

Functions may be graphed in a coordinate plane.

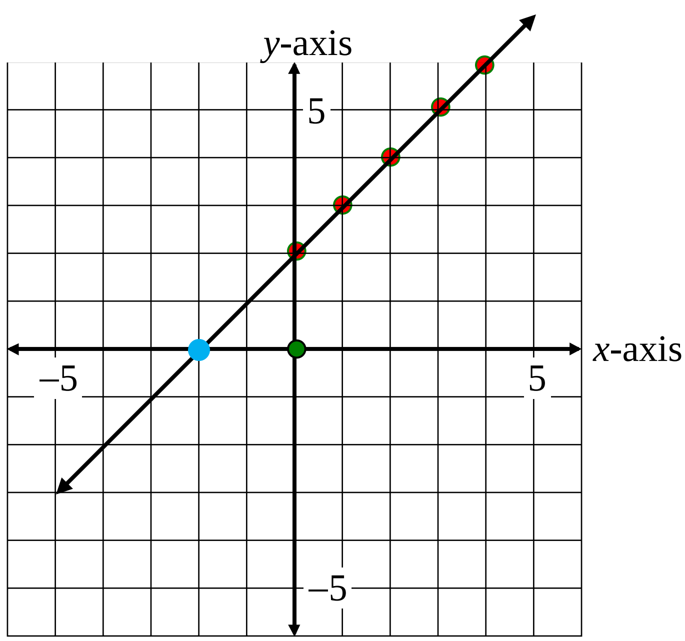
Example: Determine the graph of the function $\boldsymbol{y = 2 + x}$ using the following values for x: 0, 1, 2, 3, and 4. Make a table to organize and display the results.

Input a number (***x***) through the function rule (**2 +** ***x***) to get the output (***y***).

Write the input and the output as a set of ordered pairs to prepare for graphing the function.

$y = 2 + x$			
y **= 2 plus** ***x***			
Input (*x*)	Function rule (2 + *x*)	Output (*y*)	Ordered pairs
0	2 + 0	*2*	(0, 2)
1	2 + 1	*3*	(1, 3)
2	2 + 2	*4*	(2, 4)
3	2 + 3	*5*	(3, 5)
4	2 + 4	*6*	(4, 6)

Use the ordered pairs to plot the points.



Once the ordered pairs are graphed, draw a line through the points and beyond. Any ordered pair that can be written from the line will follow the function rule for output.

The ordered pair for the blue point is **(–2, 0)**.

$y = x + 2 \longrightarrow$ Function rule

$y = -2 + 2 \longrightarrow$ Input –2

$y = 0 \longrightarrow$ Output 0

Since the graph of this function forms a straight line, the function is considered **a linear function.**

Graphing Linear Equations

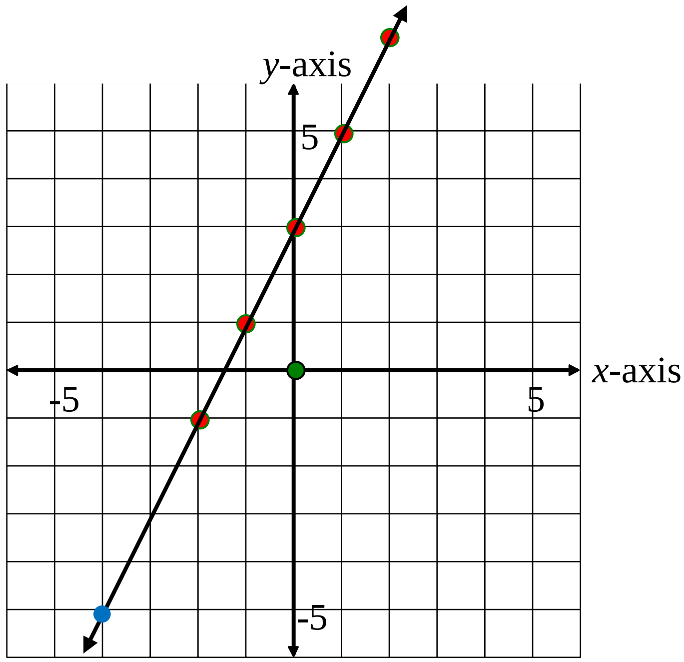
The graph of a linear equation is a straight line.

Graph $\boldsymbol{y = 2x + 3}$

- First organize the data in a table.
- You may choose several values for x. In this example we chose –2 through 2.
- Substitute the values for x in the equation to find y.
- Make ordered pairs with the results.
- Graph the points, draw a straight line through the points.

$y = 2x + 3$			
y = 2 times x plus 3			
x	Substitution	y	Ordered pairs (x, y)
–2	$y = 2 \times (-2) + 3$	–1	(–2, –1)
–1	$y = 2 \times (-1) + 3$	1	(–1, 1)
0	$y = 2 \times 0 + 3$	3	(0, 3)
1	$y = 2 \times 1 + 3$	5	(1, 5)
2	$y = 2 \times 2 + 3$	7	(2, 7)

Use the ordered pairs to graph the function.



Check

Write the ordered pair for a different point on the line by reading the coordinates where the line crosses through the point.

We will use the blue point which is not listed in the table, but falls on the line. Its coordinates are **(–4, –5)**. Substituting –4 for *x* and –5 for *y* in the equation gives the following results.

$$\mathbf{y = 2x + 3}$$
$$\mathbf{-5 = 2 \times (-4) + 3}$$
$$\mathbf{-5 = -8 + 3}$$
$$\mathbf{-5 = -5} \quad \textit{Checks}$$

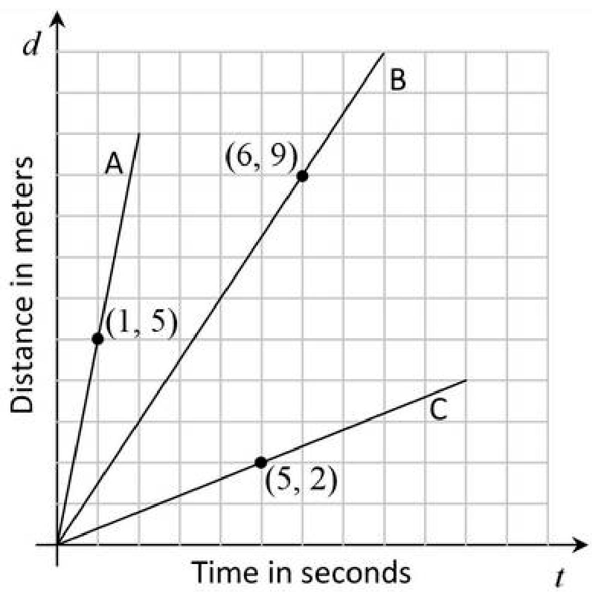
We have the correct graph for the linear equation, y = 2x + 3

Example: Carli's class built some solar-powered robots. They raced the robots in the parking lot of the school. The graphs below are all line segments that show the distance d , in meters, that each of three robots traveled after t, seconds.

1. Each graph has a point labeled. What does the point tell you about how far that robot has traveled?

2. Carli said that the ratio between the number of seconds each robot travels and the number of meters it has traveled is constant. Is she correct? Explain.

3. How fast is each robot traveling? How did you compute this from the graph?



Answer

1. The point (1, 5) tells that robot A traveled 5 meters in 1 second.
   The point (6, 9) tells that robot B traveled 9 meters in 6 seconds.
   The point (5, 2) tells that robot C traveled 2 meters in 5 seconds.

2. Carli is correct. Whenever the ratio between two quantities is constant, the graph of the relationship between them is a straight line through (0, 0). We can also say that for each robot, the relationship between the time and distance is a proportional relationship.

3. The speed can be seen as the d -coordinate of the graph when $t = 1$.

   This is the robot's unit rate:

   Robot A traveled 5 meters per second, as shown by the point (1, 5) on its graph.

   Robot B traveled 1.5 meters per second, as shown by the point (1, 1.5) on its graph.

   Robot C traveled 0.4 meters per second, as shown by the point (1, 0.4) on its graph.