VOLUME

When working with 3-D shapes we can find volume of the shapes. Volume is capacity that the shape can hold. It is measured in cubic units.

To determine the volume of a rectangular prism, find the area of the base by multiplying the length times the width, and then find the volume by multiplying by the height.

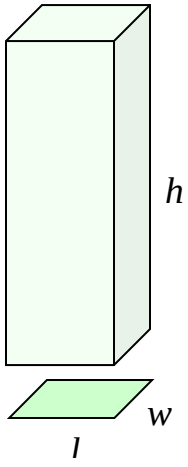
Prisms come in many shapes and sizes, so a good rule of thumb for finding the volume of a prism is to calculate the area of the base and then multiply that area by the height of the prism.

To determine the volume of a cylinder, find the area of the base which is a circle, and then multiply that area by the height of the cylinder for volume.

Doubling dimensions may have unexpected results on the new volume. We'll look at how the doubling of dimensions on a 3-D shape effects its volume.

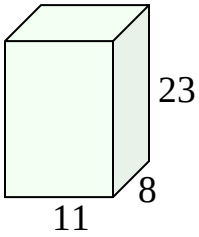
Volume of a Rectangular Prism

To calculate the volume of a **rectangular prism**, multiply the area of the base (length times width) times height.



V = Area of Base × Height
V = (length × width) × height
$V = l \times w \times h$

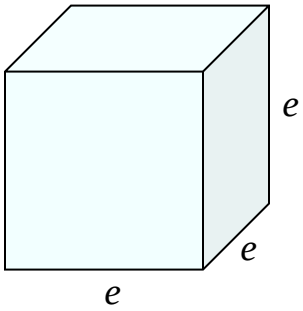
Example 1: Calculate the volume of a rectangular prism with a length of 11 feet, width of 8 feet, and a height of 23 feet.



$V = l \times w \times h$
$V = 11 \times 8 \times 23$
$V = 2024$

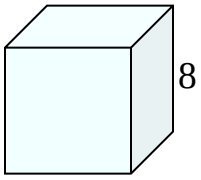
The volume of the rectangular prism is 2024 cubic feet.

To calculate the volume of a **cube**, multiply the edge times itself three times.



V = Area of Base × Height
V = (length × width) × height
$V = (e \times e) \times e$
$V = e^3$

Example 2: Calculate the volume of a cube with an edge length of eight feet.



$V = e^3$
$V = 8^3$
$V = 8 \times 8 \times 8$
$V = 512$

The volume of the cube is 512 cubic feet.

If the volume is given, use substitution and equations to determine the measurement of the desired dimension.

Example 3: If the volume of a cube is 343 cubic yards, what is the length of one edge of the cube?

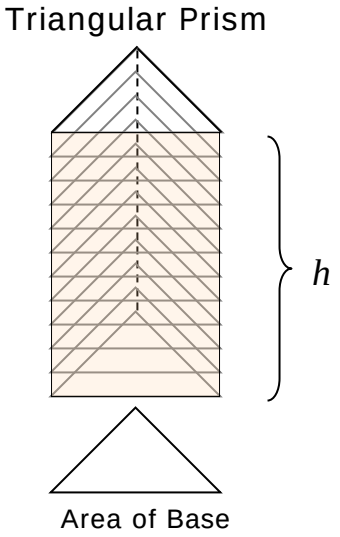
$V = e^3$	
$343 = e^3$	Substitute ($V = 343$)
$343 = e \times e \times e$	Use trial and error to find a number times itself three times that equals 343.
$e = 7$	$(7 \times 7 \times 7 = 343)$

The length of the edge of the cube is 7 yards.

*Since the answer is the length of one edge of the cube, a linear measure, use yards (not cubic yards) to label the answer.

Volume of a Prism

The **volume of a prism** is the amount the prism can hold and is measured in cubic units. To calculate the volume of a prism, multiply the area of its base times its height.



V = Area of Base × height
$V = B \times h$

Example: Find volume of a triangular prism with a height of 16 inches, and the dimensions of the triangular base are a base of six inches and a height of four inches.

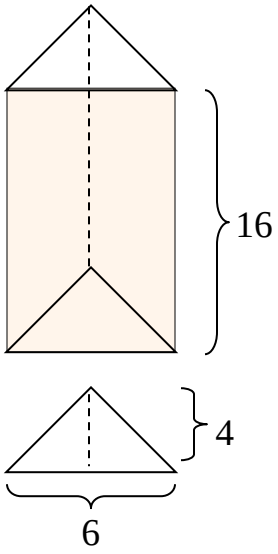
Step 1: First, find the area of the triangular base.

$$A = \frac{1}{2} b \times h$$

$$A = \frac{1}{2} \times 6 \times 4$$

$$A = \frac{1}{2} \times 24$$

A = 12 square inches (in^2)



Step 2: Let ***B*** represent the area of the base (12 in^2) and ***h*** represent the height (16 in) of the prism**.**

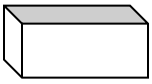
$V = B \times h$
$V = 12 \times 16$
$V = 192$

**Reminder: Volume is measured in cubic units.*

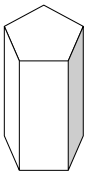
The volume of the triangular prism is 192 cubic inches (in^3).

There are many kinds of prisms. Some prisms are named for the shape of their bases. The formula, $\boldsymbol{V = B \times h}$, may be used to determine the volume of prisms.

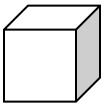
Here are some examples of prisms:



Rectangular Prism



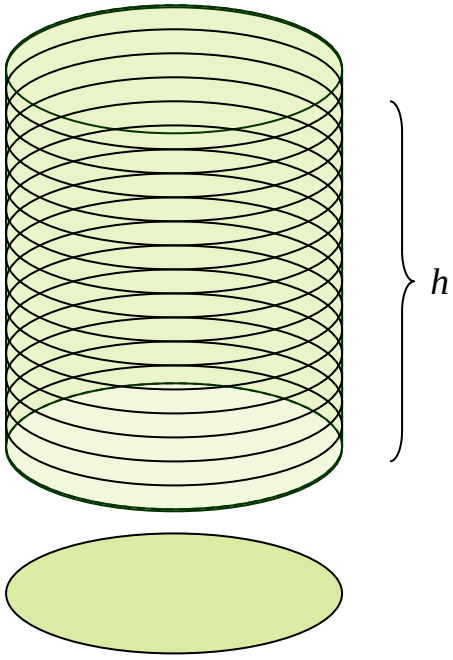
Pentagonal Prism



Cube

Volume of a Cylinder

The **volume of a cylinder** is the amount a cylinder can hold and is measured in cubic units. To calculate the volume of a cylinder, multiply the area of the base times the height.



V = Area of Base × Height

The base of a cylinder is a circle. To calculate the area of a circle, use $A = \pi \times r^2$.

$V = \quad (\pi \times r^2) \quad \times \quad h$

Example 1: Find the volume of a cylinder with a radius of 12 centimeters and a height of 23 centimeters. (Use 3.14 for "pi".)

$V = \pi \times r^2 \times h$

$V = 3.14 \times 12^2 \times 23$

$V = 3.14 \times 144 \times 23$

$V = 10{,}399.68$

**Reminder:* *Volume is measured in cubic units.*

The volume of the cylinder is 10,399.68 cubic centimeters (cm^3).

Example 2: If the volume of a cylinder is 628 cubic feet and the height of the cylinder is eight feet, what is the radius of the cylinder? (Use 3.14 for "pi".)

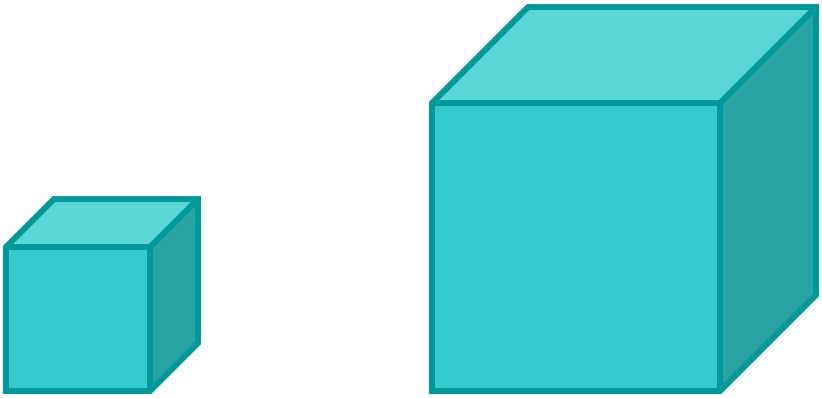
$V = \pi \times r^2 \times h$	
$628 = 3.14 \times r^2 \times 8$	Substitute $(V = 628 \text{ and } h = 8)$
$628 = 3.14 \times 8 \times r^2$	Apply the commutative property to switch the positions of 8 and r^2.
$628 = 25.12 \times r^2$	Simplify $(3.14 \times 8 = 25.12)$
$\frac{628}{25.12} = \frac{\cancel{25.12} \times r^2}{\cancel{25.12}}$	Divide both sides by 25.12.
$25 = r^2$	Simplify.
$r = 5$	Since $r^2 = 25$, take the square root of 25 to determine r.

The radius of the cylinder is 5 feet.

*Since the answer is the length the radius, a linear measure, use feet (not cubic feet) to label the answer.

Doubling Dimensions – Effects on Volume

Study the cubes and notice that the edge of the larger cube is double the length of the smaller cube. How many times larger is the volume of the larger cube than the volume of the smaller cube.



3 in

6 in

$V = e^3$

$V = 3^3$

$V = 27$ cu in

$V = e^3$

$V = 6^3$

$V = 216$ cu in

Dividing 216 by 27, we get 8. The volume of the larger cube is **8 times** larger when we double the length, width, and height of a cube.

Note: $8 = 2 \times 2 \times 2$ or 2^3