PROBABILITY: THEORETICAL AND ACTUAL

In the real world, some events happen by chance. In mathematics, we study those possibilities through probability. First, we'll look at simple probability.

Theoretical probability is the way the outcomes should happen ideally. We'll examine the outcomes and theoretical probability of tossing two coins. Then, we'll compare the theoretical probability with actual outcomes.

Simple Probability



The probability of selecting a gold star out of the entire container of stars is 4 out of 11.

Probability can be written as a **ratio** as the **number of outcomes desired : all possible outcomes**. The probability of selecting a gold star can be written as **4 : 11**.

Probability can be written as a fraction.

Number of outcomes desiredAll possible outcomes

The probability of selecting a **gold star** can be written as $\frac{4}{11}$.

Another way to write out a probability statement is to say $P(\text{gold star}) = \frac{4}{11}$

What is the probability of selecting a **black star**? Since there are no black stars, then the probability would be 0:11 or $\frac{0}{11}$ or **no** chance of happening.

Another way to write out a probability statement is to say P(black star) = 0.

What is the probability of selecting a **any color** of star in the jar? Since this covers all of the stars, then the probability would be 11 : 11 or $\frac{11}{11}$ or 1 a **sure thing** that it would happen.

Another way to write out a probability statement is to say P(any star) = 1.

Theoretical Probability

When flipping two coins, list all possible outcomes.

Solve by making a chart and filling in all possibilities.

| Theoretically, what is | s the probability of |
|------------------------------------|----------------------|
| tossing two "heads"? | Since there is only |
| one outcome out of four | r with two "heads", |
| the probability is $\frac{1}{4}$. | |

What is the probability of tossing a head and a tail? Since there are two outcomes out of four a "head" and a "tail", the probability $is\frac{2}{4}or\frac{1}{2}$.

What is the probability of tossing two

tails? Since there is only one outcome out of four with two "tails", the probability is $\frac{1}{4}$.

| Coin 1 | Coin 2 | Theoretical Outcome Possibility (as a fraction) |
|--------|--------|---|
| Н | Н | $\frac{1}{4}$ |
| Т | Н | $\frac{1}{4}$ |
| Н | Т | $\frac{1}{4}$ |
| Т | Т | $\frac{1}{4}$ |

Note: A head tossed on Coin 1 and a tail tossed on Coin 2 is a different outcome than a tail tossed on Coin 1 and a head tossed on Coin 2.

| Summar | | |
|---------------|----------------------------|---|
| Possibilities | Outcome (as a fraction) | |
| Two Heads | $\frac{1}{4}$ | $(\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = \frac{1}{2})$ |
| Heads/Tails | 1 2 | `4 ['] 4 4 2' |
| Two Tails | $\frac{1}{4}$ | |



Theoretical Probability Compared to Actual Results

Knowing the theoretical probability of tossing two coins, April predicted that if she tossed two coins 100 times, she would get a tail/head combination 50 times. She organized her data in the table below. Was her prediction accurate?

| Possibilities | Theoretical Outcome as a Fraction and Decimal | Actual Tally | Actual Outcome as a Fraction and Decimal |
|---------------|--|---|---|
| Two Heads | $\frac{1}{4} = 0.25$ | ++++ ++++ ++++ | $\frac{23}{100} = 0.23$ |
| Heads/Tails | $\frac{1}{5} = 0.50 \text{ or } 0.5$ | ++++ ++++ ++++ ++++ ++++ ++++ ++++ +++ | $\frac{51}{100} = 0.51$ |
| Two Tails | $\frac{1}{4} = 0.25$ | ++++ ++++ ++++ ++++ | $\frac{26}{100} = 0.26$ |

Even though the actual prediction was not exactly the same as the theoretical outcomes, the actual tosses closely approached the value of the theoretical probability.

A summary of the results would be that if two coins were tossed, the most likely outcome is a head/tail combination.

