EXPONENTS, SQUARE ROOTS, AND PLACE VALUE

Numbers may be raised to powers. A number is said to be “square” when it is raised to the second power. Numbers raised to the third power are said to be “cubed”. The power of a number is called its exponent.

First, we’ll review positive exponents. Then, we’ll look at powers of ten and the meaning of negative powers of ten. Powers of ten are used in expanded notation and scientific notation.

We will extend our understanding of powers by examining other bases with positive exponents, negative exponents and zero as an exponent.

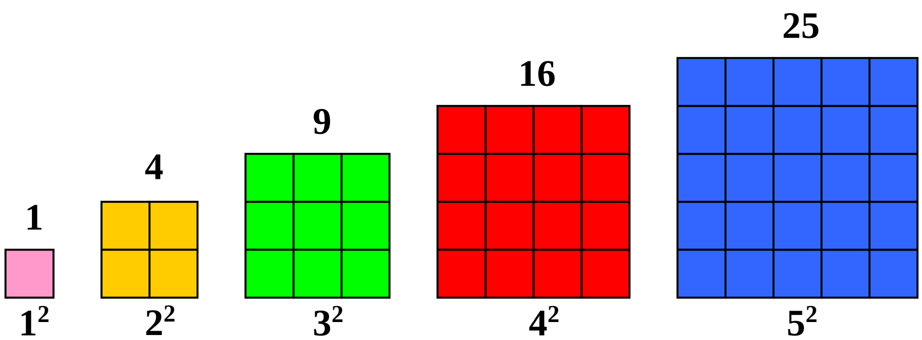
We will apply our knowledge of exponents to place value of whole numbers and place value of decimals. To express the value of each digit in a number, we write the expanded notation. Expanded notation can be expressed in standard notation or exponential notation.

Perfect Squares and Square Roots

Squares

Perfect squares are numbers that are squares of whole numbers.

base → 5^2 ← exponent



Whole Number	Perfect Square
1^2 (1 squared)	**1**
2^2 (2 squared)	**4**
3^2 (3 squared)	**9**
4^2 (4 squared)	**16**
5^2 (5 squared)	**25**

Square Roots

Square roots are numbers that when multiplied by themselves once make perfect squares.

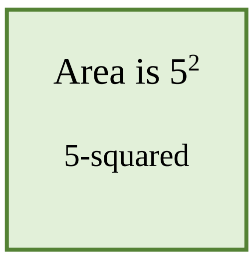
$\sqrt{}$ square root symbol

The **square root of 25 is 5** *written as* $\sqrt{25} = 5$

The **square root of 49 is 7** *because 7 × 7 or* $7^2 = 49$ $\sqrt{49} = 7$

Factors and Powers

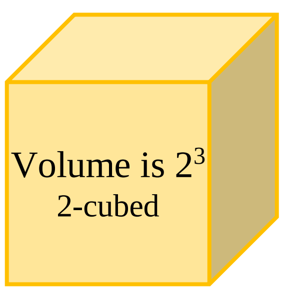
5 to the second power, 5^2, equals 5 × 5 and can be read "5 squared". Think of the area of a square whose side is 5 units long. $5 \times 5 = 5^2$ or "5-squared" or 25.



Side = 5 units in length

$A = l \times w$
$A = 5 \times 5$
$A = 5^2$ or 25 square units

2 to the third power, 2^3, equals 2 × 2 × 2 and can be read "2 cubed". Think of the volume of a cube whose side is 2 units long.
$2 \times 2 \times 2 = 2^3$ or "2-cubed" or 8.



Side = 2 units in length

$V = l \times w \times h$
$V = 2 \times 2 \times 2$
$V = 2^3$ or 8 cubic units

Example: Evaluate 3^6.

3^6, read 3 to the sixth power, equals 3 × 3 × 3 × 3 × 3 × 3 or 729.

$3^6 = 729$

Powers of Ten

Using patterns we will explore powers of ten.

Let's start with 10,000	10,000	=	$10 \times 10 \times 10 \times 10$	or	10^4
Divide 10,000 by 10 to get 1,000	1,000	=	$10 \times 10 \times 10$	or	10^3
Divide 1,000 by 10 to get 100	100	=	10×10	or	10^2
Divide 100 by 10 to get 10	10	=	10	or	10^1
Divide 10 by 10 to get 1	1	=	1	or	10^0
Divide 1 by 10 to get 0.1	0.1	=	$\frac{1}{10}$ or $\frac{1}{10^1}$	or	10^{-1}
Divide $\frac{1}{10}$ by 10 to get 0.01	0.01	=	$\frac{1}{100}$ or $\frac{1}{10^2}$	or	10^{-2}
Divide $\frac{1}{100}$ by 10 to get 0.001	0.001	=	$\frac{1}{1{,}000}$ or $\frac{1}{10^3}$	or	10^{-3}
Divide $\frac{1}{1{,}000}$ by 10 to get	0.0001	=	$\frac{1}{10{,}000}$ or $\frac{1}{10^4}$	or	10^{-4}

Study the chart. Notice that $10^0 = 1$. *Exponent Rule*: Any number raised to a power of 0 equals 1.

In general terms: $a^0 = 1$

Notice that $10^{-1} = \frac{1}{10^1}$ Notice that $10^{-2} = \frac{1}{10^2}$

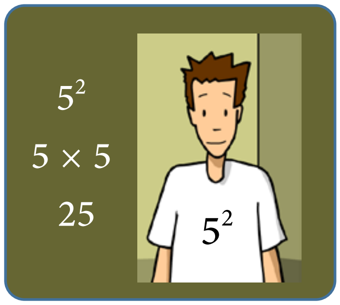
Notice that $10^{-3} = \frac{1}{10^3}$ Notice that $10^{-4} = \frac{1}{10^4}$

Exponent Rule: For a negative exponent, write the expression in a fraction form so that the numerator is 1 and the denominator is the number to the positive power.

In general terms: $a^{-b} = \frac{1}{a^b}$ $a \neq 0$ *"a" cannot be zero because division by 0 is undefined.*

Exponents

Example 1: Evaluate 5^2.



Solution: $5^2 = 25$ ($5 \times 5 = 25$)

Example 2: Evaluate 5^0.

Solution: $5^0 = 1$ (Any number to the zero power equals 1)

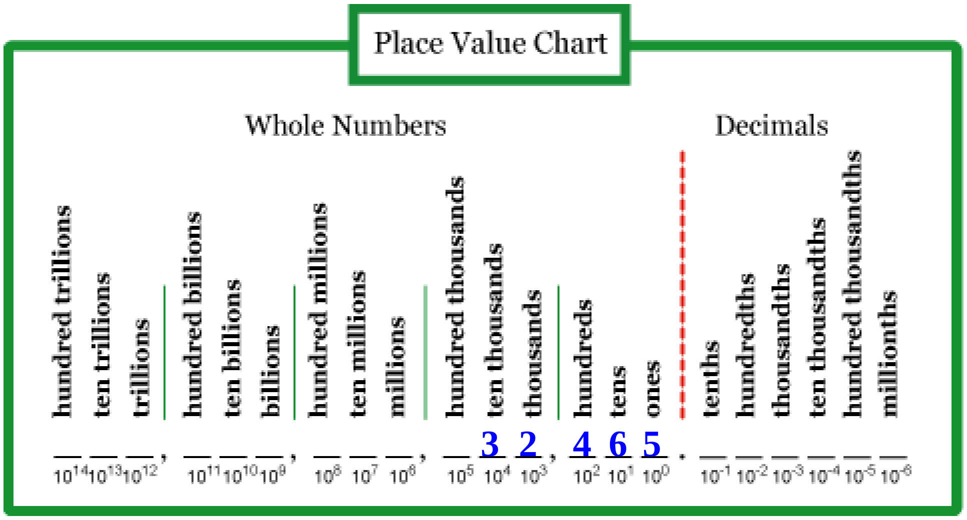
Example 3: Evaluate 5^{-2}.

Solution: $5^{-2} = \frac{1}{5^2} = \frac{1}{5 \times 5} = \frac{1}{25}$

(Numbers raised to negative powers are written as an expression in a fraction form such that the numerator is 1 and the denominator is the number to the positive power.)

Place Value – Whole Numbers

To show the value of each digit in a numeral, write its expanded notation.



Write the standard expanded notation for 32,465.

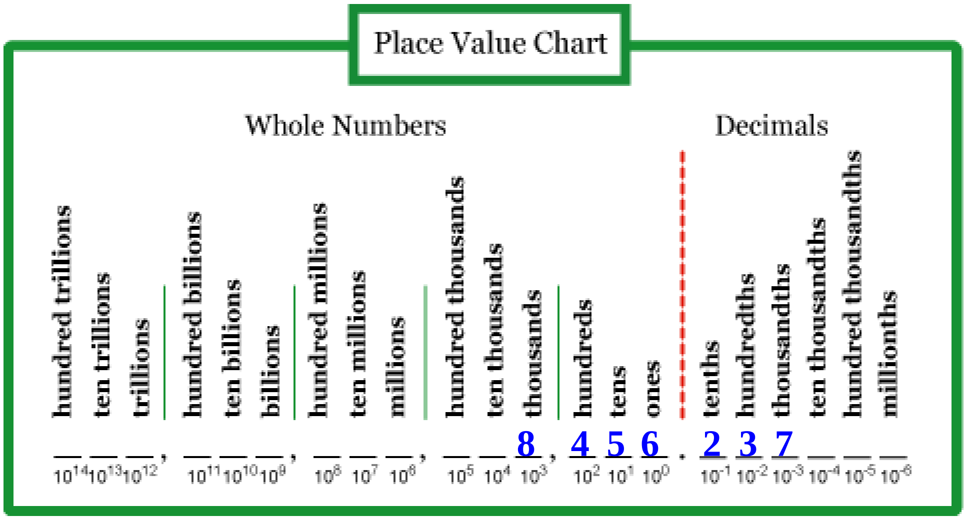
3 ten thousands	+	2 thousands	+	4 hundreds	+	6 tens	+	5 ones
$3 \times 10{,}000$	+	2×1000	+	4×100	+	6×10	+	5×1

Write the expanded notation for 32,465 in exponential form.

3 ten thousands	+	2 thousands	+	4 hundreds	+	6 tens	+	5 ones
3×10^4	+	2×10^3	+	4×10^2	+	6×10^1	+	5×10^0

Place Value – Decimals

To show the value of each digit in a numeral, write its expanded notation.



Write the expanded notation for 8,456.237.

8 thousands + 4 hundreds + 5 tens + 6 ones + 2 tenths + 3 hundredths + 7 thousandths

$$8 \times 1{,}000 + 4 \times 100 + 5 \times 10 + 6 \times 1 + 2 \times \frac{1}{10} + 3 \times \frac{1}{100} + 7 \times \frac{1}{1000}$$

Write the expanded notation for 8,456.237 in exponential form.

8 thousands + 4 hundreds + 5 tens + 6 ones + 2 tenths + 3 hundredths + 7 thousandths

$$8 \times 10^{3} + 4 \times 10^{2} + 5 \times 10^{1} + 6 \times 10^{0} + 2 \times 10^{-1} + 3 \times 10^{-2} + 7 \times 10^{-3}$$