VARIABLES, EXPRESSIONS, AND FUNCTIONS

A mathematical expression is a combination of numbers, letters, and symbols. The letters are called variables and represent a numeric quantity. The symbols represent operations such as add, subtract, multiply, and divide. When evaluating expressions we substitute numbers for the variables.

In a function, the output is related to the input through a function rule and for each input value there is exactly one output value. When evaluating functions, numbers will be substituted in for the variable and give unique answers based on the function rule.

Variables and expressions are useful to represent quantities and relationships in problem-solving situations.

PROPERTIES: COMMUTATIVE, ASSOCIATIVE, AND DISTRIBUTIVE

The commutative, associative, and distributive properties can also be used to simplify addition and multiplication. We can apply these properties to help justify steps in mathematical processes and developing formulas.

Evaluating Expressions

In the study of algebra sometimes expressions are evaluated. Here are some expressions to evaluate for a given value assigned to the variable.

4x is a mathematical expression that means **4 times *x***.

What would be the value of ***4x*** when $x = 2.5$?

Solution:
4x means *4 times x*
Substitute 2.5 in for *x*
$4x = 4 \times 2.5 = 10$

(a + b)(a – b) is a mathematical expression that means **the quantity of *a* + *b* times the quantity of *a* – *b***

What would be the value of ***(a+b)(a – b)*** when $a = 10$ and $b = 7$?

Solution:
$(a + b)(a – b) = (10 + 7) \times (10 – 7)$
$(a + b)(a – b) = \quad 17 \quad \times \quad 3 \quad = \mathbf{51}$

Evaluate ***xy*** when $x = \frac{2}{3}$ and $y = \frac{3}{2}$.

xy means ***x*** times ***y***.

$xy = \frac{2}{3} \times \frac{3}{2} = \frac{2}{3} \times \frac{3}{2} = 1$

Evaluate $\frac{m}{n}$ when $m = 10$ and $n = \frac{5}{11}$.

$\frac{m}{n}$ means m divided by *n*

$\frac{m}{n} = 10 \div \frac{5}{11} = \frac{10}{1} \times \frac{11}{5} = \frac{\cancel{10}^{2}}{1} \times \frac{11}{\cancel{5}_{1}} = 22$

Functions

Look at the function $y = 5x$.

Input a number (x) through the function rule ($5x$) to get the output (y).

Below is a chart for several input values, the application of the function rule, and the output values.

$y = 5x$		
y = 5 times x		
Input (x)	Function rule ($5x$)	Output (y)
1	5 times 1	*5*
2	5 times 2	*10*
3	5 times 3	*15*
4	5 times 4	*20*
5	5 times 5	*25*

Look at the function $y = x + 4$.

Input a number (x) through the function rule ($x + 4$) to get the output (y).

Below is a chart for several input values, the application of the function rule, and the output values.

$y = x + 4$		
y = x plus 4		
Input (x)	Function rule ($x + 4$)	Output (y)
1	12 + 4	*16*
2	13 + 4	*17*
3	14 + 4	*18*
4	15 + 4	*19*
5	16 + 4	*20*

Variables and Expressions

A phone company offers a phone card for calling long distance. The cost for a phone call is 4 cents per minute. Study the table an find a pattern for cost.

M (minutes)	C (cost)
1	.04
2	.08
3	.12
4	.16
5	.20
6	.24

M and C may be called variables. A **variable** represents a quantity. In this case the variable M would represent minutes and C would represent Cost.

An **expression** is a combination of variables, numbers and symbols. An expression that represents the relationship of the numbers in the table is **C = $.04 × M.**

The cost of 60 minutes would be:

$$C = \$0.04 \times M$$
$$C = \$0.04 \times 60$$
$$C = \$2.40$$

Let's try another example.

A number plus 13 equals 28.

First set up an equation using the clue you learn in the Evaluating Expressions section.

$$x + 13 = 28$$

What number plus 12 equals 28? We can find out by subtracting 13 from 28.

$$28 - 13 = 15$$

We can check our answer by adding 13 and 15.

$$13 + 15 = 28$$

Commutative, Associative and Distributive Properties

Associative Property

The way the numbers are grouped to add does not change the sum.

$$(a + b) + c = a + (b + c)$$
$$(5 + 2) + 3 = 5 + (2 + 3)$$
$$7 + 3 = 5 + 5$$
$$10 = 10$$

Commutative Property

The numbers may be added in any order giving the same sum.

$$a + b = b + a$$
$$6 + 7 = 7 + 6$$
$$13 = 13$$

Distributive Property

Distribute a number through multiplication over the sum of two numbers.

$$a \times (b + c) = a \times b + a \times c$$
$$4 \times (5 + 2) = 4 \times 5 + 4 \times 2$$
$$= 20 + 8$$
$$= 28$$

Applying the Distributive, Commutative, and Associative Properties

Example 1:

Multiplication of whole numbers can be achieved by using the distributive property. Solve **7 × 54** using the distributive property.

Statement	***Explanation***
7 × (50 + 4)	Write 54 as 50 + 4
(7 × 50) + (7 × 4)	Distribute the 7 over the 50 and the 4
350 + 28	Multiply the numbers inside parenthesis
378	Add

Example 2:

To find the perimeter of a rectangle, there are several formulas that can be used and developed through the commutative, associative, and distributive properties. Find the formulas.

Method One
To find perimeter, add up the length of all four sides.

W

P = L + W + L + W

L

Method Two
To find perimeter, sum up the lengths, sum up the widths, and then add the two sums.

Use the commutative property of addition and switch around the center variables.

P = L + **L + W** + W

Use the associative property to regroup the L's and W's.

P = **(L + L)** + **(W + W)**

Method Three

To find perimeter, double the length, double the width, then add.

Collect like terms (variables from Method Two). In this case, collect the L's and the W's.

P = **2L** + **2W**

Method Four

To find perimeter, add the length and width, then double that sum by multiplying by two.

Using the **distributive property**, **2L + 2W** may be written as **2 (L+ W)**

P = **2 × (L + W)**

Through the commutative, associative, and distributive properties, four methods for finding perimeter of a rectangle have been shown.

Which method do you prefer?