# DECIMALS: EQUIVALENCE AND MULTIPLICATION

To multiply decimals, we multiply and then count the number of decimal places to the right of the decimal point for each decimal number. We then add them together to find the total number of decimal places needed in the answer. We'll first look at modeling multiplication of decimals to find out why we place the decimal point in the answer this way. We will use decimal squares to create the model.

After investigating decimal models, we will practice multiplying decimals and extending our knowledge with more decimal multiplication using paper and pencil.

When computing with decimals some answers can be simplified. We will examine equivalent decimals and find ways to simply answers.

Modeling Multiplication of Decimals
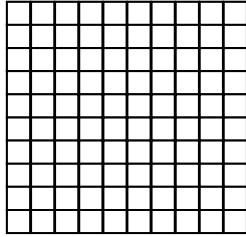
10 by 10	1 by 10	1 by 1
represents 1	represents	represents
	0.1 or one tenth	0.01 or one hundredth

To Show 0.9 x 0.6

Use ones square (10 by 10).

Step 1
Shade in 0.9 (9 one-tenth strips) of one color (yellow) horizontally all the way across the ones block.

Step 2
In the same ones block, shade in 0.6 (6 one-tenth strips) using a different color (blue) vertically all the way to the top.

The Result
The hundredths squares that are colored with both blue and yellow, resulting in green is the area that is the answer to the multiplication problem.
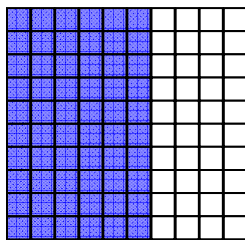
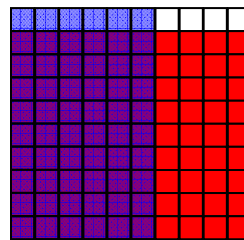| **Step 1**	**Step 2**	**The Result**
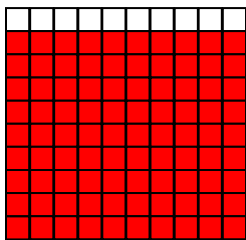
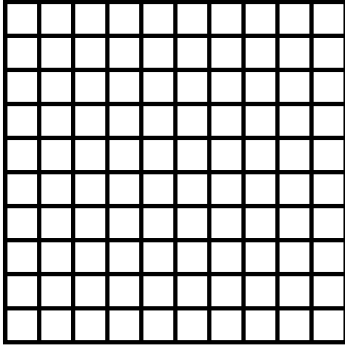| *0.9*	x	*0.6*	=	*0.54*
				(54 hundredths)

By the rule for placing the decimal point in the answer, count the places to the right of the decimal in 0.9, which is one, and also there is one in 0.6, totaling two decimals needed for the answer 0.54.

Notice in the model that 54 of the one hundredths squares are shaded green.

Decimal Squares

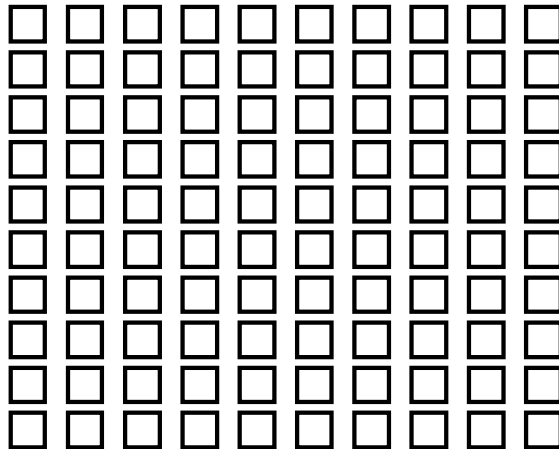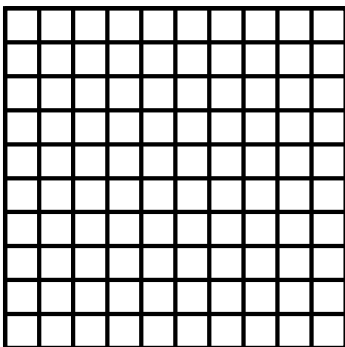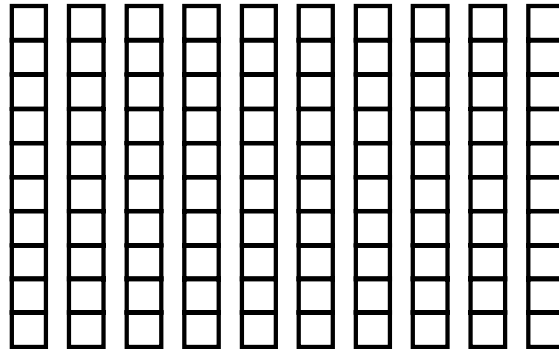1 Whole or $\dfrac{100}{100ths}$ $\dfrac{1}{10th}$
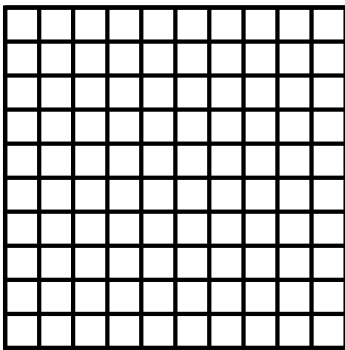
$\dfrac{1}{100th}$

0.01

1   **0.1**

Multiply Decimals

Multiplying Decimals Less Than One

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).

Multiply 0.7 × 0.9   *Estimate: 1 × 1 = 1*

one decimal place $\longrightarrow$ **0.7**

one decimal place $\longrightarrow$ **× 0.9**

total – two decimal places $\longrightarrow$ **0.63**

Why two places?
Write both as fractions and multiply.
$$\frac{7}{10} \times \frac{9}{10} = \frac{63}{100} = 0.63$$

Answer: 63 hundredths

Check: The estimate, 1, is close to 0.63.

Multiply 0.12 × 0.36   *Estimate: 0 × 0 = 0*

two decimal places $\longrightarrow$ **0.12**

two decimal places $\longrightarrow$ **× 0.36**

72

36

total – four decimal places $\longrightarrow$ **0.0432**

Why four places?
Write both as mixed fractions and multiply.
$$\frac{12}{100} \times \frac{36}{100} = \frac{432}{10000} = 0.0432$$

Use 0 as a placeholder before the numerals to give 4 decimal places.

Answer: 432 ten thousandths

Check: The estimate, 0, is close to 0.0432.

More Decimal Multiplication

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).

Multiply 5.23 × 7.9   *Estimate: 5 × 8 = 40*

two decimal places ⟶ **5.23**
one decimal place ⟶ **× 7.9**
4707
3661
total – three decimal places ⟶ **41.317**

Answer: 41 and 317 thousandths

Check: The estimate, 40, is close to 41.317.

Why three places?
Write both as mixed fractions and multiply.

$$5\frac{23}{100} \times 7\frac{9}{10} =$$

$$\frac{523}{100} \times \frac{79}{10} = \frac{41317}{1000} =$$

$$41\frac{317}{1000} = 41.317$$

Multiply 46 × 2.8   *Estimate: 50 × 3 = 150*

zero decimal places ⟶ **46**
one decimal place ⟶ **× 2.8**
368
92
total – one decimal place ⟶ **128.8**

Answer: 128 and 8 tenths

Check: The estimate, 150, is close to 128.8.

Why one decimal place?
Write both as mixed fractions and multiply.

$$\frac{46}{1} \times 2\frac{8}{10} =$$

$$\frac{46}{1} \times \frac{28}{10} = \frac{1288}{10} = 128.8$$

Equivalent Decimals

To make **equivalent decimals**, you may **add on zeros** as needed.
The zeros do not change the value of the decimal, just its appearance.

$$2.4 = 2.4\textcolor{blue}{0} = 2.4\textcolor{blue}{00}$$

2 and 4 tenths *equals* **2 and 40 hundredths** *equals* **2 and 400 thousandths**

$$2\frac{4}{10} = 2\frac{40}{100} = 2\frac{400}{1000}$$

$$\left(\begin{array}{c} 2\dfrac{40}{100} \div \dfrac{10}{10} = 2\dfrac{4}{10} \\[2ex] 2\dfrac{400}{1000} \div \dfrac{100}{100} = 2\dfrac{4}{10} \end{array}\right)$$

To make **equivalent decimals**, you may **take off zeros** as needed.
The zeros do not change the value of the decimal, just its appearance.

$$2.4\textcolor{blue}{00} = 2.4\textcolor{blue}{0} = 2.4$$