# RATIOS & PROPORTIONS

A ratio is a comparison of two quantities. Ratios can be written several ways. A ratio of 7 to 6 may be written 7:6 or could be written 7/6. The word ratio is pronounced "rash – e – o" with a "long a" in "rash".

A very useful problem solving technique is to learn how to write a proportion for information given in a problem. This is called solving problems using the ratios and proportions.

# ESTIMATE REASONABLE SOLUTIONS WITH FRACTIONS AND DECIMALS

We'll now switch gears a little and look at working with fractions and decimals. Often times all that we need is an estimate of the actual amount. We will use rounding to nearest 1/2 for fractions and nearest 0.5 for decimals to estimate a reasonable solution.

# Ratios

These leaves can be compared many ways.

The ratio of yellow leaves to green leaves is **4 to 3** which can also be written as **4 : 3** or $\frac{4}{3}$.

The ratio of red leaves to brown leaves is **2 to 3** or **2 : 3** or $\frac{2}{3}$.

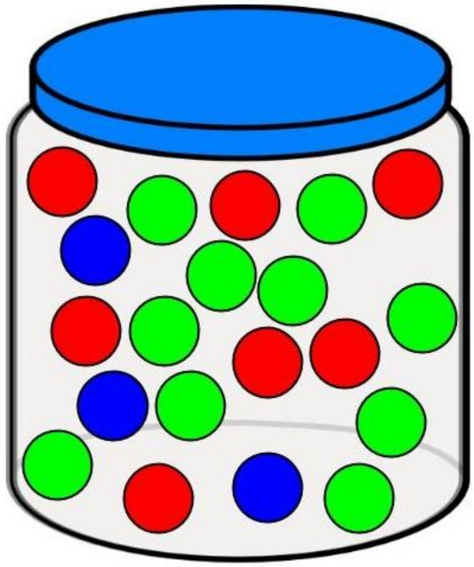
The ratio of green to all the leaves is **3 to 12** or **3 : 12** or $\frac{3}{12}$ .

The ratio of all to red is **12 to 2** or **12:2** or $\frac{12}{2}$.

Ratios such as 4:3, $\frac{4}{3}$ remain in fraction form (do not simplify to mixed number).

Ratios such as 3:12, $\frac{3}{12}$ may be reduced to its simplest form $\frac{1}{4}$, 1:4.

Ratios such as 12:2, $\frac{12}{2}$, may be reduced to $\frac{6}{1}$, 6:1 (do not simplify to a whole number).



Now let's compare the different colored marbles.

The ratio of blue marbles to green marbles is **3 to 10** which can also be written as **3:10**, or $\frac{3}{10}$.

The ratio of red marbles to blue marbles is **7 to 3**, **7:3**, or $\frac{7}{3}$.

The ratio of green marbles to all the marbles are **10 to 20**, **10:20**, or $\frac{10}{20}$.

*Notice that the above ratio can be reduced to **1 to 2**, **1:2**, or $\frac{1}{2}$.

The ratio of all the marbles to the blue marbles is **20 to 3**, **20:3**, or $\frac{20}{3}$.

## Unit Rate

A unit rate is a special type of ratio. It compares two **different** measurements or units. You will label your answer with a compound unit. This will include the numerator measurement, the world *per,* and then the denominator measurement. For example a compound unit is speed, it can be written as miles per hour, miles/hour, or mph. When finding a unit rate we want to see how many objects go with **ONE** of something else. Like **1** camp counselor has **10** campers. So the denominator of a unit rate can be simplified to 1.

Let's look at an example. The class field trip this year is to the zoo. You have 32 peanuts to feed the elephants. You want to give each elephant the same amount of peanuts so it is fair. There are 4 elephants. How many peanuts does each elephant get?

Set up a ratio of the number of peanuts to the number of elephants.

$$\frac{number\ of\ peanuts}{number\ of\ elephants}$$

Now substitute the numbers into the equation.

$$\frac{number\ of\ peanuts}{number\ of\ elephants} = \frac{32\ peanuts}{4\ elephants}$$

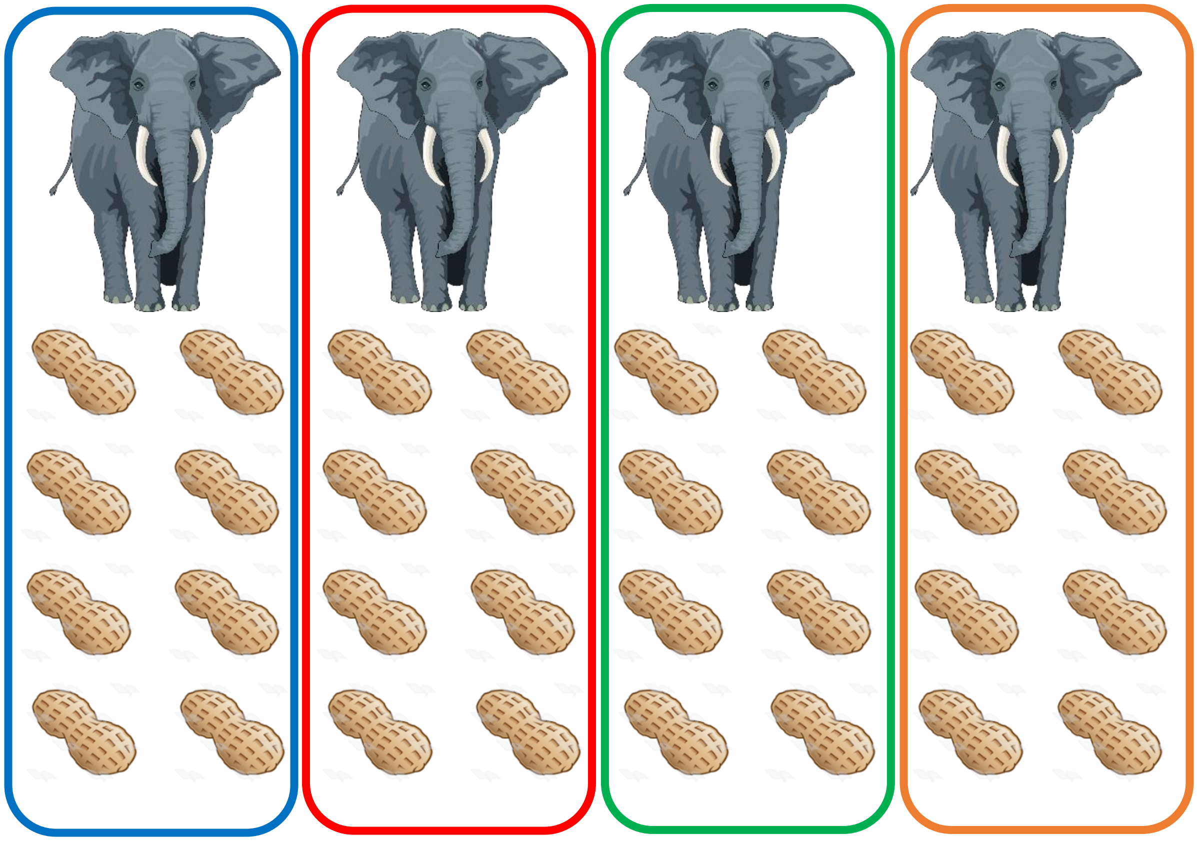
To get a 1 in the denominator we need to divide it by 4. Since we divided the denominator by 4 we have to do the same thing to the numerator.

$$\frac{number\ of\ peanuts}{number\ of\ elephants} = \frac{32\ peanuts}{4\ elephants} \div \frac{4}{4}$$

By dividing by four, you simplify the expression.

$$\frac{number\ of\ peanuts}{number\ of\ elephants} = \frac{32\ peanuts}{4\ elephants} \div \frac{4}{4} = \frac{8\ peanuts}{1\ elephant}$$

So each elephant gets 8 peanuts.  We can also show this visually by setting up a box for each elephant, so you will have 4 boxes. Begin to place one peanut in each box until you run out of peanuts.  When you are done each box will have 8 peanuts.



Speed is a great example of a unit rate. Let's say you are driving to the beach. The beach is 1,000 miles away from your house. It takes you 20 hours to get there. How fast were you going?

You can find out by setting up a ratio of distance to time.

$$\frac{Distance}{Time}$$

The distance in this example is miles and the time is hours.

$$\frac{Distance}{Time} = \frac{Miles}{Hours}$$

Now just insert the number of miles and hours into the formula.

$$\frac{Distance}{Time} = \frac{Miles}{Hours} = \frac{1{,}000\ miles}{20\ hours}$$

To get a 1 in the denominator we need to divide it by 20. Since we divided the denominator by 20 we have to do the same thing to the numerator.

$$\frac{Distance}{Time} = \frac{Miles}{Hours} = \frac{1{,}000\ miles}{20\ hours} \div \frac{20}{20}$$

This simplifies the expression to get the following ratio.

$$\frac{Distance}{Time} = \frac{Miles}{Hours} = \frac{1{,}000\ miles}{20\ hours} \div \frac{20}{20} = \frac{50\ miles}{1\ hour}$$

Our units with our answer include the both measurements, miles and hours. The answer can be written in different ways.

50 miles per hour or 50 miles/hour or 50 mph or $\frac{50\text{ miles}}{1\text{ hour}}$

Now that you found a unit rate, you can use it to find other information.

At 300 miles your family decides to stop for lunch. How long were you in the car for? Take the number of miles you were in the car for and divide it by your unit rate.

$$\frac{300\ miles}{50\ miles/hour}$$

The miles cancel out leaving you with hours

$$\frac{300\ \cancel{miles}}{50\ \cancel{miles}\ per\ hour} = 6\ hours$$

You were in the car 6 hours before your first stop.

We can also put this information into a ratio table.

You know that in 1 hour you travel 50 miles. So in 2 hours you go 2 x 50 miles, which equals 100. Continue taking the number of hours in the car, multiplied by 50 to get the number of miles traveled in that many hours.

$$1 \times 50 = 50$$

$$2 \times 50 = 100$$

$$3 \times 50 = 150$$

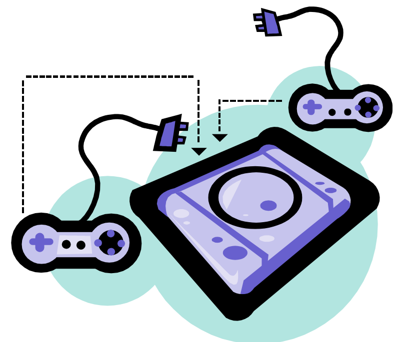
$$4 \times 50 = 200$$

$$5 \times 50 = 250$$

$$6 \times 50 = 300$$

| | 1 hour | 2 hours | 3 hours | 4 hours | 5 hours | 6 hours |
|---|---|---|---|---|---|---|
| Miles | 50 | 100 | 150 | 200 | 250 | 300 |

## Applications of Proportions



Cindy surveyed the sixth grade class and found that 11 out of 25 sixth-grade students have a video game console. There are 500 students in the class. Based on the data Cindy collected; predict how many sixth grade students may have a videogame console.

To solve, first find the comparison ratio.

$$\frac{11\,(\text{have a video game console})}{25\,(\text{all students surveyed})}$$

Then, set up a proportion where both numerators represent the same type of quantity and both denominators represent the same type of quantity.

*Setting up the problem in a word ratio first can be very helpful.*

For this problem, the word ratio would be: $\dfrac{\text{have a video game console}}{\text{all students in the group}}$.

Thus, the proportion would be: $\dfrac{11}{25} = \dfrac{n}{500}$.

Solve.

$$\frac{11}{25} = \frac{n}{500} \xrightarrow[\text{multiply}]{\text{cross}} \frac{11}{25} = \frac{n}{500}$$

$$\begin{aligned} 25 \times n &= 11 \times 500 && \text{Simplify.} \\ 25n &= 5500 && \text{Divide.} \\ n &= 220 \end{aligned}$$

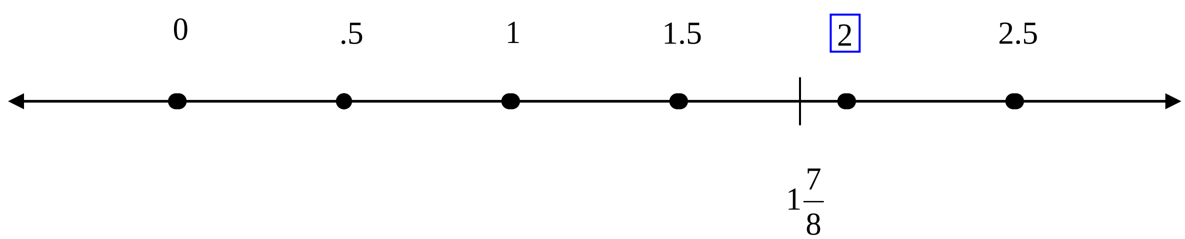
**Cindy predicted that about 220 sixth grade students have a video game console.**

# Estimate Reasonable Solutions with Fractions and Decimals

Estimate a reasonable solution for $1\frac{7}{8}$ times 0.87.

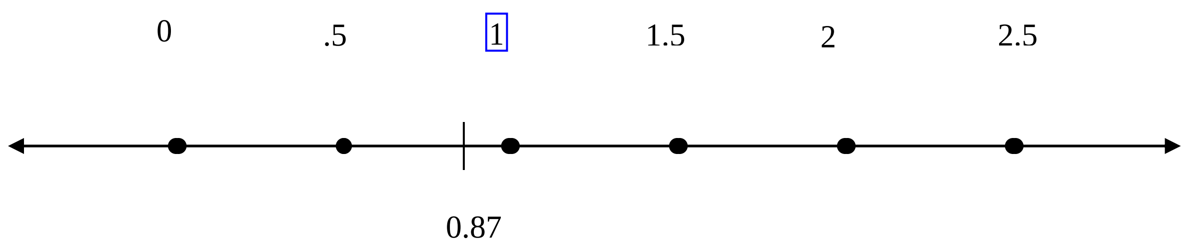
Round to nearest $\frac{1}{2}$. (For a detailed explanation refer back to Lesson 3.)

$1\frac{7}{8}$ is close to **2** on the number line.




**Round to nearest 0.5. (For a detailed explanation refer back to Unit 8.)**

0.87 is close to 1 on the number line.



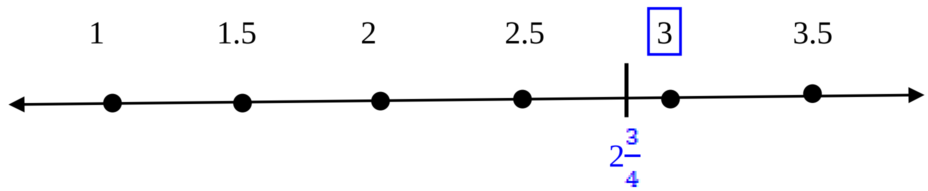

$$1\frac{7}{8} \times \mathbf{0.87} \approx \mathbf{2} \times \mathbf{1} \approx \mathbf{2}$$

Note: $\approx$ is a symbol that means "approximately equal to".

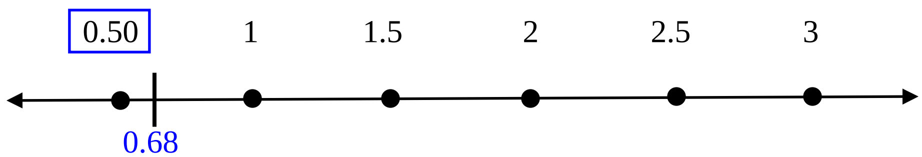
Let’s try another example.

Estimate a reasonable example for $2\frac{3}{4}$ plus 0.68.

$2\frac{3}{4}$ is close to **3** on the number line.




0.68 is close to **0.50** on the number line.




$2\frac{3}{4} + 0.68$

$3 + 0.50 = 3.50$

$2\frac{3}{4} + 0.68 \approx 3.50$