CONGRUENT AND SIMILAR SHAPES

Polygons whose angles and sides measure the same are congruent. We will look at the congruent sides and angles for congruent triangles.

Similar polygons have same shape, but are different in size. Their corresponding sides are proportional and corresponding angles are congruent.

When polygons are congruent or are similar, depending on the given information, some of the missing angles can be determined. We'll look at how to determine missing angles for both congruent and similar shapes.

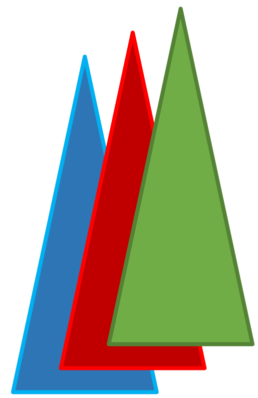
We'll look at how to determine missing lengths for similar shapes by using proportions.

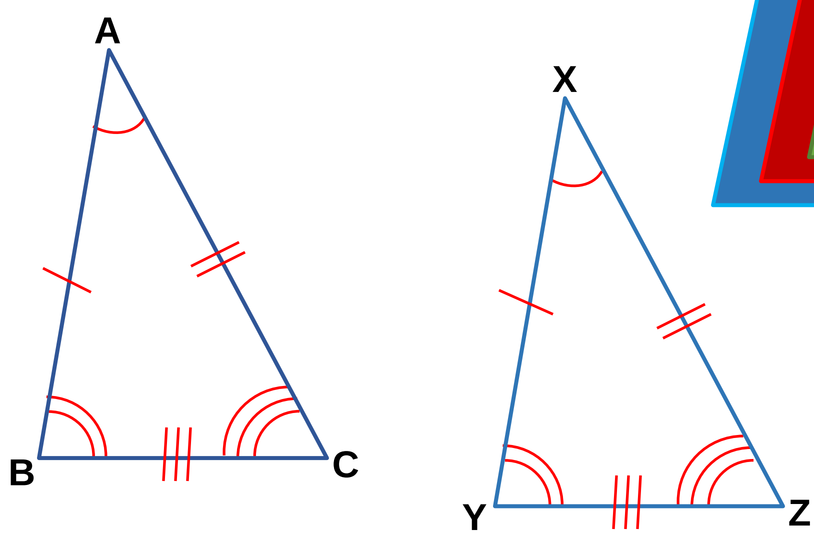
PYTHAGOREAN THEOREM

A special theorem, the Pythagorean Theorem, was developed for right triangles by a Greek mathematician named Pythagoras. When two of the lengths of the sides of a right triangle are known, the length of the third side can be determined. Right triangles and diagonal lengths are all around us and are especially useful in the building industry to firm up building structures.

Congruent Figures

In congruent triangles, the corresponding angles are the same size and the corresponding sides are the same length.





The red line hash marks are used to mark the corresponding congruent sides.
The red curved line hash marks are used to mark corresponding congruent angles.

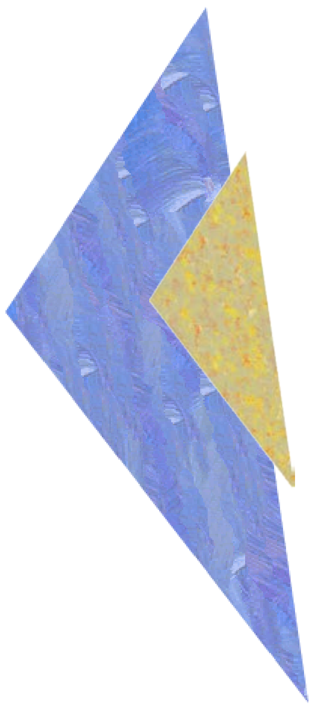
$\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$ $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$

The symbol, $\cong$, means “is congruent to”.

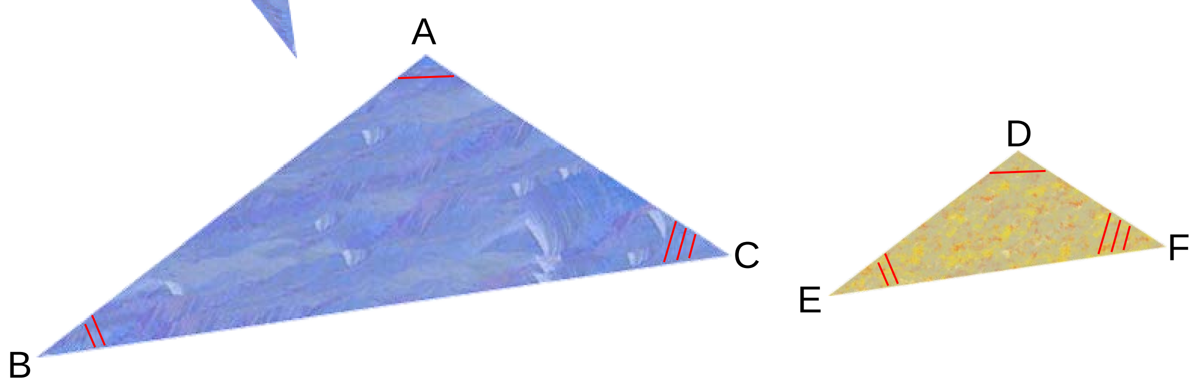
The symbol, $\overline{TS}$, is read “line segment TS”.

Similar Figures

Similar Figures have the similar shape and size. Their sides are in proportion with each other**.**



In similar triangles, the lengths of the corresponding sides are proportional (connect with the same ratio) and the corresponding angles are congruent (equal in measure).



$\triangle ABC \sim \triangle DEF$ (The symbol $\triangle$ means triangle. The symbol ~ means **similar**.)

Corresponding Angles

$\angle A \cong \angle D,\ \angle B \cong \angle E,\ \angle C \cong \angle F$ (The symbol ≈ means congruent.)

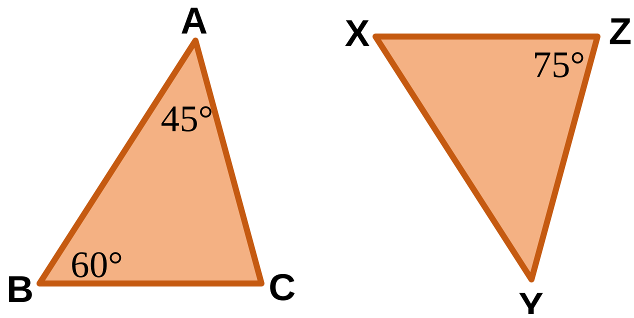
Corresponding Sides

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Congruent and Similar Shapes – Missing Angles

Example 1: $\triangle ABC \cong \triangle XYZ$. Find the measures of angles *C*, *X* and *Y*.

*$\triangle ABC \cong \triangle XYZ$ is read "Triangle ABC is congruent to triangle XYZ."



Since the triangles are congruent, the corresponding angles are congruent.

$\angle A \cong \angle Y$, $\angle B \cong \angle X$, $\angle C \cong \angle Z$

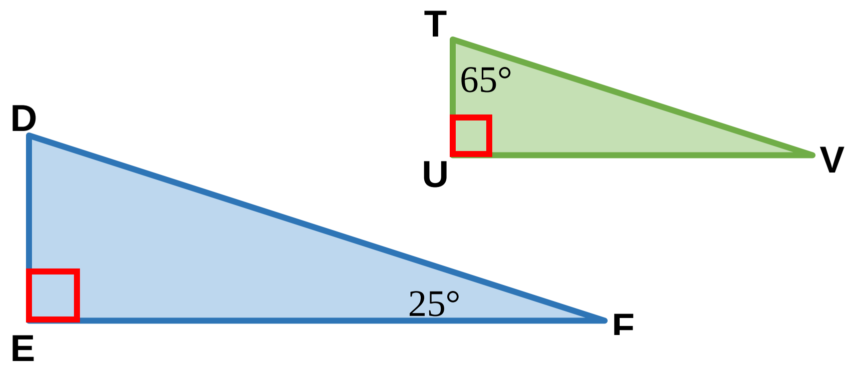
$m\angle Y = m\angle A = 45°$, $m\angle X = m\angle B = 60°$, $m\angle C = m\angle Z = 75°$

*$m\angle Y = m\angle A = 45°$ *is read "the measurement of angle Y is 45 degrees"*.

Angle *C* measures 75°, angle *X* measures 60°, and angle *Y* measures 45°.

Example 2: △DEF ~ △TUV. Find the measures of angles *D* and *V*.

*△DEF ~ △TUV is read "Triangle DEF is similar to triangle TUV."



The *red* *square designates that the angle is a right angle.*

In similar triangles the corresponding angles are congruent.

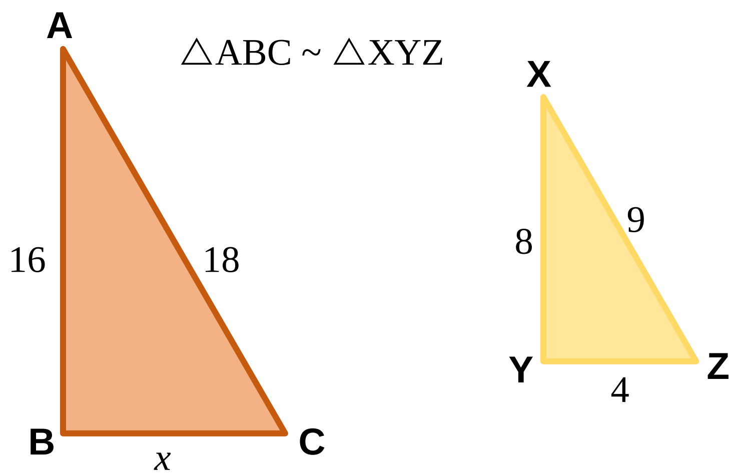
$\angle D \cong \angle T,\ \angle E \cong \angle U,\ \angle F \cong \angle V$

$m\angle D = m\angle T = 65°, \quad m\angle E = m\angle U = 90°, \quad m\angle F = m\angle V = 25°$

The measurement of angle *D* is 65° and the measurement of angle *V* is 25°.

Similar Triangles – Missing Lengths

$\triangle ABC \sim \triangle XYZ$. Find the length of segment BC that is labeled with *x*.



$\triangle ABC \sim \triangle XYZ$ is read, "Triangle ABC is similar to triangle XYZ." Since the triangles are similar triangles and the scale factor is the same between any pair of corresponding sides in similar triangles, you can write equivalent ratios to find *x* using the corresponding sides.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \qquad \frac{16}{8} = \frac{x}{YZ} = \frac{18}{9}$$

To solve for *x*, set up a proportion based on two of the ratios. One proportion could be $\frac{AB}{XY} = \frac{BC}{YZ}$.

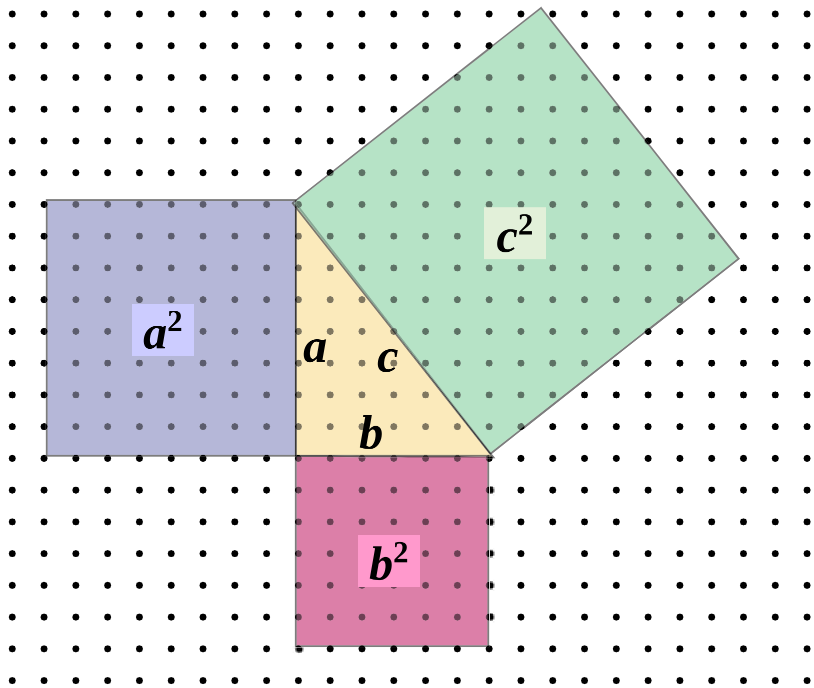
$\frac{16}{8} = \frac{x}{4}$	Substitute values for side lengths.
$8x = 64$	Cross multiply and write an equation.
$x = 8$	Divide.

Another proportion could be $\frac{BC}{YZ} = \frac{AC}{XZ}$.

$\frac{x}{4}=\frac{18}{9}$	Substitute values for side lengths.
$9x = 72$	Cross multiply and write an equation.
$x = 8$	Divide.

The length of the missing side, *x*, is 8 units.

Pythagorean Theorem



Special names are given to the sides of a right triangle. The two sides that make up the right triangle are called "**legs**" and the side opposite the right angle is called the "**hypotenuse**".

A special relationship exists between the sides of a right triangle. The sum of the squares of the two legs equals the square of the hypotenuse.

$$c^2 = a^2 + b^2$$

In the example above, the legs measure 6 and 8 units. What does the diagonal measure?

$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = \sqrt{100}$$
$$c = 10$$