VARIABLES, FORMULAS, AND ALGEBRAIC EXPRESSIONS

Variables are mathematical representations of quantities. First, we'll look expressions with variables.

Formulas use variables to make mathematical statements. Formulas give mathematical instructions to solve problems. We will examine the interest formula closely.

To write algebraic expressions, a common agreement amongst mathematicians is to use certain keywords to signal mathematical operations. Some keywords and phrases often used are sum, difference, product, quotient, more, less, increased by, and decreased by. We'll examine which operations match which expressions.

Variable Expressions

In the study of algebra, often times expressions must be evaluated. Here are some expressions to evaluate for the given value assigned to the variable.

Example 1: Evaluate $4x$ when $x = 2.5$.

4x means *4 times x*
Substitute 2.5 in for x
$4x = 4 \times 2.5 = \mathbf{10}$

Example 2: Evaluate $(a + b)(a - b)$ when $x = 10$ and $b = 7$.

$(a + b)(a - b)$ means the quantity of $a + b$ times the quantity of $a - b$
$(a + b)(a - b) = (10 + 7) \times (10 - 7)$
$(a + b)(a - b) = \quad 17 \quad \times \quad 3 \quad = \mathbf{51}$

Example 3: Evaluate xy when $x = \dfrac{2}{3}$ and $y = \dfrac{3}{2}$.

xy means *x* times *y*

$$xy = \frac{2}{3} \times \frac{3}{2} = \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{\cancel{2}_{1}} = \mathbf{1}$$

Example 4: Evaluate $\dfrac{m}{n}$ when $m = 5$ and $n = \dfrac{10}{11}$.

$\dfrac{m}{n}$ means *m* divided by *n*

$$\frac{m}{n} = 10 \div \frac{5}{11} = \frac{10}{1} \times \frac{11}{5} = \frac{\overset{2}{\cancel{10}}}{1} \times \frac{11}{\cancel{5}_{1}} = \mathbf{22}$$

Formulas

A **formula** shows how certain quantities are related. You may use a formula to find an unknown quantity if you know the other quantities.

Distance

Method:
Step 1: Write the appropriate formula.
Step 2: Replace each variable with the appropriate number.
Step 3: Solve.

Example 1: William travels 825 miles to St. Helen's Mountain. If he travels at an average rate of 55 miles per hour, how long will it take?

The formula that relates distance, rate or speed, and time is $d = rt$. The variable "*d*" represents distance, "*r*" represents rate or speed, and "*t*" represents time.

Step 1: Write the appropriate formula.

$$d = rt$$

Step 2: Replace d with 800, *r* with 55.

$$825 = 55t$$

Step 3: Solve.

$$\frac{825}{55} = \frac{55}{55}t$$
$$15 = t$$

It will take 15 hours to drive to St Helen's Mountain.

Interest

Interest is the amount of money that can be earned or charged on a banking account. If the account is a savings account, then the interest will be added on to the savings periodically. If the account is a loan, the interest is the extra charges that are added on to the loan amount. The formula for simple interest is $I = prt$.

I = p r t		
I	Interest	Amount of money earned if the account is savings *or* Amount of money charged if the account is a loan
p	Principal	Original amount of money placed in the account *or* Amount of the loan
r	Rate	Interest rate (a percent expressed as a decimal)
t	Time	Time (in years)

Example 2: Jack has $2,000 in his savings account that pays simple interest yearly at 3%. How much interest will be earned at the end of the first year?

To solve this problem use the simple interest formula. Replace each variable with the appropriate number, and then solve.

$$\begin{aligned} I &= p\ r\ t \\ I &= 2000 \times \% \times 1 \\ I &= 2000 \times 0.03 \times 1 \\ I &= \$60 \end{aligned}$$

The interest earned in one year is $60.

Example 3: Jenna's bank figures interest semi-annually; that is every 6 months. How much interest will $3000 earn at 3.5% for 6 months?

In this problem we must figure what part of a year 6 months is. Six months is $\frac{6}{12}\,or\,\frac{1}{2}$ of a year. Replace each variable with the appropriate number, and then solve.

$$\begin{aligned} I &= p\ r\ t \\ I &= 3000 \times 3.5\% \times \frac{1}{2} \\ I &= 3000 \times 0.035 \times \frac{1}{2} \\ I &= \frac{3000}{1} \times \frac{0.035}{1} \times \frac{1}{2} \\ I &= \frac{3000 \times 0.035 \times 1}{2} \\ I &= \$52.50 \end{aligned}$$

The interest earned for six months is $52.50.

Writing Algebraic Expressions

In writing phrases into algebraic expressions, we let a **variable** represent the **unknown quantity**. We'll take a look at several algebraic expressions and the phrases that translate to the expression.

$x + 5$

x plus 5
5 added to *x*
5 more than *x*
the sum of *x* and 5
x increased by 5
the total of *x* and 5

$5 + x$

5 plus *x*
x added to 5
x more than 5
the sum of 5 and *x*
5 increased by *x*
the total of 5 and *x*

$y - 4$

y minus 4
4 subtracted from *y*
4 less than *y*
the difference between *y* and 4
y decreased by 4

$4 - y$

4 minus *y*
y subtracted from 4
y less than 4
the difference between 4 and *y*
4 decreased by *y*

$7m$

7 times *m*
the product of 7 and *m*
7 multiplied by *m*
$m + m + m + m + m + m + m$

$m7 \longrightarrow 7m$

m times 7
the product of *m* and 7
m multiplied by 7

We do not leave the expression as ***m*7** but rather use the **commutative property** and write it as **7*m***.

$\frac{p}{9}$ means $\boldsymbol{p \div 9}$

p divided by 9
the quotient of *p* and 9

$\frac{9}{p}$ means $\boldsymbol{9 \div p}$

9 divided by *p*
the quotient of 9 and *p*

Example: Tom wants to buy some protein bars and magazines for a trip. He has decided to buy three times as many protein bars as magazines. Each protein bar costs \$0.70 and each magazine costs \$2.50. The sales tax rate on both types of items is 6½%. How many of each item can he buy if he has \$20.00 to spend?

Answer 1: Using a ratio table

The table below shows the cost for the protein bars and magazines in a 3 : 1 ratio.

Number of protein bars	**3**	**6**	**9**	**12**
Value of the magazines	\$2.50	\$5.00	\$7.50	\$10.00
Value of the protein bars	\$2.10	\$4.20	\$6.30	\$8.40
Value of both magazines and candy bars	\$4.60	\$9.20	\$13.80	\$17.40
Cost with tax	\$4.90	\$9.80	\$14.70	\$19.60

Looking at the last column of the table, we can see that Tom can buy 4 magazines and 12 protein bars for \$20, and that he cannot afford 5 magazines and 15 protein bars.

Answer 2: 1 magazine and 3 protein bars as a single unit

Tom's decision to buy three times as many protein bars as magazines can be thought of as deciding to buy in a unit consisting of 1 magazine AND 3 protein bars.

The cost of a unit then is \$2.50 + 3 × (\$0.70), which is \$4.60.

With sales tax, this would be \$4.60 × 1.065, which when rounded to the nearest cent would be \$4.90, or just under \$5.00.

There are four groups of five in 20 and $4 \times 4.899 = 19.596$. This leaves \$0.40 in change. So, with \$20, he can buy 4 magazines and 12 protein bars, with \$0.40 in change.