RECIPROCALS AND DIVISION OF FRACTIONS

Sometimes when we divide fractions, the answer will be larger than either of the numbers being divided. That seems strange for division, but when fractions are involved, it can happen.

In division we use reciprocals. The reciprocal of a fraction is its inverse. The product of a fraction and its reciprocal is one. For example: the reciprocal of 2/3 is 3/2 because 2/3 times 3/2 equals one.

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

Write fraction answers using the form in these examples.

Example 1: two-thirds is written as 2/3.
Example 2: five and three fourths is written as 5 3/4.

Reciprocals

The **reciprocal** (inverse) of a fraction is the fraction that is multiplied by the original fraction to equal one (1).

For example, the **reciprocal** of $\frac{1}{2}$ is $\frac{2}{1}$ because $\frac{1}{2} \times \frac{2}{1} = \frac{1}{\cancel{2}^1} \times \frac{\cancel{2}^1}{1} = \frac{1}{1} = 1$.

*To find the reciprocal of a proper fraction, just "flip" the fraction.

> *Example 1*: What is the reciprocal of 3/5?

> Flip the fraction: $\frac{3}{5} \rightarrow \frac{5}{3}$.

> The reciprocal of 3/5 is 5/3.

*To find the reciprocal of a mixed number, first write the mixed number as an improper fraction, and then "flip" the fraction.

> *Example 2*: What is the reciprocal of 2 3/4?

> First, write the fraction as an improper fraction.

$$2\frac{3}{4} = \frac{4 \times 2 + 3}{4} = \frac{11}{4}$$

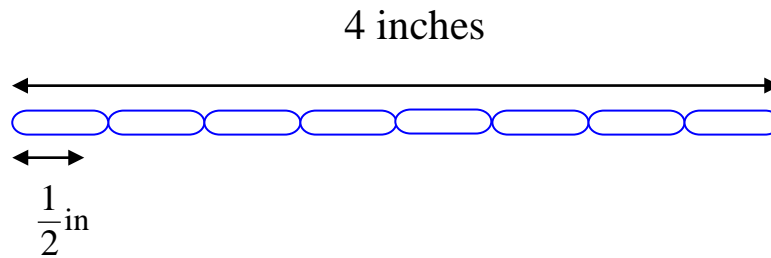> Then, flip the fraction: $\frac{11}{4} \rightarrow \frac{4}{11}$.

> The reciprocal of 2 3/4 is 4/11.

Division of Fractions

Introduction to Division of Fractions

A child's bracelet is four (4) inches long. Each of the links is a half-inch long. How many links are in the chain?

4 inches

$$\frac{1}{2}\text{in}$$

Count the 1/2-inch links to determine the number of links in the bracelet. There are eight. Four divided by one-half equals eight.

$$4 \div \frac{1}{2} = 8$$

It takes eight half-inch links to make up a bracelet four inches long.

*Recall that the answer to a division problem is called the **quotient**, the number being divided is the **dividend**, and the number that the dividend is being divided by is the **divisor**.

$$4 \div \frac{1}{2} = 8$$

*Notice, that in this division problem of fractions, the dividend (4) and the divisor (1/2) are both smaller than the quotient (8). Interesting!!!

To divide fractions, multiply by the **reciprocal or inverse** of the second fraction.

Let's revisit the problem above, and solve it using the algorithm (mathematical process) for dividing fractions.

Example 1: Find the quotient of 4 divided by 1/2.

*Note: When entering a division problem via the keyboard, it would look like 4 / (1/2) . The first slash (/) represents division and the second slash is part of the fraction.

$$4 \div \frac{1}{2} \; =$$

$$= \frac{4}{1} \div \frac{1}{2} \qquad \left[\text{Write 4 as a fraction } (\tfrac{4}{1}). \right]$$

$$= \frac{4}{1} \times \frac{2}{1} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{4 \times 2}{1} = \frac{8}{1} = 8 \qquad \left[\begin{array}{l} \text{Multiply the numerators} \\ \text{and the denominators.} \end{array} \right]$$

Dividing Fractions and Mixed Numbers

Now, we'll take a look at several examples of division of fractions and mixed numbers.

Example 2: Find the quotient of 3/4 divided by 4/5.

$$\frac{3}{4} \div \frac{4}{5} \; =$$

$$= \frac{3}{4} \times \frac{5}{4} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{3 \times 5}{4 \times 4} \qquad \left[\begin{array}{l} \text{Multiply the numerators} \\ \text{and the denominators.} \end{array} \right]$$

$$= \frac{15}{16}$$

Example 3: Find the quotient of 2/3 divided by 8/9.

$$\frac{2}{3} \div \frac{8}{9} =$$

$$= \frac{2}{3} \times \frac{9}{8} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{2^1}{3^1} \times \frac{9^3}{8^4} \qquad \left[\text{Use cancelling.} \right]$$

$$= \frac{1 \times 3}{1 \times 4} = \frac{3}{4}$$

Example 4: Find the quotient of 12 divided by 3/5.

$$12 \div \frac{3}{5} =$$

$$= \frac{12}{1} \times \frac{5}{3} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{12^4}{1} \times \frac{5}{3^1} \qquad \left[\text{Use cancelling.} \right]$$

$$= \frac{4 \times 5}{1 \times 1} = \frac{20}{1} = 20$$

Example 5: Find the quotient of 2 2/7 divided by 6 2/3.

$$2\frac{2}{7} \div 6\frac{2}{3} =$$

$$= \frac{16}{7} \div \frac{20}{3}$$
Change both mixed numbers to improper fractions.

$$= \frac{16}{7} \times \frac{3}{20}$$
Multiply by the reciprocal.

$$= \frac{\cancel{16}^{4}}{7} \times \frac{3}{\cancel{20}^{5}}$$
Use cancelling.

$$= \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$$

Click on the gears below to play a game.

Divide
Fractions