

RATE OF CHANGE

Tables and graphs may be used to show relationships between two variables. Equations can be used to represent the relationship between two variables. For example, the equation, $d = 65t$ could represent the amount of miles traveled (d) by a car on the Interstate which would be related to the time (t) the car traveled. We'll examine some equations, tables, and graphs that show relationships between two variables.

Some software programs, such as Microsoft Excel and Microsoft Works, have built-in graphs or chart wizards that will help in constructing graphs. In this link "Using Technology to Graph Data" are steps that demonstrate how to create a graph using Excel's chart wizard. Every software program works a little differently, but generally follows the same general principle for creating graphs.

Graphs can show rates of change. Sometimes the data will change at a constant rate while times the data's rate of change will vary. Graphs give a visual way to view rates of change both constant and erratic.

INEQUALITIES

We will now switch gears and extend our knowledge of equations to math statements called inequalities. In place of the "equals" sign, inequalities use "greater than" or "less than" signs to make a math statement. When listing answers to inequality problems, there may be several answers.

Graphs of Relationships between Two Variables

Admission to an amusement park costs \$2.00. Amusement park rides cost \$0.75 per ride. Write an equation to show how much it is going to cost to go to the amusement park. Notice that we are now working with **two** variables, the total cost of going to the amusement park and the number of rides. Let's have **R** represent the number of rides and **C** represent the total cost.

We know it costs \$2.00 for admission to the park **plus** the amount of money spent on rides. To find the costs for rides, take the number of rides, **R** and multiply it by the cost of the rides.

$$0.75 \times \mathbf{R}$$

Now we need to add in the cost of admission.

$$0.75 \times \mathbf{R} + 2$$

This equation will equal the total cost of the amusement park.

$$0.75 \times \mathbf{R} + 2 = \mathbf{C}$$

The **R** in this equation is your **independent variable**. This means that the number is fixed and does not change or depend on other variables.

The **C** is the **dependent variable** because the cost depends on how many rides the person goes on.

Think of the equation like a machine. The independent variable is the number put into the equation. This is considered the **input**. The dependent variable is what comes out of the machine. This is called the **output**. To learn more about these terms, click the video link below.

Input and Output (2:01)

$$0.75 \times \mathbf{R} + 2 = \mathbf{C}$$

Let's input a 1 for **R** and see what we get for **C** as an output.

$$0.75 \times \mathbf{1} + 2 = \mathbf{2.75}$$

Now input the numbers to 2 - 6.

$$0.75 \times \mathbf{2} + 2 = \mathbf{3.50}$$

$$0.75 \times \mathbf{3} + 2 = \mathbf{4.25}$$

$$0.75 \times \mathbf{4} + 2 = \mathbf{5.00}$$

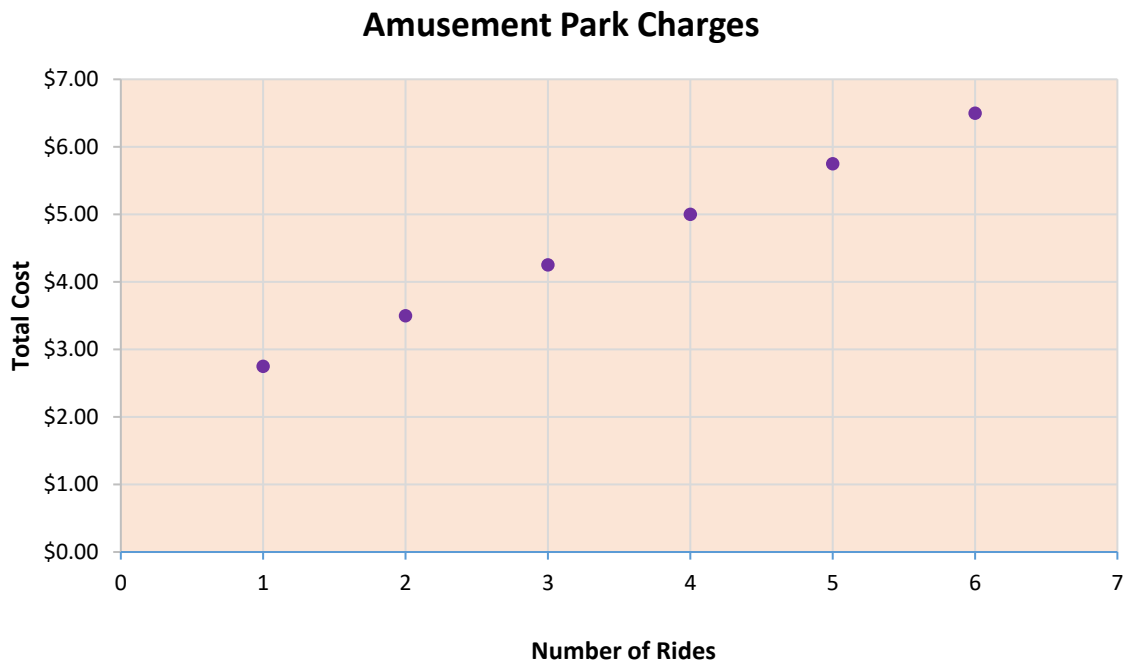
$$0.75 \times \mathbf{5} + 2 = \mathbf{5.75}$$

$$0.75 \times 6 + 2 = 6.50$$

This information can be represented in a table. One column for the input and one for the output.

Number of Rides (R)	Total Cost (C) $C = \$0.75 \times R + \2.00
1	2.75
2	3.50
3	4.25
4	5.00
5	5.75
6	6.50

A graph can be made to show the **relationship** between the number of rides and the total cost. The equation, table, and graph all show the same information, just in different ways.



The relationship is that as the number of rides increases, the total cost increases.

Using Technology to Graph Data

With Excel, a spreadsheet program, you can create some very nice graphs easily. Other software programs also include spreadsheet programs like Microsoft Works.

Here are the steps to use Excel to create a graph. Most spreadsheet programs work in a similar way.

Step One – Type in the data and select the data that is to be graphed, then select the graph icon on the menu bar. The chart wizard will pop up. Select the type of graph. Pressing the bar “Press and Hold to View the Sample” will give you a preview of what your graph is going to look like.

The screenshot shows the Microsoft Excel interface. The data in cells B1:B4 is selected and highlighted in yellow. The Chart Wizard dialog box is open, showing the 'Standard Types' tab. The 'Line' chart type is selected. A preview of the chart is shown, displaying a line with markers for each data point. The 'Press and Hold to View Sample' button is highlighted in yellow. The 'Next >' button is also highlighted with a yellow arrow.

Row	Column A	Column B
1	1	70
2	2	140
3	3	210
4	4	280

Chart Wizard - Step 1 of 4 - Chart Type

Standard Types | Custom Types

Chart type:

- Column
- Bar
- Line
- Pie
- XY (Scatter)
- Area
- Doughnut
- Radar
- Surface
- Bubble
- Stock

Sample:

Line with markers displayed at each data value.

Press and Hold to View Sample

Cancel < Back Next > Finish

Step Two – For a simple graph, just press “next” in the chart wizard.

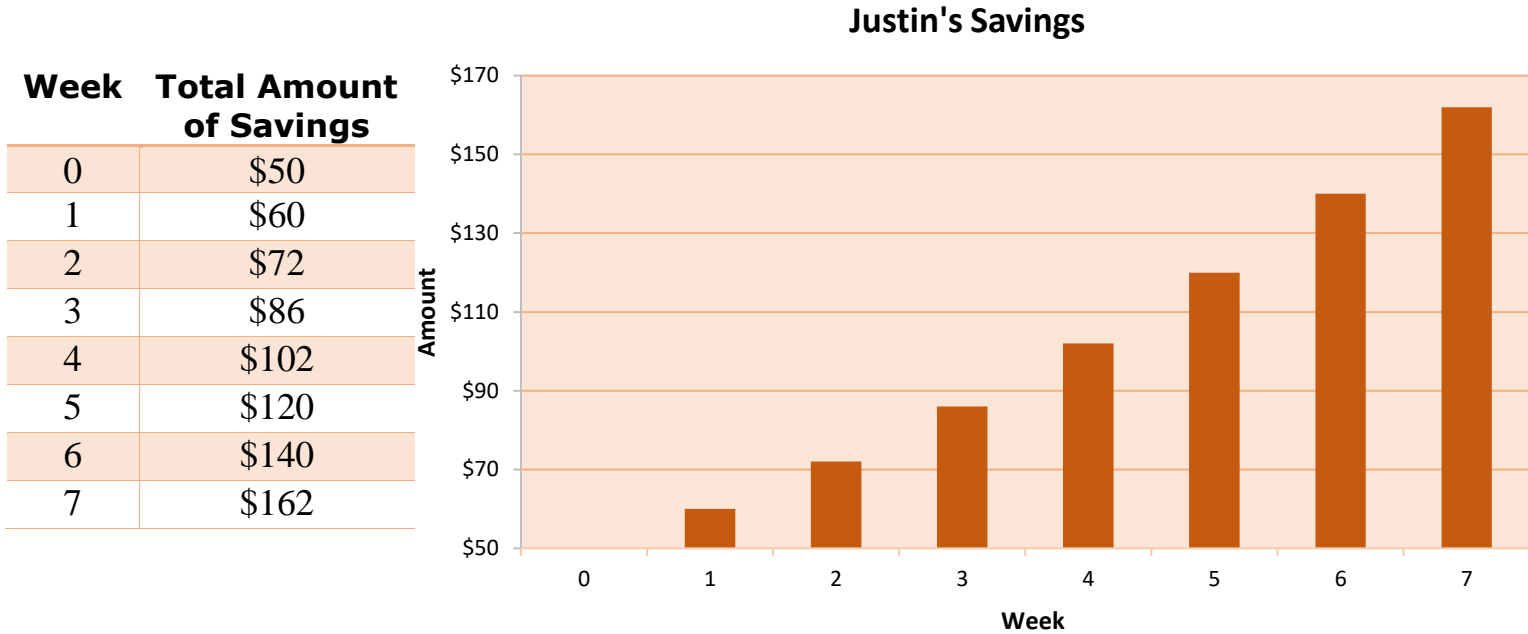
The screenshot shows the Microsoft Excel interface with the Chart Wizard dialog box open. The dialog box is titled "Chart Wizard - Step 2 of 4 - Chart Source Data". It has two tabs: "Data Range" and "Series". The "Data Range" tab is active, showing a line chart with four data points. The data range is set to "=Sheet1!\$B\$1:\$B\$4". The "Series in:" section has "Columns" selected. A yellow arrow points to the "Next >" button.

Row	Column A	Column B
1	1	70
2	2	140
3	3	210
4	4	280

Constant and Varying Rates of Change

Justin is saving for a new bicycle. His grandpa gave him \$50 to begin his savings. The bicycle he would like to have costs \$150. He adds \$10 the first week, \$12 the second week, \$14 the third week and continues in this pattern of savings.

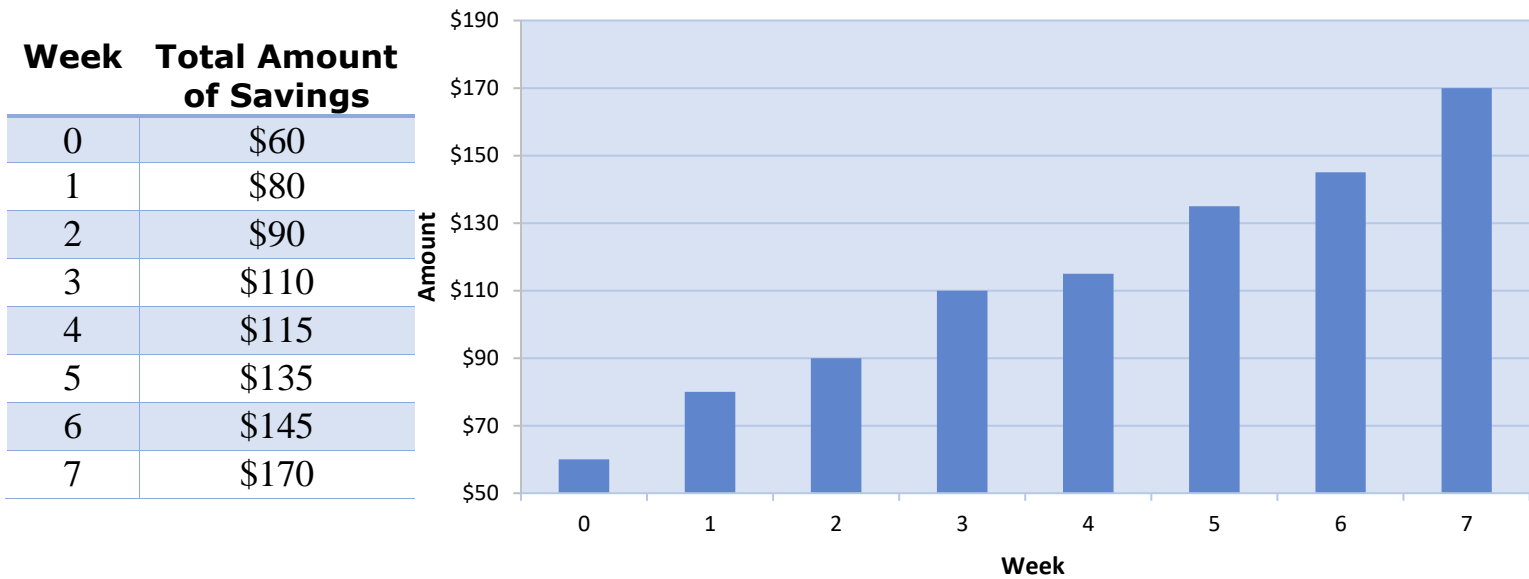
Make a chart to record Justin's weekly savings, and then graph it.



Chris is also saving for a new bicycle. His dad gave him \$60 to begin his savings. Chris mows lawns to earn money for his savings; therefore his savings varies from week to week depending on the weather. His weekly savings is as follows: Week 1 - \$20, Week 2 - \$10, Week 3 - \$20, Week 4 - \$5, Week 5 - \$20, Week 6 - \$10, Week 7 - \$25.

Make a chart to record Chris's weekly savings, and then graph it.

Chris's Savings



Comparing the two boys' savings, **Justin's savings** grows constantly each week with two dollars more than the week before. This kind of growth is called a **constant rate of change**. **Chris's savings** varies from week to week depending on the amount he earns. This kind of growth is called a **varying rate of change**.

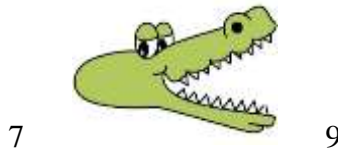
Inequalities

Inequalities are math sentences that make a statement using the less than sign ($<$), the less than or equal to sign (\leq), the greater than sign ($>$), or the greater than or equal to sign (\geq).

Say I have 7 pieces of candy and you have 9 pieces of candy. We can write an inequality for this situation. We know 9 is a bigger number than 7 or 7 pieces is less than 9 pieces.

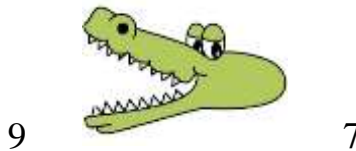
$$7 < 9$$

To remember which sign to use we can think of a hungry alligator. The alligator's mouth is always going to be open to the bigger number.



Our example can also be written as...

$$9 > 7$$



This is the same thing because the alligator's mouth is still open to the bigger number.

Inequalities are solved very similar to our normal equations, but there is a difference with the signs or symbols. In equations we use an equal sign ($=$), that tells us both sides of the equation equal each other. Inequalities tells us that both sides of the equation do not equal the same thing.

Example 1:

Find all whole numbers that when three is added, the result is less than 8.

The **inequality** would be: $x + 3 < 8$

There are two methods for solving inequalities.

Method One: Through trial and error, substitute values in for x to make the math statement true.

Therefore, the solution is “all whole numbers less than 5”, or $\{0, 1, 2, 3, 4\}$.

$$0 + 3 < 8 \text{ since } 3 < 8$$

$$1 + 3 < 8 \text{ since } 4 < 8$$

$$2 + 3 < 8 \text{ since } 5 < 8$$

$$3 + 3 < 8 \text{ since } 6 < 8$$

$$4 + 3 < 8 \text{ since } 7 < 8$$

$$5 + 3 \text{ is not } < 8 \text{ since } 8 = 8$$

$$6 + 3 \text{ is not } < 8 \text{ since } 9 > 8$$

Method Two: The rules for **solving equations** are the **same** for **solving inequalities**. Keep both sides of the inequality sign in balance by undoing what’s been done to the **unknown**.

$$x + 3 < 8$$

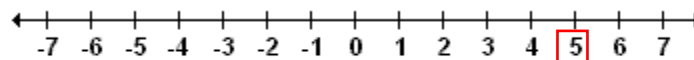
$$x + 3 - 3 < 8 - 3$$

$$x < 5$$

x is all numbers less than 5

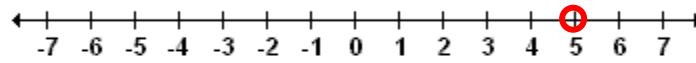
Let’s graph our answer on a number line.

First find the number 5 on the number line.



Number Line

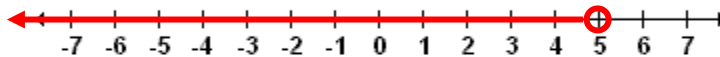
The sign in our answer is less than ($<$) so we need to put an open circle. This open circle tells us that the number 5 is NOT included. 5 is not part of the answer.



Number Line

**Note that when you see a ($<$) or a ($>$) sign you use an open circle on the number line. \circ
When you see a (\leq) or a (\geq) sign you use a closed or shaded in circle. \bullet

For the final step we draw an arrow through all the numbers less than 5.



Number Line

Example 2:

Shawn needs at least 14 tickets to get a prize at the fair. The more tickets he has, the bigger the prize. He already has 8 tickets. Set up an inequality to represent this scenario.

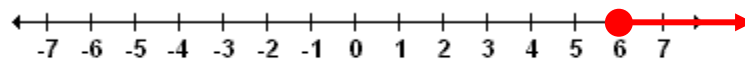
$$x + 8 \geq 14$$

We can write it this way because his number of tickets CAN equal 14 or be more than 14.

Next solve the inequality

$$\begin{aligned} x + 8 &\geq 14 \\ x + 8 - 8 &\geq 14 - 8 \\ x &\geq 6 \end{aligned}$$

Finally graph the inequality.



Number Line

The circle is closed because the inequality had the sign (\geq). We draw the arrow towards the 7 because 7 is greater than 6.