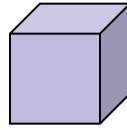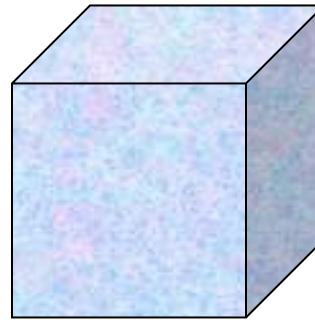# SURFACE AREA

Surface area is total area of all the surfaces of a shape. Surface area is measured in square units. In this unit you will examine finding the surface area of a cube (multiply the area of one face by six), surface area of a rectangular prism (find the sum of the areas of the top, bottom, front, back, and both sides), and the surface area of a cylinder (area of the two circular bases plus the area of the body). In addition, you will examine the unexpected effects on surface area when dimensions of the figures are doubled.

Surface Area of a Cube

A cube is a special rectangular prism with all of its edges measuring the same and all of its faces having the same area. The surface area of a cube is the total area of all of the square faces measured in square units.

Square Unit

e

The area of one face of a cube is a square.

Let e represent the length of one edge.

Then, the area of one face can be represent by $e \times e$ or e^2.

Since a cube has six faces, the total surface area of a cube is $6 \times e^2$.

"T" will be used to represent surface area in the formulas developed in this unit.

The formula for the surface area of a cube is:

The surface area (T) of a cube is:

$$T = 6e^2$$

Example 2: Find the surface area of a cube with an edge measuring seven inches.

$$T = 6e^2$$
$$T = 6 \times 7^2$$
$$T = 6 \times 49$$
$$T = 294$$

Check: Area of one face is 7×7 or 49 square inches.

Area of six faces is 49×6 or 294 square inches.

The surface area of the cube is 294 square inches.

Surface Area of a Rectangular Prism

When asked to find surface area, find the areas of the faces of the solid by using formulas previously discovered, and then add the areas of the faces together to find the total. Surface is measured in square units.
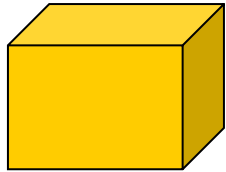
Square Unit

Example 1: Compute the surface area of a rectangular prism with a width of four feet, a length of seven feet, and a height of five feet.
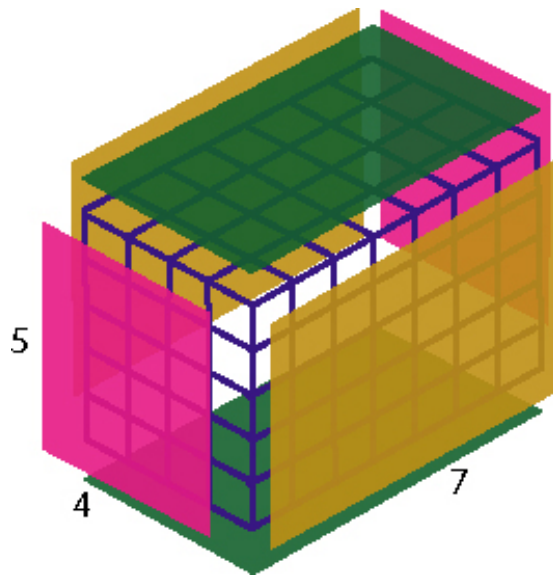
Face	Computation	Area
Front 7 ft across by 5 ft tall	7×5	35 sq ft
Back (same as front, hidden from view) 7 ft across by 5 ft tall	7×5	35 sq ft
Bottom (side the box is sitting on) 7 ft across by 4 ft wide	7×4	28 sq ft
Top (same as the bottom) 7 ft across by 4 ft wide	7×4	28 sq ft
Left Side 5 ft wide by 4 ft tall	5×4	20 sq ft
Right Side (same as left side, hidden from view) 5 ft wide by 4 ft tall	5×4	20 sq ft
Total Surface Area (Add together all of the areas of the six faces.)		**166 sq ft**

The total surface area of the rectangular prism is **166 square feet**.
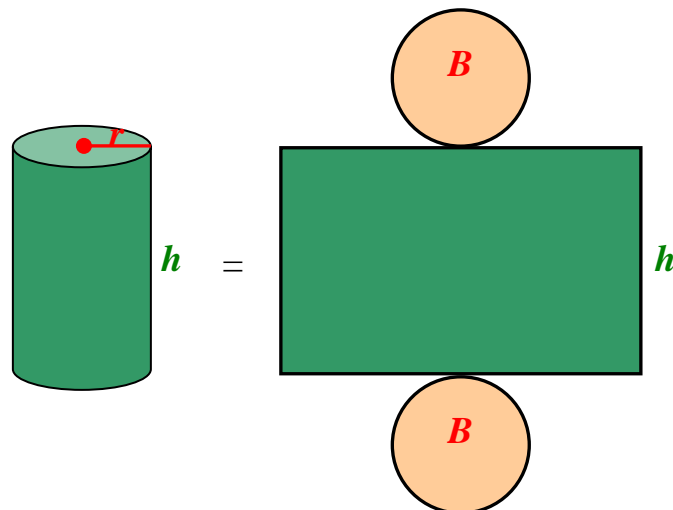
Surface Area of a Cylinder

To find the surface area of a cylinder, a little more thinking is involved.

The top and bottom of a cylinder are circles.

The side of a cylinder is one continuous curved surface. When it is laid flat, it is shaped like a rectangle.

STOP Notice that the length of the rectangle is the same as the circumference (distance around) of the circular base.

The area of the rectangular face is determined by multiplying the circumference of the base circle (length of the rectangle) by the height of the cylinder (width of the rectangle).

The formula for the surface area of a cylinder can be developed as follows:

$$T = \text{Circular Face} \quad + \quad \text{Circular Face} \quad + \quad \text{Curved Surface (Rectangle)}$$

$$T = \qquad \pi r^2 \qquad + \qquad \pi r^2 \qquad + \qquad C \times h$$

$$T = \qquad \pi r^2 \qquad + \qquad \pi r^2 \qquad + \qquad 2\pi r \times h$$

$$T = 2\pi r^2 + 2\pi rh$$

The surface area (T) of a cylinder is:

$$T = 2\pi r^2 + 2\pi rh$$

Example 5: Find the surface area of a cylinder that has a radius of three inches and a height of eight inches.

Step 1: Write the formula for the surface area of a cylinder.

$T = 2\pi r^2 + 2\pi rh$, where $\pi = 3.14$

$r = 3$ in

8 in

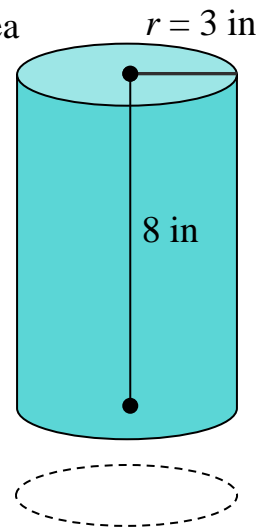*Step 2:* Substitute the given information in the formula and simplify.

$T = 2\pi r^2 + 2\pi rh$
$T = 2(3.14)(3^2) + 2(3.14)(3)(8)$
$T = 2(3.14)(9) + 2(3.14)(3)(8)$
$T = 56.52 + 150.72$
$T = 207.24$

The surface area of the cylinder is 207.24 square inches.

Doubling Dimensions – Effects on Surface Area

The edge of a smaller cube measures 3 inches. A larger cube has an edge that is twice the length of the smaller cube (6 inches). Predict how many times larger the surface area of the larger cube is than the surface area of the smaller cube. Now compute to check your answer.

How many times larger is the surface area of the larger cube than the surface area of the smaller cube?
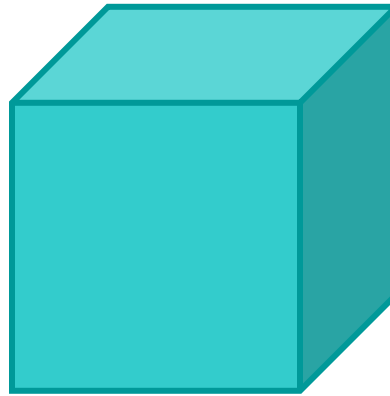
The surface area (T) of a cube is: $\boldsymbol{T = 6e^2}$

3 in   6 in

$T = 6e^2$ $T = 6e^2$

$T = 6 \times 3^2$ $T = 6 \times 6^2$

$T = 6 \times 9$ $T = 6 \times 36$
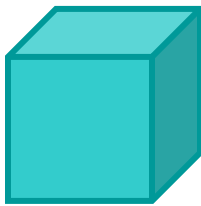
$T = 54$ sq in $T = 216$ sq in

Dividing 216 by 54, we get 4. The surface area of the larger cube is **4 times** larger when we double the length, width, and height of a cube.

Note: $4 = 2 \times 2$ or 2^2.