## COMPARI NG I NTEGERS

Integers expand the number system to include negative numbers. The integers are the whole numbers and their opposites and also zero. First, we'll investigate the number line and compare integers.

## ABSOLUTE VALUE

Absolute value is a method used to express the positive value of a number. The absolute value of a number is the number's distance from zero expressed as a positive number.

## I NTEGER OPERATI ONS: ADDI NG AND SUBTRACTI NG I NTEGERS

To compute with integers we must pay attention to the sign of the integer. We can use the number line to add integers, but to streamline the computation we will develop the rules for adding integers.

The definition for subtracting integers is to "add the opposite". We can use the number line to subtract integers, but we can speed up the computation for subtraction by learning to write the subtraction problem as an addition problem where we change the second integer to its opposite. From there, we proceed by using the addition rules for integers.

## Comparing Integers



Integers are used to show positive and negative quantities. Integers are a union of the natural numbers (counting numbers), " 0 ", and the natural numbers' opposites. Thus, integers are $\{\ldots-4,-3,-2,-1,0,1,2,3,4, \ldots\}$.

Starting anywhere on the number line, a number to the right will be of higher value than the one on the left.

## Example:

In the game of Jeopardy, Jennifer's score was -100 and Jeremy's score was 400. Which person is closer to getting back into the positive numbers?
$\{\ldots-800,-700,-600,-500,-400,-300,-200,-100,0,100,200,300,400,500,600,700,800 \ldots\}$
Since Jennifer's score is to the right of Jeremy's score, her score is higher than his, and she is closer to getting back into the positive numbers.

$$
-100>-400
$$

-100 is greater than -400

## Other Examples:

$$
600>200
$$

600 is greater than 200
(600 is to the right of 200 on the number line.)

$$
300>-700
$$

300 is greater than -700
( 300 is to the right of -700 on the number line.)
(Any positive number will be greater than a negative number.)

To compare integers, think about the number line and which one is located to the right of the other. The integer on the right will be the larger of the two integers.

## Absolute Value

Absolute value of an integer is the distance the integer is from zero.

$$
\{\ldots-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8 \ldots\}
$$

Absolute value is represented by two vertical bars around the number.

The absolute value of -5 is 5 because -5 is 5 units from $0 . \quad|-5|=5$ The absolute value of 5 is 5 because 5 is 5 units from 0 . $5=5$

## I nteger Operations

The integers are a set of numbers that contain the whole numbers and their opposites. There are no decimals or fractions in the set of integers.

$$
\text { Integers: }\{\ldots-4,-3,-2,-1,0,1,2,3,4 \ldots\}
$$

## Addition of I ntegers

Rule 1: When the signs are the same, ADD the values and use the same sign.

Example 1: Find the sums.

$$
\begin{array}{ll}
-4+(-5)=-9 & \begin{array}{l}
\text { The signs are the same (both are negative), } \\
\text { so ADD, and the answer will be negative. }
\end{array} \\
21+45=66 & \begin{array}{l}
\text { The signs are the same (both are positive), } \\
\text { so ADD, and the answer will be positive. }
\end{array}
\end{array}
$$

Rule 2: When the signs are not the same, SUBTRACT and take the sign of the number that is farthest from zero on the number line.

Example 2: Find the sums.
$8+(-3)=-5 \quad$ The signs are different, so SUBTRACT and the answer will be positive because 8 is farther from 0 than -3 on the number line.
$-7+4=-3 \quad$ The signs are different, so SUBTRACT and the answer will be negative because -7 is farther from 0 than 4 on the number line.
$5+(-12)=-7 \quad$ The signs are different, so SUBTRACT and the answer will be negative because -12 is farther from 0 than 5 on the number line.
$-11+15=5 \quad$ The signs are different, so SUBTRACT and the answer will be positive because 15 is farther from 0 than -11 on the number line.

## Subtraction of Integers

Rule: To subtract integers, ADD the opposite.
In other words, change the sign of the second integer, and then apply the addition rules of integers.

Example 3: Find the differences.

$$
\begin{array}{ll}
-6-(-8) & =-6+(+8)=-6+8=2 \\
-10-4 & =-10+(-4)=-14 \\
16-(-2) & =16+(+2)=16+2=18 \\
5-11 & =5+(-11)=-6 \\
13-7 & =6 \text { (Just use normal whole number subtraction.) }
\end{array}
$$

Click on the log below to play a game.

$$
3+(-5) \quad \text { Add and Subtract Integers }
$$

## Word Problems using Real-World Contexts

For example:

Aakash, Bao Ying, Chris and Donna all live on the same street as their school, which runs from east to west.

- Aakash lives $51 / 2$ blocks to the west.
- Bao Ying lives $41 / 4$ blocks to the east.
- Chris lives 2 3/4 blocks to the west.
- Donna lives $61 / 2$ blocks to the east.
a. Draw a picture that represents the positions of their houses along the street.
b. Find how far each house is from every other house?
c. Represent the relative position of the houses on a number line, with the school at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers.
d. How can you see the answers to part (b) on the number line? Using the numbers (some of which are positive and some negative) that label the positions of houses on the number line, represent these distances using sums or differences.
a. There are many ways to draw a picture that represents this situation. Here is one:

b. Here is a table that shows the distances between each of the student's houses.

|  | Bao Ying | Chris | Donna |
| :--- | :---: | :---: | :---: |
| Aakash | $93 / 4$ | $23 / 4$ | 12 |
| Bao Ying |  | 6 | $1 / 4$ |
| Chris |  |  | $91 / 4$ |

c. The colors show which point corresponds to which person in the first picture:

d. The distance between the houses is represented by the distance between the points that correspond to the houses on the number line. This can be computed by subtracting the numbers that represent the
position of the house relative to the school. For example, to find the distance between Bao Ying and Chris, we subtract $-23 / 4$ from $41 / 4$ :

$$
4^{1 / 4}-(-23 / 4)
$$

We can communicate this more clearly by labeling the distance between the points with the difference of the numbers on the number line:


