NEGATIVE NUMBERS AND GRAPHING IN THE COORDINATE PLANE

In the real world often times there is a need to represent numbers below zero which we call negative numbers. Negative numbers are an extension of our numbering system. Integers, a group of numbers we will work with in this lesson, include whole numbers, 0, and the numbers opposite the whole numbers which are negative numbers.

Integers

{...-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10...}

We will apply our knowledge of integers by graphing in the coordinate plane which has lots of practical uses. One use is mapping locations in a coordinate graphing system on travel maps.

A mathematical coordinate grid includes four quadrants. First we will look at graphing coordinates in Quadrant I where all numbers are positive, and then we will extend the coordinate plane to include graphing points in Quadrants II, III, and IV which include negative numbers.

Functions may also be graphed. To graph a function we write the input value as the first coordinate and the output value as the second coordinate.

Exploring Negative Numbers

{…-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8...}

Integers are used to show positive and negative quantities.

On Monday, the high temperature was 8° F. On Tuesday the high **dropped 10°.** What was the high temperature on Tuesday?

A number sentence to represent this problem is

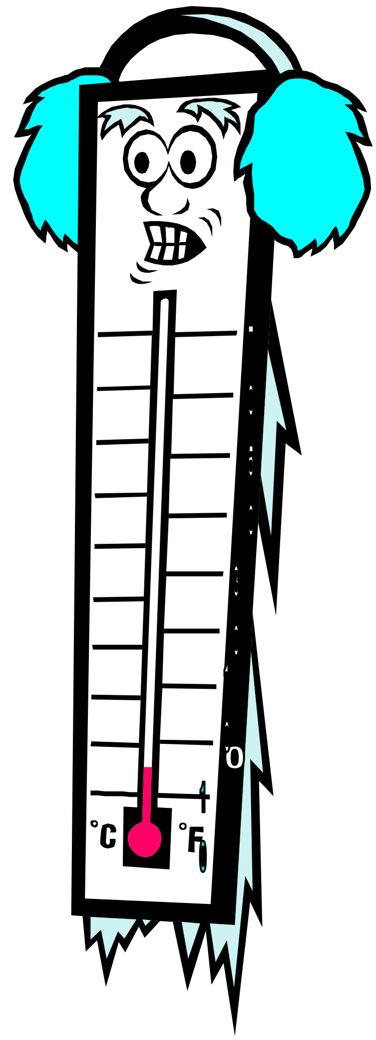
8 – 10 = N

-10

{…-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8...}

-2° = N

The temperature high on Tuesday was a -2° or 2° below zero.



On Wednesday the temperature was a high of -5° F. On Thursday, the high **rose 12°**. What was the high temperature on Thursday?

A number sentence to represent this problem is

-5 + 12 = N

{…-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8...}

12

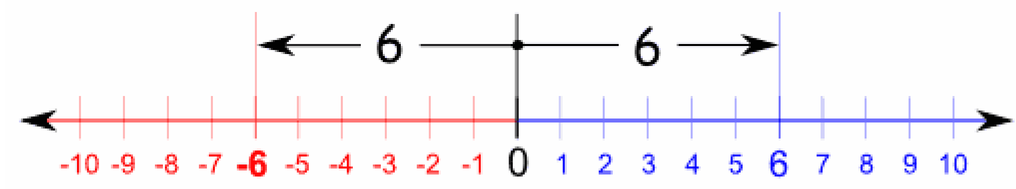
7 = N

The temperature high on Thursday was 7°.

Absolute Value

The absolute value of a number tells us how far away that number is from zero.

For example 6 is 6 units away from zero. -6 is also 6 units away from zero.



Two bars around the number indicate that it's an absolute value.

$$|6|$$

This reads "the absolute value of 6".

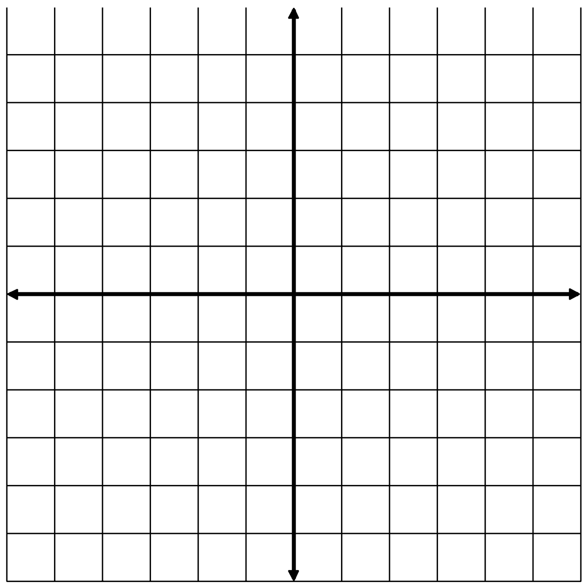
$$|6| = 6$$
$$|-6| = 6$$

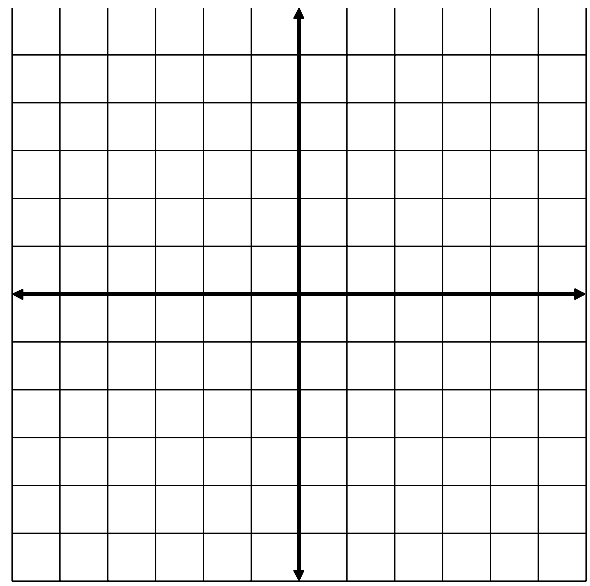
The absolute value of a number is ALWAYS positive.

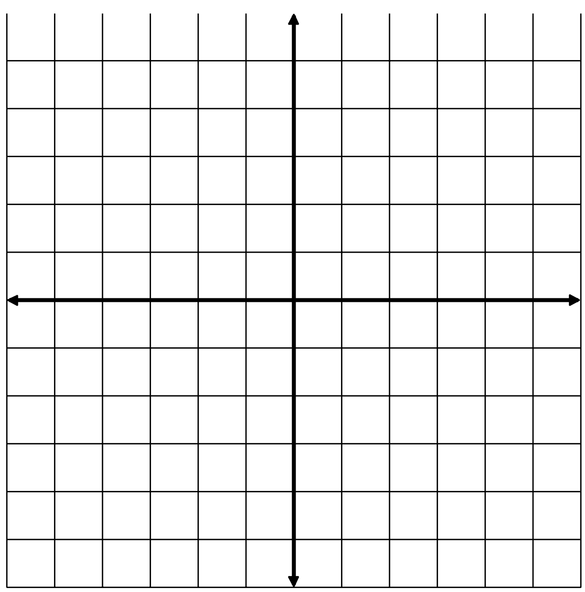
Since the absolute value of a number tells us the distance from 0 the absolute value of 0 is 0.

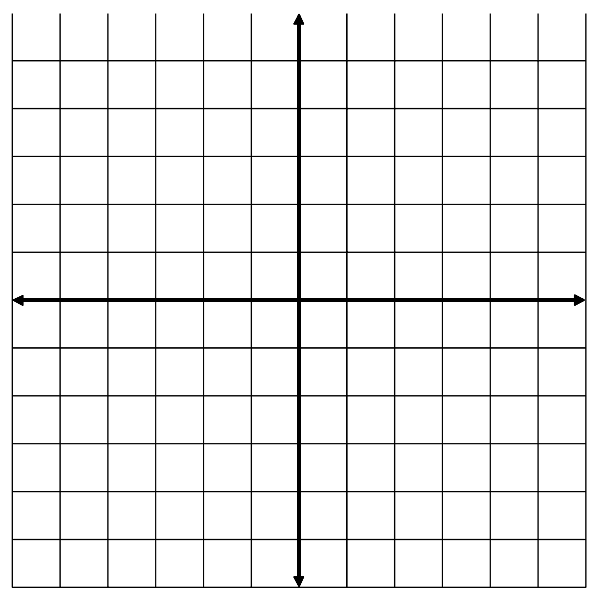
$$|0| = 0$$

Coordinate Planes



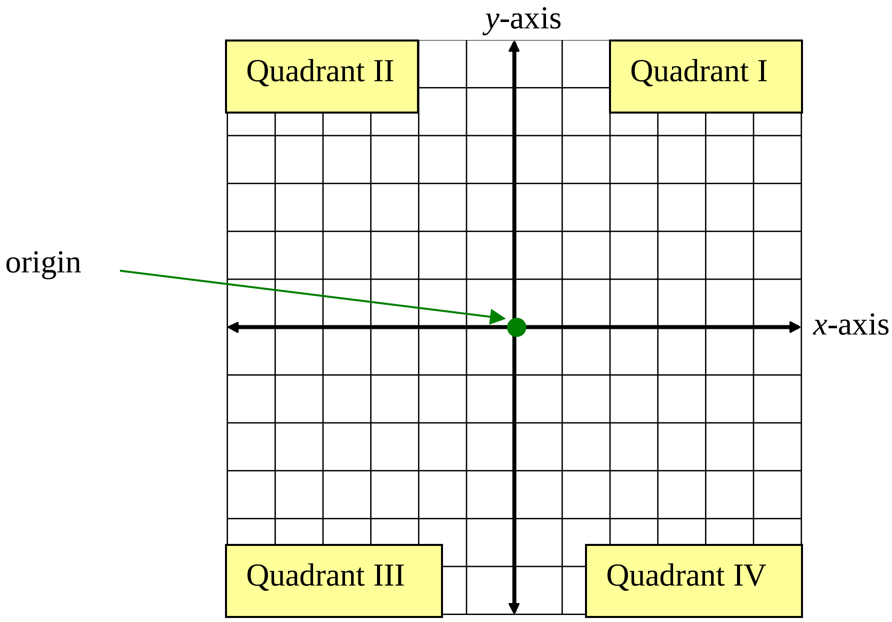






Graphing in Quadrant I of the Coordinate Plane

In a **coordinate plane**, points may be located by **plotting** them. The coordinate plane is divided into **four quadrants** by the ***x*–axis** and the ***y*-axis**. The starting point, the **origin**, is the center, or point where the *x* and *y* axis intersect (cross).



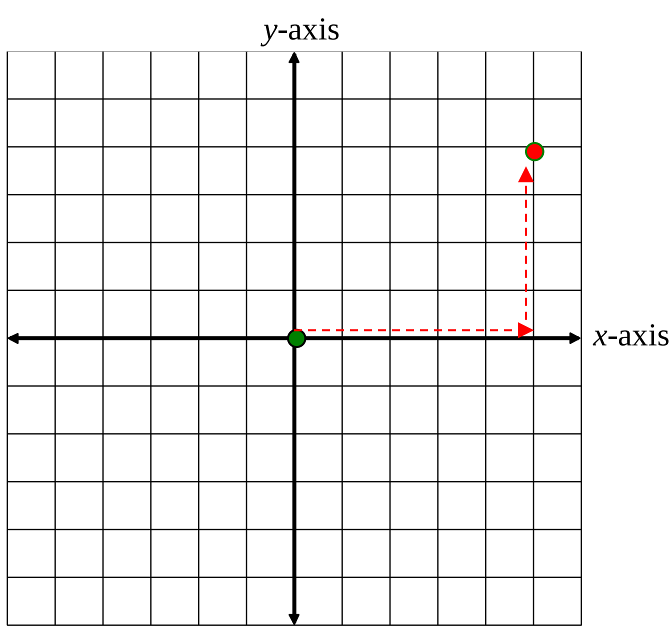
A point is designated by both an ***x*-coordinate** and a ***y*-coordinate.** The origin's coordinates would be (0, 0). The *x*-coordinate is the first number and the *y*-coordinate is the second number.

The ***x*-coordinate** is how far you count **right or left** of the origin. The ***y*-coordinate** is how far you then count **up or down**. A point's location is written as an **ordered pair (*x*, *y*)**.

Plot (5, 4)

When plotting points, start at the origin. Count right if the *x*-coordinate is positive; count left if it is negative. Then count up if the *y*-coordinate is positive; count down if it is negative.

Starting at the origin, count 5 units to the right, and then count 4 units up.



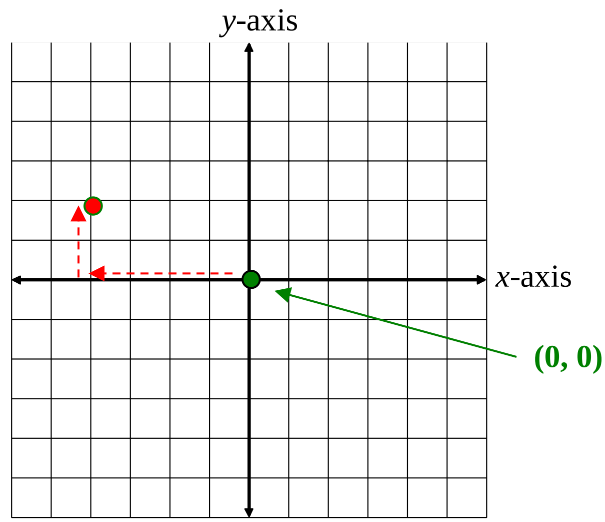
Graphing in Quadrants II, III, and IV

In these graphs, each space represents one unit. The starting point is the origin whose coordinates are (0, 0)

Plot (–4, 2)

When plotting points, start at the origin. Count right if the *x*-coordinate is positive; count left if it is negative. Then count up if the *y*-coordinate is positive; count down if it is negative.

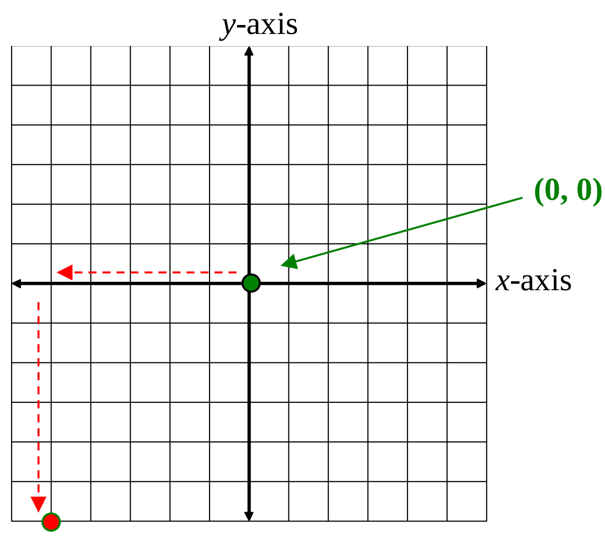
Starting at the origin, count 4 units to the left, and then count 2 units up.



Plot (–5, –6)

When plotting points, start at the origin. Count right if the *x*-coordinate is positive; count left if it is negative. Then count up if the *y*-coordinate is positive; count down if it is negative.

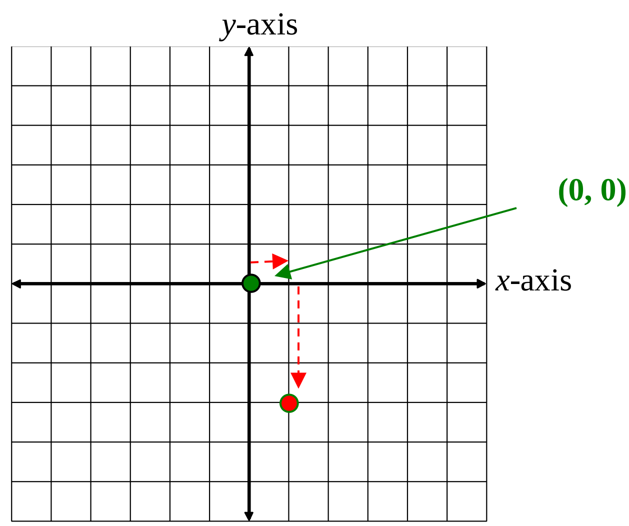
Starting at the origin, count 5 units to the left, and then count 6 units down.



Plot (1, –3)

When plotting points, start at the origin. Count right if the *x*-coordinate is positive; count left if it is negative. Then count up if the *y*-coordinate is positive; count down if it is negative.

Starting at the origin, count 1 unit to the right, and then count 3 units down.



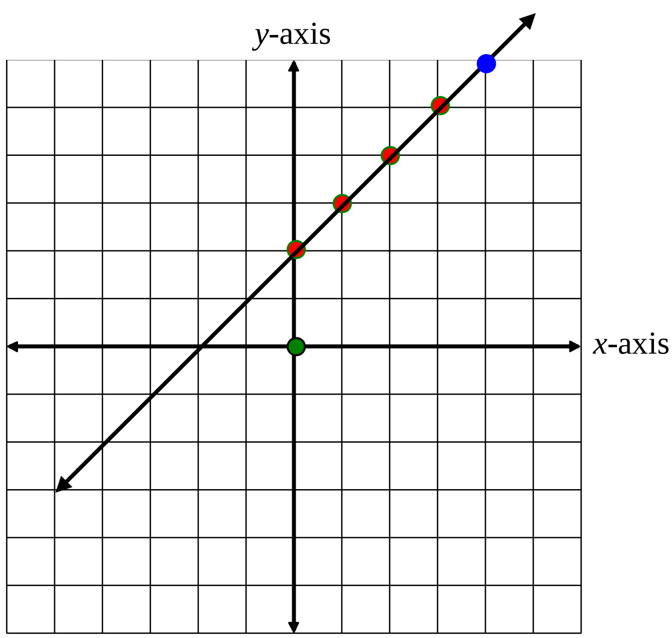
Graphing Functions

Functions may be graphed in a coordinate plane. Use the input as the *x*-coordinate and the output as the *y*-coordinate and write ordered pairs (*x*, *y*). Use the ordered pairs to graph the function.

$y = 2 + x$			
y* = 2 plus *x			
Input (*x*)	Function rule (2 + *x*)	Output (*y*)	Ordered pairs
0	2 + 0	*2*	(0, 2)
1	2 + 1	*3*	(1, 3)
2	2 + 2	*4*	(2, 4)
3	2 + 3	*5*	(3, 5)

Once the ordered pairs are graphed, a line is drawn through the points and beyond. Any ordered pair that can be written from the line will follow the function rule for output.

*Notice the blue point is not in the table but its ordered pair still follows the function rule.



The ordered pair for the **blue** point is (**4,6**).

$y = 2 + x$ ⟶ function rule

$y = 2 + 4$ ⟶ input 4

$y = 6$ ⟶ output 6

Let's see if we can find the rule given the input and output.

Input (x)	Function Rule	Output (y)	Ordered Pairs
0		3	(0,3)
1		4	(1,4)
2		5	(2,5)
3		6	(3,6)

We need to find a pattern between the input and output. The pattern can include addition, subtraction, multiplication, or division.

What do we do to 0 to get 3?

$0 + 3 = 3$

Now look at the rest of the numbers.

$1 + 3 = 4$

$2 + 3 = 5$

$3 + 3 = 6$

So the function is $x + 3 = y$

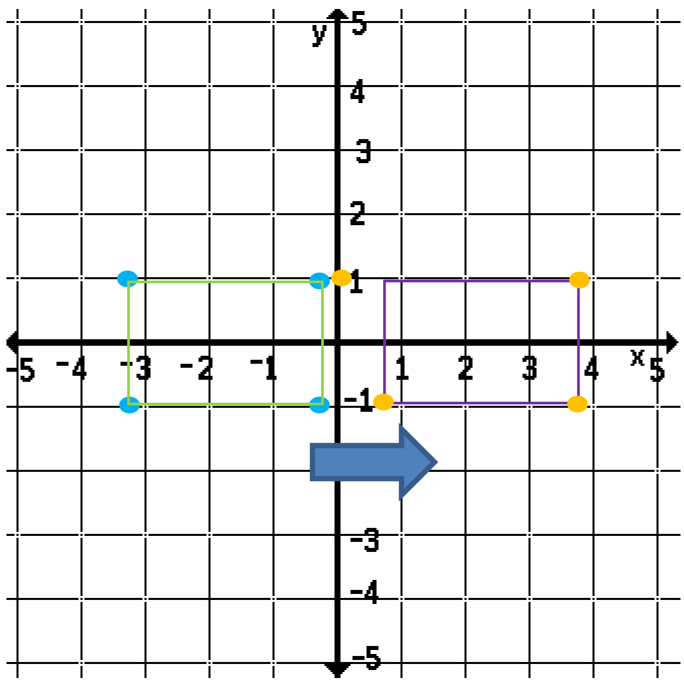
Input (x)	Function Rule	Output (y)	Ordered Pairs
0	0 + 3	3	(0,3)
1	1 + 3	4	(1,4)
2	2 + 3	5	(2,5)
3	3 + 3	6	(3,6)
x	$x + 3$	y	

Graphing Transformations

In Unit 17 we graphed different shapes on a coordinate plane. Now we will look at what happens to the ordered pairs when those shapes are transformed. To review the different type of transformations look back to Unit 19.

The first transformation is **Translation**, this is when the figure slides from one place to another. There are be vertical slides or horizontal slides. The rectangle below has vertices at the coordinates **(-4, 1), (-1, 1), (-1, -1), and (-4, -1).**

Let's translate the rectangle 4 units to the right. So every point in the rectangle gets moved to the right 4 units.

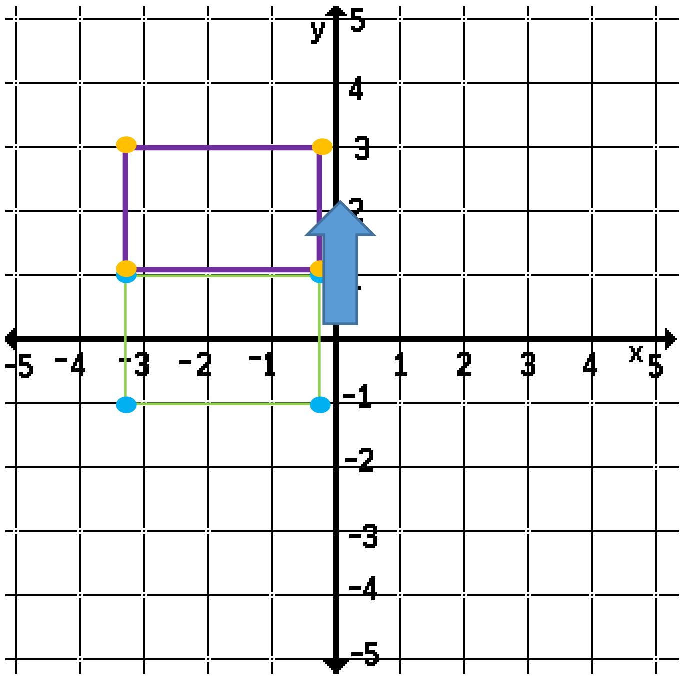


Now let's compare the coordinates between the original rectangle (green) and the translated one (purple).

Original Rectangle	Translated Rectangle
(-4,1)	(0,1)
(-1,1)	(3,1)
(-1,-1)	(3,-1)
(-4,-1)	(0,-1)

Notice the *y*-coordinate in the order pair stays the same and the *x*-coordinate increases by 4. It **increases** because we moved it to the **right**. If we were to move the rectangle to the **left** the *x*-coordinate would **decrease** by that number. This is a horizontal shift, so only the *x*-coordinate changes.

Take the original rectangle and slide it up 2 units

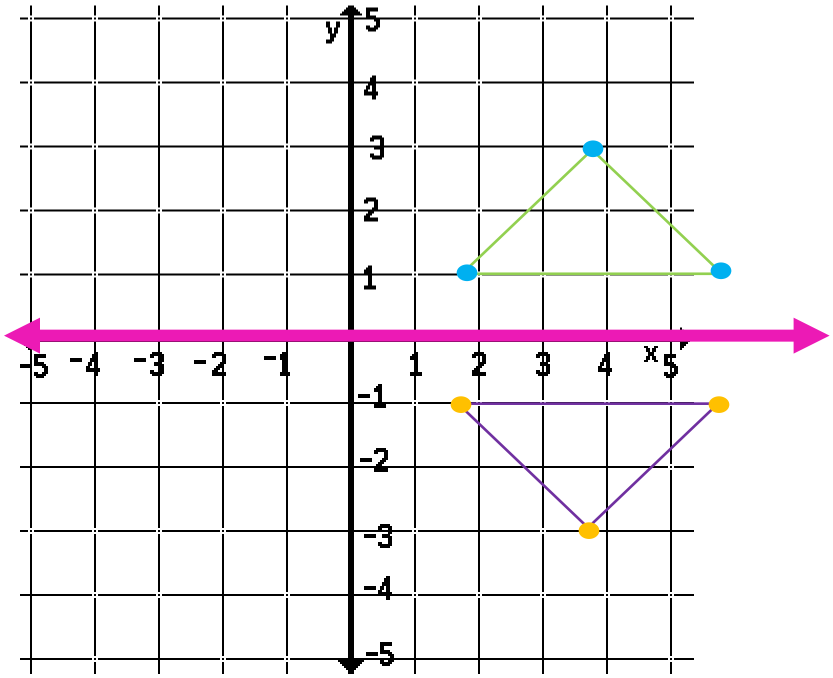


Original Rectangle	Translated Rectangle
(-4,1)	(-4,3)
(-1,1)	(-1,3)
(-1,-1)	(-1,1)
(-4,-1)	(-4,1)

Again notice that the *x*-coordinate stays the same and the *y*-coordinate increases by 2. It **increases** because we move the rectangle **up**. If we were to move the rectangle to the **down** the *y*-coordinate would **decrease** by that number. This is a vertical translation, so only the *y*-coordinate changes.

Now let's look at **Reflections**. This is when we find the mirror image of a figure. We learn about reflections over the *x*-axis and *y*-axis. The triangle below has the vertices **(1, 1), (3, 3), and (5, 1).**

Reflect the triangle over the *x*-axis.

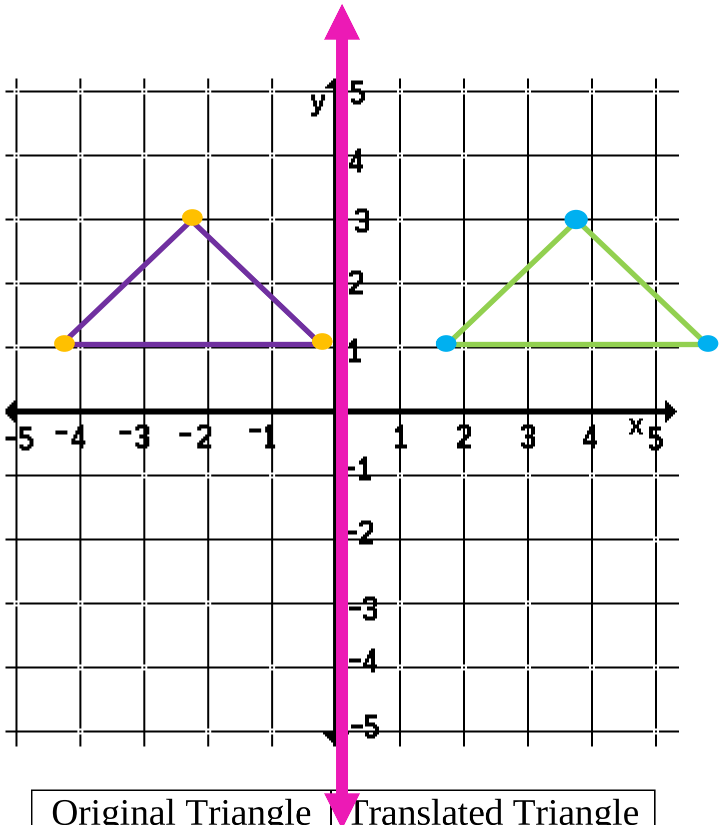


Now let's compare the coordinates between the original triangle (green) and the reflected one (purple).

Original Triangle	Translated Triangle
(1,1)	(1,-1)
(3,3)	(3,-3)
(5,1)	(5,-1)

Notice that the *x*-coordinates stay the same and the *y*-coordinates are opposite each other. This is because the vertical position has changed but not the horizontal. If we could fold our graph along the *x*-axis (pink line), the triangles will match up perfectly.

Reflect the original triangle across the *y*-axis this time.



Original Triangle	Translated Triangle
(1,1)	(-1,1)
(3,3)	(-3,3)
(5,1)	(-5,1)

When reflected over the *y*-axis, the *y*-coordinates stay the same and the *x*-coordinates are now the opposite. This is because the horizontal position has changed but the vertical has not. If we could fold our graph along the *y*-axis (pink line), the triangles will match up perfectly.