

RATIOS, PROPORTIONS, RATES

A ratio is a comparison of two quantities. Ratios can be written several ways. A ratio of 7 to 6 may be written $7:6$ or could be written $7/6$.

A very useful problem solving technique is writing a proportion for numerical information that can be written in ratio form. Once a proportion is written, cross multiplication is used to write an equation and solve the problem.

Some measurements are a combination of two units such as miles per hour or meters per second called “derived measurements”. We will solve some word problems that use derived measurements.

SCALE FACTORS AND MODELS

A scale factor is a factor that can be used to multiply the measurements of an object for the purpose of finding the measurements of a second object that is in proportion to the original object.

Scale models are used in many industries. In constructing buildings and making new vehicles, scale models are used as a guide for measuring building materials needed and assembling the parts.

Ratios

A ratio is a **comparison** of two quantities.

The colors of these leaves may be compared many ways.



There are three ways to write ratios.

To compare the yellow leaves to green leaves, you can write:

4 to 3 *or* 4:3 *or* $\frac{4}{3}$

The ratio of yellow leaves to green leaves is **4 : 3**.

Example 1: Write the ratio of red leaves to brown leaves three ways.

2 to 3 *or* 2:3 *or* $\frac{2}{3}$

The ratio of red leaves to brown leaves is **2 : 3**.

Example 2: Write the ratio of green leaves to all of the leaves as a fraction in simplest form.

3 to 12 *or* 3:12 *or* $\frac{3}{12}$

The ratio of green leaves to all of the leaves is **3 : 12**.

This ratio can be simplified; so, use the fraction form of the ratio and reduce.

$$\frac{3}{12} \div \frac{3}{3} = \frac{1}{4}$$

The ratio of green leaves to all of the leaves is **1 : 4**. This means that one in four leaves is green.

Example 3: Write the ratio of all of the leaves to the red leaves as a fraction in simplest form.

$$12 \text{ to } 2 \qquad \text{or} \qquad 12:2 \qquad \text{or} \qquad \frac{12}{2}$$

The ratio of all of the leaves to the red leaves is **12 to 2**.

This ratio can be simplified; so, use the fraction form of the ratio and reduce.

$$\frac{12}{2} \div \frac{2}{2} = \frac{6}{1}$$

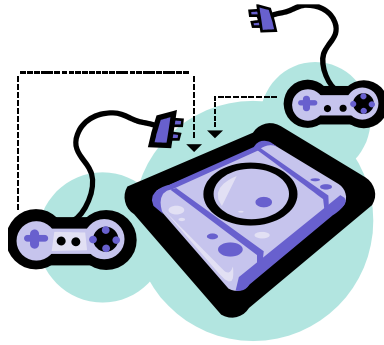
The ratio of all of the leaves to the red leaves is **6 : 1**. This means for every six leaves in the group, one is red.

*Ratios such as $6/1$ or $5/3$ remain as improper fractions when written in fraction form since a ratio is a comparison of two quantities.

Using Ratios to Solve Proportions

Using ratios to set up a proportion is a very useful problem-solving technique that can be used in a variety of everyday problems.

Example 4: Cindy surveyed her home-room class and found that 11 out of 25 students played video games at home at least once every day of the school week. There are 500 students in the eighth grade at her school. Based on Cindy's survey, predict how many eighth grade students in the school play video games at least once every day of the school week.



To solve, first find the comparison ratio.

$$\frac{11}{25} = \frac{\text{(play a video game at least once every day of the school week)}}{\text{(all students surveyed)}}$$

Then, set up a proportion where both numerators represent the same idea and both denominators represent the same idea.



Thinking through the problem with brief words can be helpful.

11 out of 25 play video games...how many out of 500 play video games...?

The proportion is $\frac{11}{25} = \frac{n}{500}$.

$$\frac{11}{25} = \frac{n}{500} \quad \text{Cross Multiply}$$

$$25 \times n = 11 \times 500 \quad \text{Simplify}$$

$$25n = 5500 \quad \text{Divide}$$

$$n = 220$$

Cindy estimated that, based on her survey, about 220 eighth grade students play a video game at least once every day of the school week.

Derived Measurements

Rates / Unit Rates

rate – a ratio that is a comparison of two quantities that are different kinds of units

unit rate – rate of one (The rate is expressed with a denominator of one.)



Sample Rates:

miles per hour (**mph**)

miles per gallon (**mpg**)

revolutions per minute (**rpm**)

Example: Find the average unit rate of speed for a car traveling 700 miles in 12 hours.

Set up a proportion comparing miles to hours on both sides.

$$\frac{700}{12} = \frac{n}{1}$$

To solve for unit rate, solve for ONE.

Cross multiply.

$$12n = 700$$

Divide by 12 to find n .

$$n = 58.3 \text{ mph}$$

The rate of speed was 58.3 miles per hour.

Scale Factors

Determine the scale factor for the scale.

$$\text{Scale} \\ \frac{1}{2} \text{ in} = 3 \text{ ft}$$

To determine the scale factor, simplify the scaled to actual, expressing each part in the **same unit**.

$$3 \text{ ft} = 36 \text{ inches} \qquad \frac{\frac{1}{2} \text{ scaled inches}}{36 \text{ actual inches}}$$

$$\frac{\frac{1}{2}}{36} \text{ means } \frac{1}{2} \div 36$$

$$\frac{1}{2} \div 36 = \frac{1}{2} \div \frac{36}{1} = \frac{1}{2} \times \frac{1}{36} = \frac{1}{72}$$

$$\frac{1}{72} \leftarrow \text{Scale factor}$$

Scale Models

Sebastian is a friendly and lovable Rottweiler. Using the scale in the picture, determine Sebastian's actual height in inches.

His height in the picture is $1\frac{1}{8}$ in.



Method 1: Set up a proportion comparing scale length to actual length.

$$\frac{\frac{1}{4} \text{ scaled inches}}{8 \text{ actual inches}} = \frac{1\frac{1}{8} \text{ scaled inches}}{n}$$

$$\text{(Cross Multiply)} \quad \frac{1}{4}n = 8 \times 1\frac{1}{8}$$

$$\left(\frac{8}{1} \times \frac{9}{8} = 9\right) \quad \frac{1}{4}n = 9 \quad \text{(Multiply both sides by 4.)}$$
$$n = 36 \text{ in}$$

Method 2: We can also simplify the scale factor first, then solve. It may make the problem simpler to solve.

$$\frac{\frac{1}{4} \text{ scaled inches}}{8 \text{ actual inches}} = \frac{1}{4} \div 8 = \frac{1}{4} \div \frac{8}{1} = \frac{1}{4} \times \frac{1}{8} = \frac{1}{32} \text{ (the scale factor)}$$

Make a proportion using the scale factor.

$$\frac{1}{32} = \frac{1\frac{1}{8} \text{ scaled inches}}{n}$$

Cross Multiply.

$$n = 32 \times 1\frac{1}{8}$$

Multiply.

$$n = \frac{32}{1} \times \frac{9}{8} = \frac{\overset{4}{\cancel{32}}}{1} \times \frac{9}{\cancel{8}_1} = \frac{36}{1}$$

$$n = 36 \text{ in}$$

Making Equations Using Tables and the Constant of Proportionality

Dora has been asked to make cupcakes for the entire swim team. She knows that when she makes 1, 2, or 3 cupcakes how much icing she needs. How much icing will she need for the whole team of 45 swimmers?



The table below shows the ratio of cupcakes to tablespoons of icing. Dora knows how much icing she will need for up to three cupcakes.

number of cupcakes	1	2	3	...	45
tablespoons of icing	3	6	9	...	?

We can tell that this is a proportional relationship because:

- the number of cupcakes increases by 1 each time
- the number of tablespoons of icing increases by 3 each time
- the relationship between the number of cupcakes and tablespoons of icing is multiplying the number of cupcakes by three each time to get the number of tablespoons of icing

The relationship between cupcakes and icing tells the constant of proportionality. The constant of proportionality is three.

Let's look at this table in a different way. Let's put in a middle column and call it a "working column".

number of cupcakes	"working column"	Tbs. of icing
1	$1 \times 3 = 3$	3
2	$2 \times 3 = 6$	6
3	$3 \times 3 = 9$	9
4	$4 \times 3 = ?$?
10	$10 \times 3 = ?$?
45	$45 \times 3 = ?$?

(number of cupcakes) \times (constant of proportionality) = tablespoons of icing

The working column shows how the constant of proportionality really is constant! (Constant means stays the same.) No matter how many cupcakes are made, all we have to do is multiply the number by 3 and we find out how many tablespoons of icing needed. The constant of proportionality is just that - it is a number that we constantly use to keep things in proportion.

If Dora makes 4 cupcakes, then she needs 12 tablespoons of icing. If she makes 10 cupcakes, then she needs 30 tablespoons of icing.

For the whole swim team of 45 swimmers, Dora needs 45 cupcakes. Using the constant of proportionality, we multiply the 45 cupcakes by 3 to get 135 tablespoons of icing.

How do we make this data in the table into an equation? We use variables to represent the parts of the equation we do not know and put in numbers for the parts we do know.

- Number of cupcakes - this can change so we use a variable. Let the number of cupcakes = c .
- Constant of proportionality - this stays the same. It is 3.
- Tablespoons of icing - this can change so we use a variable. Let the tablespoons of icing = t .

Now we rewrite the equation:

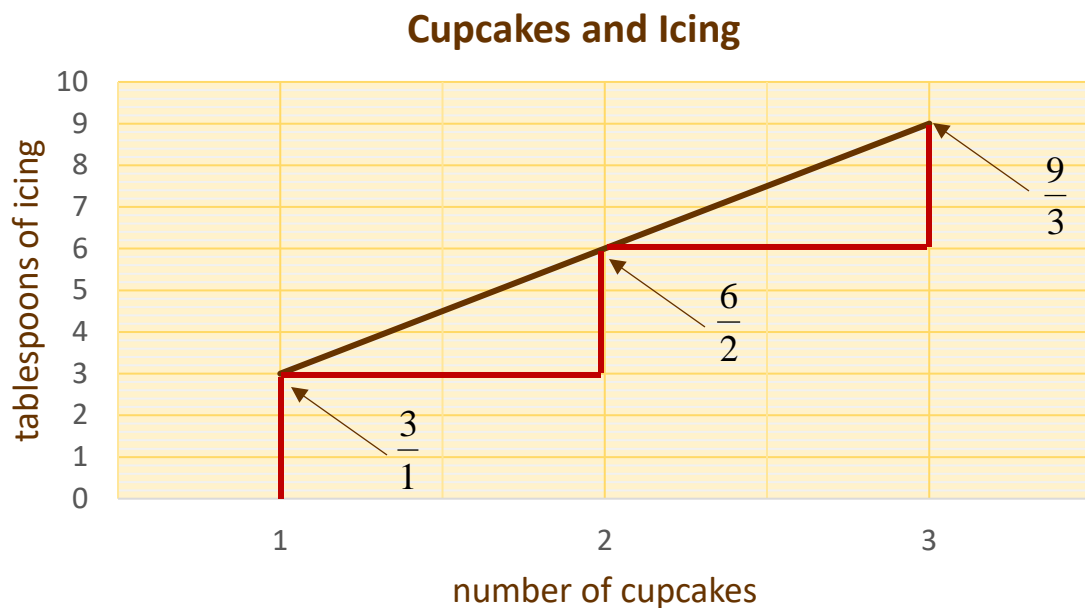
$$(\text{number of cupcakes})(\text{constant of proportionality}) = \text{tablespoons of icing} \rightarrow (c)(3) = t$$

Graphing Rates Using the Constant of Proportionality

We know that the graph of a proportional relationship must go through the origin. We know it must increase. But did you know that by graphing a proportional relationship we can find many different proportions that are also equivalent?

Let's use our cupcake and icing problem again. The graph shows each of the rates as a point on the graph. Remember, to write our table as a series of rates, we have to reverse the rates listed in the chart because the 1 has to be in the denominator of the unit rate.

number of cupcakes	1	2	3
tablespoons of icing	3	6	9



The rate $\frac{3}{1}$ can also be written as the point (1, 3). The rate of $\frac{6}{2}$ can be written as the point (2, 6). The rate $\frac{9}{3}$ can be written as (3, 9).

What if we wanted to find out how much icing is needed to frost half a cupcake? Just follow the blue line between the 0 and 1 on the horizontal axis (number of cupcakes). When you reach the red line, go back to the vertical axis (tablespoons of icing).

It will be $1\frac{1}{2}$. So a new, equivalent rate is $\frac{1\frac{1}{2}}{\frac{1}{2}}$. Looks a little different, but

easy to find using a graph.

Just like there are infinitely many points on a ray, there are infinitely many equivalent rates in a proportional relationship.