

## **TRANSFORMATIONS, TESSELLATIONS, AND SIMILAR TRIANGLES**

Many designs occur naturally or man-made and display properties of transformation. We will look at these types of transformations: translation, reflection, and rotation.

M. C. Escher, who lived from 1898 to 1972, is a famous Dutch artist who created wonderful tessellations, which were drawings and paintings of fish, birds, and reptiles, in which he intertwined mathematics with transformations. The tessellations were congruent shapes that covered the surface of the artwork and fit together like intricate puzzle pieces.

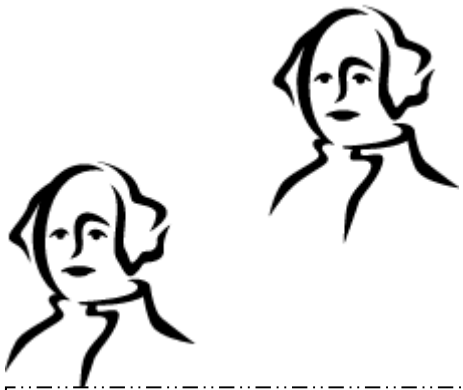
Similar triangles have the same shape but are different in size. The lengths of the corresponding sides are proportional (connect with the same ratio) and the corresponding angles are congruent (equal in measure).

## **CUSTOMARY UNITS CONVERSIONS AND COMPUTATIONS**

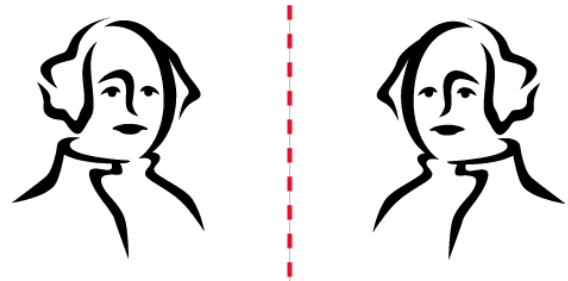
We'll switch gears and review the customary system of measurement which is used in the United States and England. We will practice conversions and computations using conversions from the customary units conversions chart.

# Transformations

Many designs occur naturally or man-made and display properties of translation, reflection, and rotation.



**Translation** – sliding a figure to another location.



**Reflection** – flipping a figure over a line of reference creating a mirror image.



**Rotate** – turning a figure around a specified point call the center of rotation.



**Translation and Reflection** – two transformations have been made. The figure has both been slid to another location and also flipped horizontally.

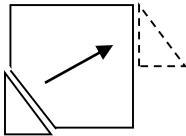
# Tessellations

M. C. Escher who lived from 1898 to 1972 is a famous Dutch artist who created wonder drawings and paintings using mathematics. One type of artwork that he created is called a “tessellation.” He made tessellations of fish, birds, and reptiles. A tessellation is a shape that when translated to another position fits exactly into the other shape. No gaps are showing and the design “covers the plane”, in other words, covers the surface.

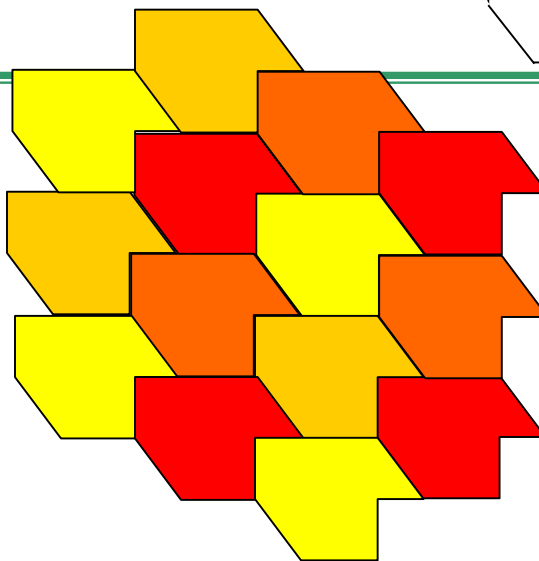
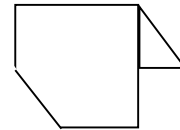
Here is an example of a shape that tessellates the plane.

Start with a square, cut away a piece of the square and translate it to the opposite side.

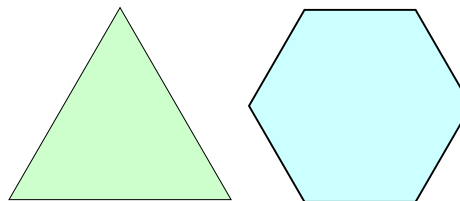
A new shape is formed.



The shape tessellates the plane.

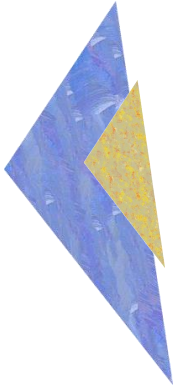


Here are some other regular shapes that tessellate a plane.

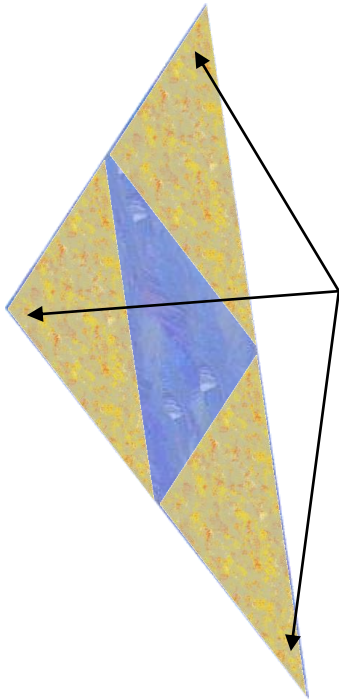


These shapes may be cut up and the pieces translated to make more interesting shapes for tessellating the plane.

## Similar Triangles

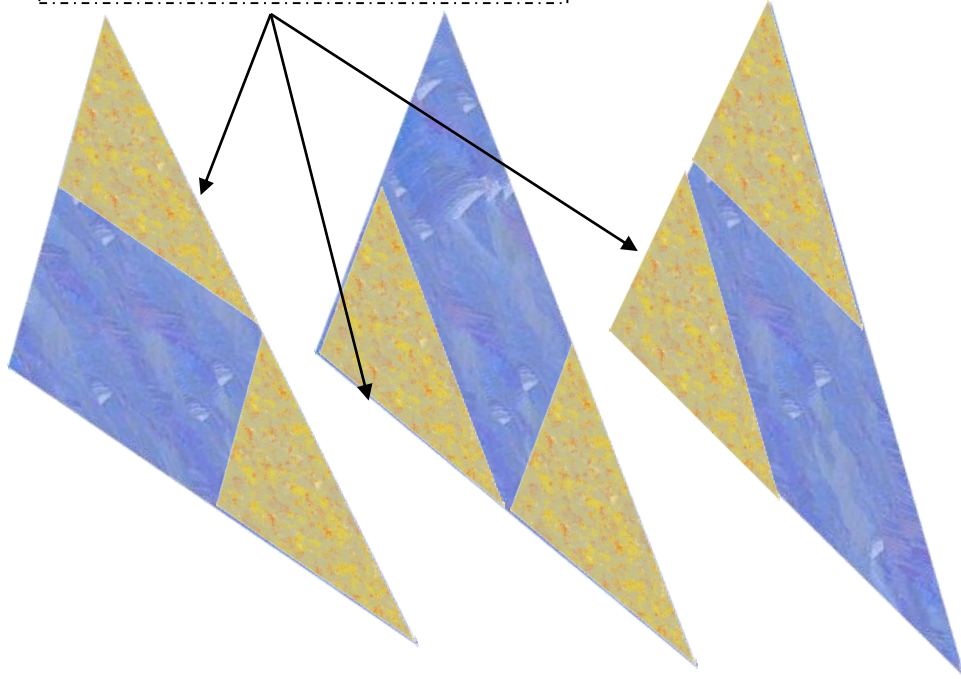


Here are two similar triangles. The larger triangle has sides that are twice as long as each of the corresponding sides for the smaller triangle which gives them a relationship of *2 to 1*.



The angles at the three corners of the larger triangle are the same size as the angles in the smaller triangle. They are **congruent**, that is they have **equal measures**.

The ratio of the lengths of the sides of the larger triangle is 2:1 to the smaller triangle. It **takes two of the lengths** of the smaller triangle to equal the length of the larger triangle



**In similar triangles, the lengths of the corresponding sides are proportional (connect with the same ratio) and the corresponding angles are congruent (equal in measure).**

# Customary Units – Conversions and Computations

To express a **larger unit as a smaller unit**, multiply by the conversion factor.

Example 1: Find how many ounces are in 7 pounds.

$$7 \text{ lb} \times 16 = 112 \text{ oz} \quad (1 \text{ lb} = 16 \text{ oz})$$

There are 112 ounces in 7 pounds.

Example 2: Find how many inches are in 5 feet 4 inches.

$$5 \text{ feet} \times 12 = 60 \text{ inches} + 4 \text{ extra inches makes } 64 \text{ inches.} \quad (1 \text{ ft} = 12 \text{ in})$$

There are 64 inches in 5 feet 4 inches.

To express a **smaller unit as a larger unit**, divide by the conversion factor.

Example 3: Find how many gallons make 18 quarts.

$$18 \text{ qt} \div 4 = 4 \frac{1}{2} \text{ gal} \quad (1 \text{ gal} = 4 \text{ qt})$$

$$\begin{array}{r} 4 \frac{2}{4} = 4 \frac{1}{2} \\ 4 \overline{)18} \\ \underline{16} \\ 2 \end{array}$$

There are 4 1/2 gallons in 18 quarts.

**Here are some sample problems for computing within the customary system of measurement.**

Example 4: Add.

$$\begin{array}{r} 5 \text{ feet } 7 \text{ inches} \\ +2 \text{ feet } 8 \text{ inches} \\ \hline 7 \text{ feet } 15 \text{ inches} \end{array} \begin{array}{l} \text{Simplify} \\ \rightarrow 7 \text{ feet } 15 \text{ in} = 7 \text{ feet} + 12 \text{ in} + 3 \text{ in} \\ \quad \quad \quad 7 \text{ feet } 15 \text{ in} = 7 \text{ feet} + 1 \text{ ft} + 3 \text{ in} \\ \leftarrow 8 \text{ feet } 3 \text{ inches} \end{array}$$

The sum of 5 ft 7 in and 2 ft 8 in equals 8 ft 3 in.

Example 5: Subtract.

Since 22 is smaller than 45, borrow to get 5 hr 82 min

$$\begin{array}{r} 6 \text{ hr } 22 \text{ min} = 5 \text{ hr } 82 \text{ min} \\ -3 \text{ hr } 45 \text{ min} = 3 \text{ hr } 45 \text{ min} \\ \hline 2 \text{ hr } 37 \text{ min} \end{array}$$

$6 \text{ hr } 22 \text{ min} = 5 \text{ hr} + 1 \text{ hr} + 22 \text{ min} =$   
 $5 \text{ hr} + 60 \text{ min} + 22 \text{ min} =$   
 $5 \text{ hr} + 82 \text{ min}$

The difference between 6 hr 22 min and 3 hr 45 min is 2 hr 37 min.

## Customary Units – Conversion Charts

Use these customary unit equivalences to make conversions to add measurements and to subtract measurements.

Units of length – inch, foot, yard, mile	
1 foot (ft)	12 inches (in)
1 yard (yd)	3 ft or 36 in
1 mile (mi)	1760 yd or 5280 ft

Units of weight – ounce, pound, ton	
1 pound (lb)	16 ounces (oz)
1 ton (T)	2000 lb

Units of capacity – ounce, cup, pint, quart, gallon	
1 cup (c)	8 fluid ounces (fl oz)
1 pint (pt)	2 c
1 quart (qt)	2 pt
1 gallon (gal)	4 qt

Units of time – seconds, minutes, hour, day, week, month, year	
1 minute (min)	60 seconds (s)
1 hour (hr)	60 min
1 day (d)	24 hr
1 week (wk)	7 d
1 year (y)	52 wk, 12 months (mo), 365 d