AREA

Area is the coverage within a closed figure. We find the number of square units it takes to cover the area of the figure. Some shapes have formulas to simplify the computations. Area is measured by square units.

First, we'll look at finding the area of rectangles and triangles. To find the area of a rectangle multiply base times height. To determine the area of a triangle, multiply base times height, and then divide by two.

Next, we'll look at finding the area of squares and parallelograms. To find the area of a square, square one side. To determine the area of a parallelogram, multiply base times height where height is the perpendicular distance between the parallel sides.

We will then look at finding the area of a circle. When we work with circles we must consider the value of "pi". For most problems we will round the value of "pi" to 3.14. To determine the area of a circle multiply the radius-squared times pi.

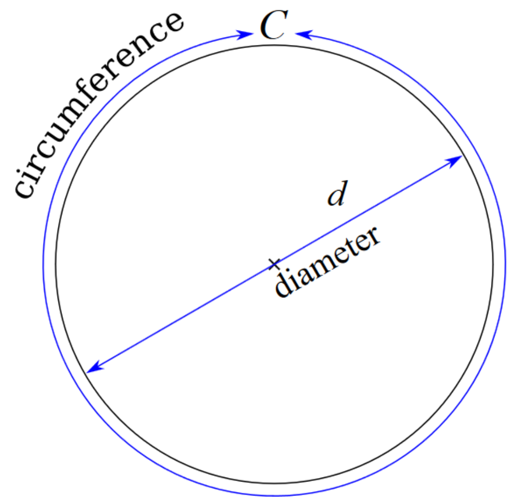
A circle sector is a portion of the circle. We will determine the area of a circle sector by finding the area of the entire circle first, and then multiplying by the fraction that represents the sector.

Composite shapes are a combination of basic shapes. We will find the areas of each of the shapes that make up the figure, and then add.

To determine the area of a trapezoid, we will find the sum of the parallel sides, multiply by the perpendicular distance between the parallel sides, and then divide by two.

Circumference of a Circle

The distance around a circle is called the circumference. The distance across a circle through the center is called the diameter. π is the ratio of the circumference of a circle to the diameter. Thus, for any circle, if you divide the circumference by the diameter, you get a value close to π. This relationship is expressed in the following formula:



$$\frac{C}{d} = \pi$$

Where C is circumference and d is diameter. You can test this formula at home with a round dinner plate. If you measure the circumference and the diameter of the plate and then divide C by d, your quotient should come close to π. Another way to write this formula is: $C = \pi \cdot d$ where $\cdot$ means multiply. This second formula is commonly used in problems where the diameter is given and the circumference is not known.

Composite shapes are a combination of basic shapes. We will find the areas of each of the shapes that make up the figure, and then add.

To determine the area of a trapezoid, we will find the sum of the parallel sides, multiply by the perpendicular distance between the parallel sides, and then divide by two.

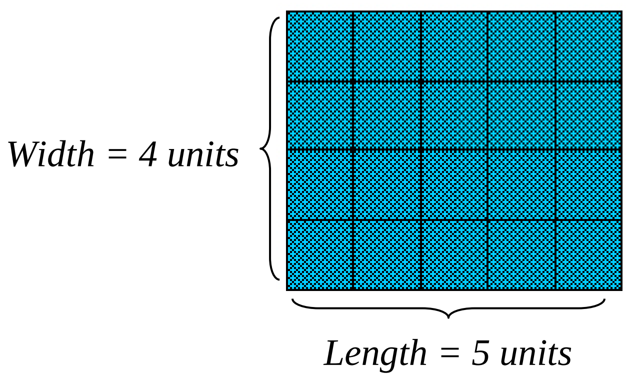
Area of a Rectangle and a Triangle

The **area of a rectangle** is the product of the length and the width.

Area is a measurement of coverage and is measured in **square units**.

$$A = lw$$

Example 1: Find the area of a rectangle that measures 5 units by 4 units.

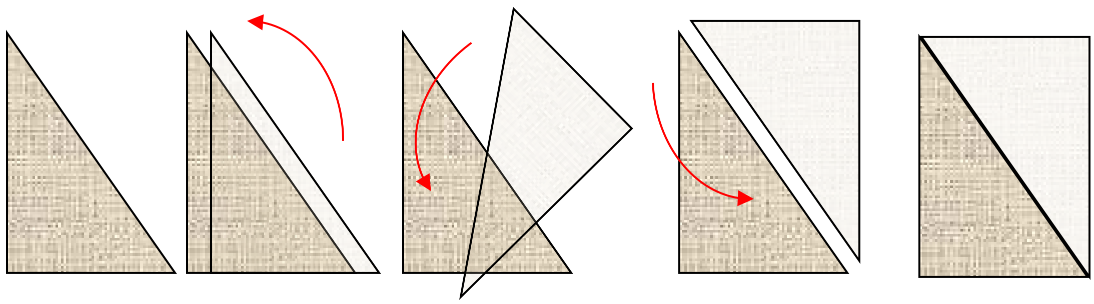


$$A = lw$$
$$A = 5 \times 4$$
$$A = 20 \text{ square units}$$

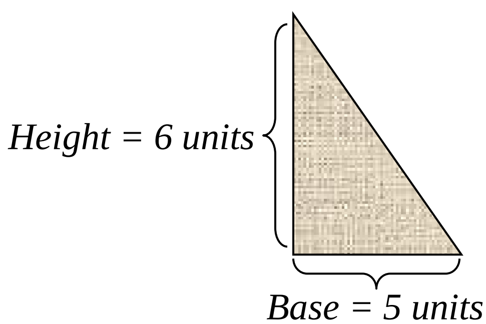
The area of the rectangle is 20 square units.

The **area of a triangle** is equal to half the area of a rectangle with the same base and height. Study the figure below and follow the arrows to see that the area is only half as much.

$$A = \frac{1}{2}bh$$



Example 2: Find the area of a triangle that measures 5 units by 6 units.



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 5 \times 6$$

$$A = \frac{1}{2} \times 30$$

$$A = 15 \text{ square units}$$

The area of the triangle is 15 square units.

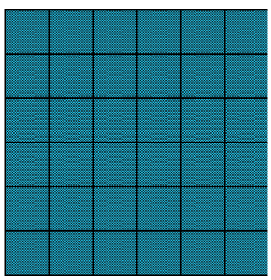
Area of a Square and a Parallelogram

Area is a measurement of coverage and is measured in **square units**.

The **area of a square** is the product of its length and width. Since squares have sides of equal length, the area of a square is the product of its length (side) and its width (side).

$$A = lw$$
$$A = s \times s$$
$$A = s^2$$

Example 1: Find the area of a square that is 6 units in length on each side.



Side = 6 units

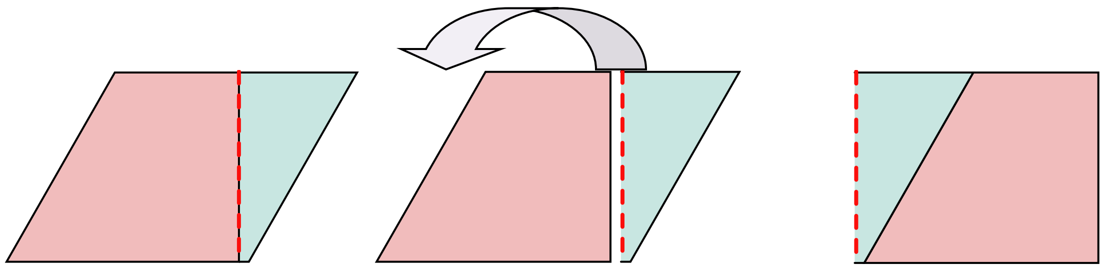
$$A = s^2$$

$$A = 6^2$$

$$A = 36 \text{ square units}$$

The area of the square is 36 square units.

The **area of a parallelogram** can be rearranged into the shape of a rectangle if the parallelogram is cut along a perpendicular height from the top to its base. Thus, a formula for the area of a parallelogram can be written based on the formula for the area of a rectangle.

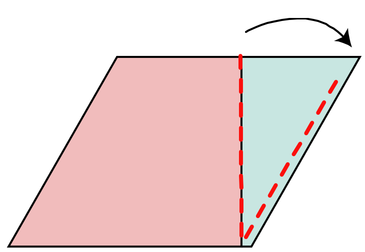


Move the triangular
piece to the left side.

The **area of a parallelogram** is the product of its base and height. The height of a parallelogram is the length of a perpendicular line from the top of the parallelogram to the base.

$$A = lw$$
$$A = bh$$

*Notice that the height of a parallelogram is shorter than the length of its side. When calculating the area of a parallelogram, be sure to use the height of the parallelogram rather than the length of the side.

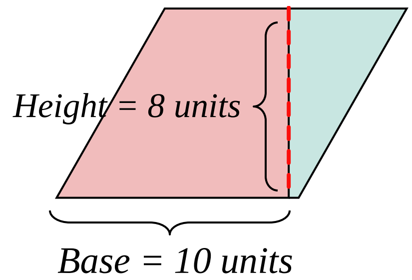


Example 2: Find the area of a parallelogram that has a base of 10 units and a height of 8 units.

$A = bh$

$A = 10 \times 8$

$A = 80$ square units



Area of a Circle

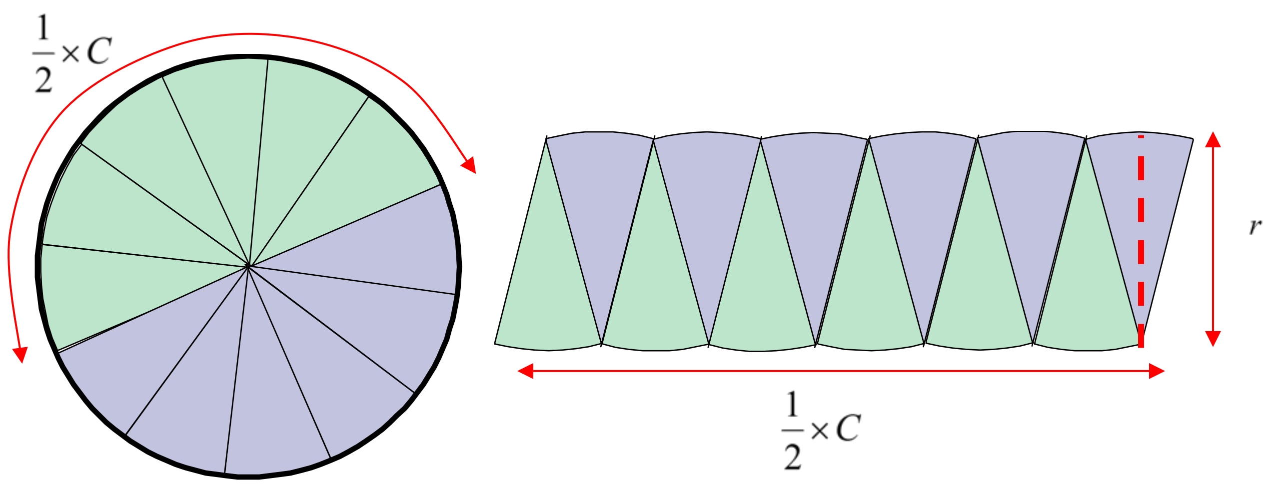
Area is a measurement of coverage and is measured in **square units**.

The area of a circle can be rearranged into a shape that approximates a parallelogram.

The length of the parallelogram is the same length as half the circle's circumference. The height of the parallelogram is the same as the radius of the circle.

Let's take a look at how this can happen.

The circle shown below is divided into 12 congruent pieces. The pieces are then laid out to make a shape that looks similar to a parallelogram.



Notice that the length of the "parallelogram" is half of the length of the circumference of the circle.

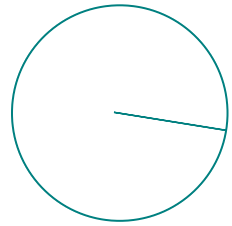
Notice that the height of the parallelogram is close to the radius of the circle.

For this theory to truly work, the circle would be divided into many, many, more pieces. When that is done, then the bottom of the parallelogram is close to a straight line and the height of the parallelogram is closer to a perpendicular line.

Now, we'll build the formula based on this theory.

Statement	Reason
$A = bh$	Formula for area of a parallelogram.
$A = (\frac{1}{2}C) \times r$	base $= \frac{1}{2}C$ height $= r$
$A = (\frac{1}{2} \times 2\pi r) \times r$	$C = 2\pi r$
$A = 1 \times \pi r \times r$	$\frac{1}{2} \times 2 = 1$
$A = \pi r \times r$	Identity Property (Any number times 1 is the number.)
$A = \pi \times (r \times r)$	Associative Property (Regrouping is allowed in multiplication.)
$A = \pi \times r^2$	$r \times r = r^2$

Example 1: Find the area of a circle that has a radius of five inches.

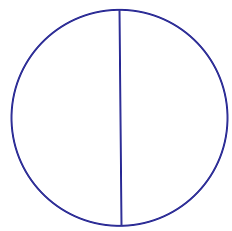


r = 5 in

$$A = \pi r^2$$
$$A = 3.14 \times 5^2$$
$$A = 3.14 \times 25$$
$$A = 78.5 \text{ square inches}$$

The area of the circle is 78.5 square inches.

Example 2: Find the area of a circle that has a diameter of twenty feet.



Diameter = 20 ft

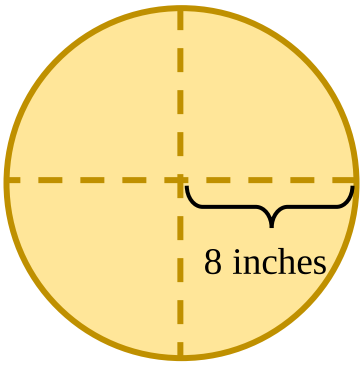
*Since a diameter is given, and a diameter equals two radii, take half of 20 to determine the radius.

$$A = \pi r^2$$
$$A = 3.14 \times 10^2$$
$$A = 3.14 \times 100$$
$$A = 314 \text{ square feet}$$

The area of the circle is 314 square feet.

Area of Circle Sectors

To find the area of a sector of a circle, first determine the area of the whole circle, and then find the fractional part that represents the circle.



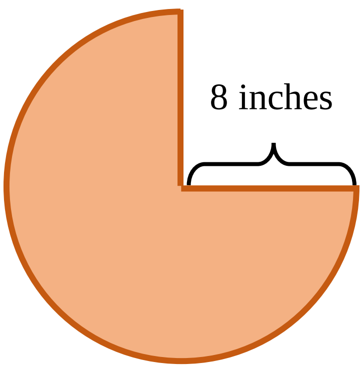
Find the area.

$A = \pi \times r^2$

$A = 3.14 \times 8 \times 8$

$A = 3.14 \times 64$

$A = 200.96$ square inches



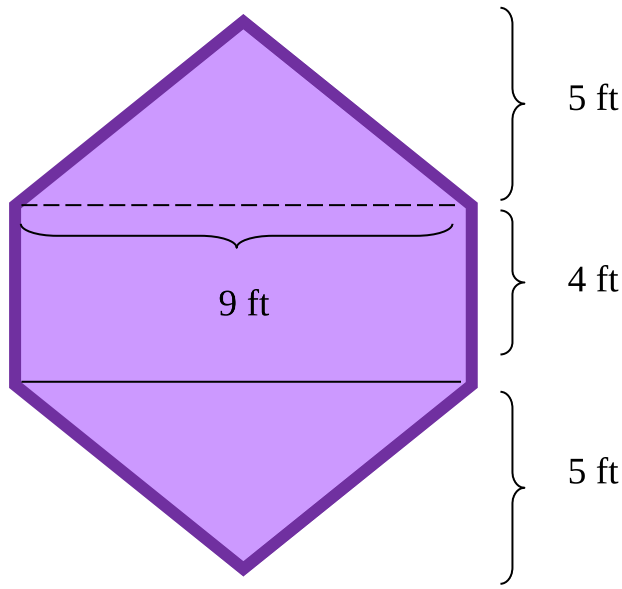
Find the area of this sector.

Since the sector is $\frac{3}{4}$ the area of the entire circle, the area of the sector would be $\frac{3}{4}$ of 200.96 square inches.

$\frac{3}{4} \times \frac{200.96}{1} = \frac{602.88}{4} = 150.72$ square inches

Area of Composite Shapes

To find the area of this hexagon, we will divide it into shapes for which we are familiar with their formulas.



Area of Triangle

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 9 \times 5$$

$$A = \frac{1}{2} \times 45$$

$$A = 22\frac{1}{2} \text{ sq ft}$$

Area of Rectangle

$$A = lw$$

$$A = 9 \times 4$$

$$A = 36 \text{ sq ft}$$

The area of the second triangle has the same area as the first triangle because it has the same dimensions.

$$A = 22\frac{1}{2} \text{ sq ft}$$

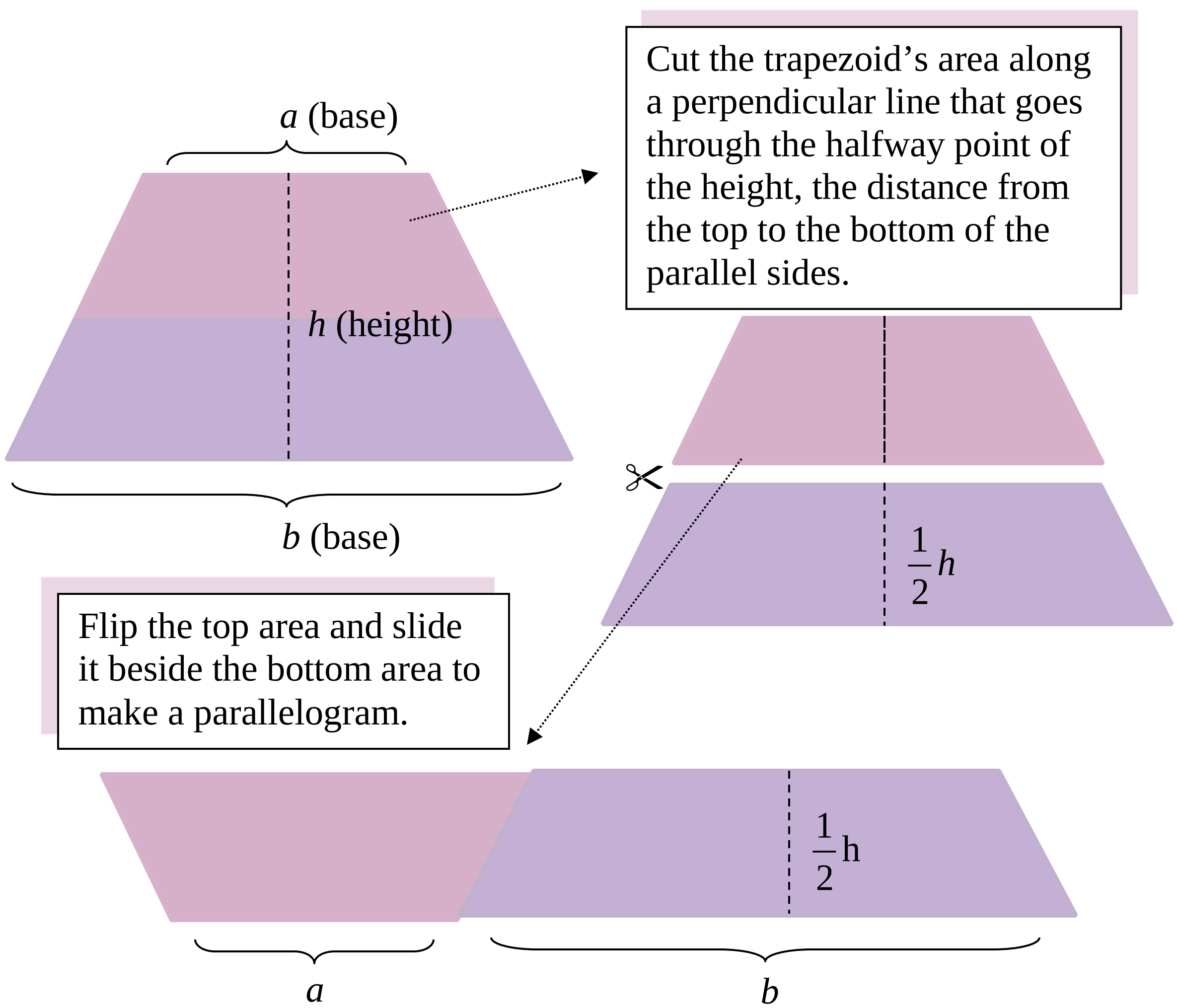
Total Area

First Triangle	=	$22\frac{1}{2}$
Rectangle	=	36
Second Triangle	=	$22\frac{1}{2}$
Area of Hexagon	=	81 sq ft

Area of a Trapezoid

Area is a measurement of coverage and is measured in **square units**.

The **area of a trapezoid** can be rearranged into the shape of a parallelogram. Let's take a look at how this can happen.



Base $= (a + b)$ Height $= \frac{1}{2}h$

Build the formula for the area of a trapezoid based on the formula for the area of a parallelogram.

$$A = bh$$

$$A = (a+b)(\frac{1}{2}h)$$

$$A = (\frac{1}{2}h)(a+b)$$ Apply the commutative property.

$$\boxed{A = \frac{1}{2}h(a+b)}$$

The area of a trapezoid equals one-half of the height times the sum of the bases.

*Note: The bases of a trapezoid are the parallel sides.

Example: Find the area of a trapezoid where the parallel sides measure 4 feet and 10 feet and the height of the trapezoid is 15 feet.

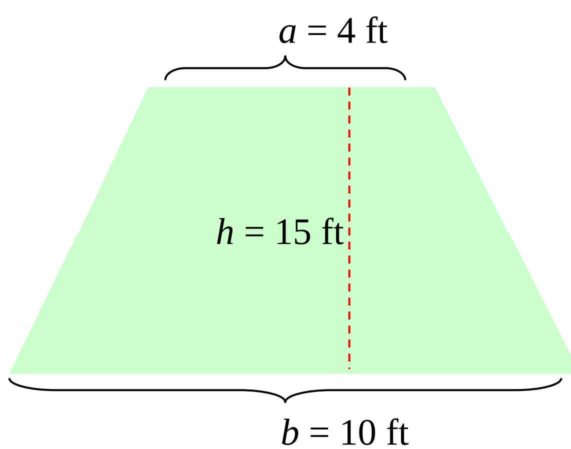
$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2} \times 15 \times (4+10)$$

$$A = \frac{1}{2} \times 15 \times 14$$

$$A = \frac{1}{2} \times 210$$

$$A = 105 \text{ sq ft}$$



The area of the trapezoid is 105 square feet.