RATIONAL NUMBERS, GCF, AND LCM

The number system that we know is called the "Real Number System". One set of numbers that are a subset of the "real numbers" is called the rational numbers. Rational Numbers are numbers that can be written in fraction form where each part of the fraction is an integer.

Prime factorization is a way to write a number as a product of prime numbers. To find the Greatest Common Factor (GCF) of two numbers, apply prime factorization by finding all the common factors in each prime factorization, then multiplying them.

To find the Least Common Multiple (LCM) of two numbers, apply prime factorization by finding all the prime factors in each prime factorization, then multiplying the highest occurrence of each different factor.

Rational Numbers

Rational numbers include integers, whole numbers, and natural numbers. Natural numbers are also called counting numbers. They start with the number 1 and continue increasing from there. Whole numbers include natural numbers, only they start with the number 0. Integers include whole numbers and their negatives. Negative numbers are the opposite of whole number is. If you have the number 1, it negative is -1. So a rational number is any number that can be written in fraction form.

Natural Numbers: 1, 2, 3, 4, 5…

Whole Numbers: 0, 1, 2, 3, 4, 5…

Integers: …-3, -2, -1, 0, 1, 2, 3…

Three-fifths, $\frac{3}{5}$, is a **rational number** because it can be written in fraction form.

Integers
{…-4, -3, -2, -1, 0, 1, 2, 3, 4

General Case

$\frac{a}{b}$ is a rational number where **a** can be any integer and **b** can be any integer except 0.

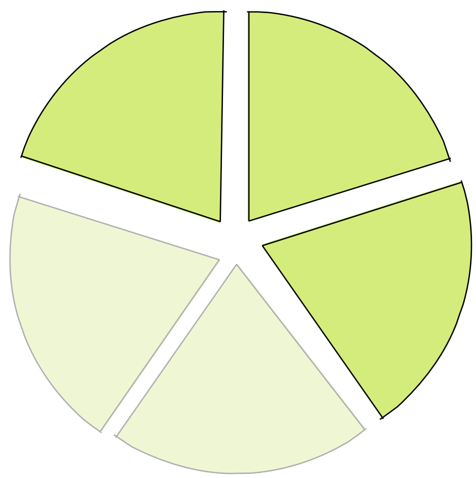
Zero is not acceptable in the denominator because division by zero is *undefined*.

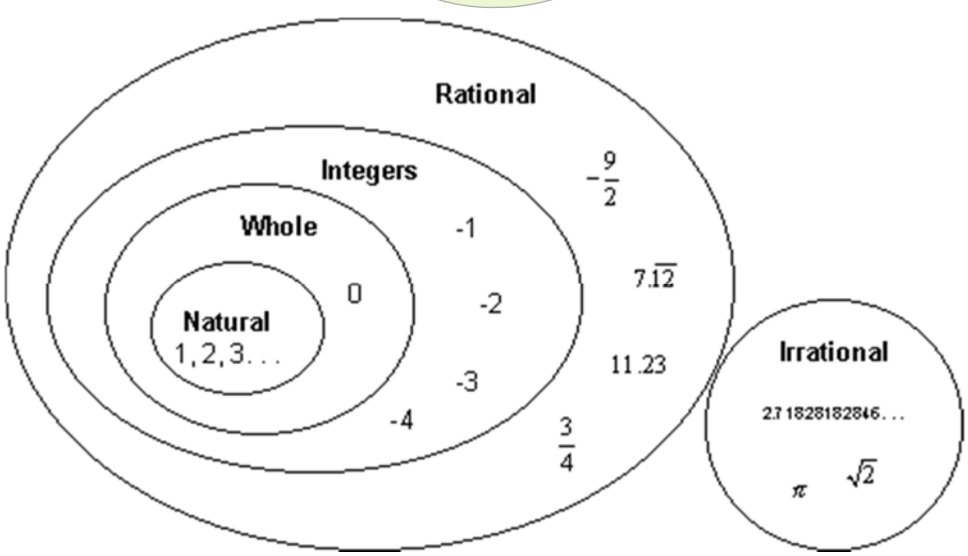
$\frac{3}{5}$ means 3 parts out of 5

$\frac{3}{5}$ means 3 $\div$ 5

$\frac{3}{5}$ means 3 to 5

When looking at a whole pie divided into equal fifths, $\frac{3}{5}$ means 3 parts that are each $\frac{1}{5}$ in size.





Prime Factorization

To find the prime factors of a composite number,
you write the number as a product of prime numbers.

Prime Numbers Under 20 →
{2,3,5,7,11,13,17,19,…}

Find the prime factorization of 40.

The steps for finding the prime factorization of 40:

40	**=**	**2**	**×**		**20**	
40	**=**	**2**	**×**	**2**	**×**	**10**
40	**=**	**2**	**×**	**2**	**×**	**2 × 5**

The prime factorization of 40 is $2 \times 2 \times 2 \times 5$ which can be expressed as $2^3 \times 5$.

Greatest Common Factor (Prime Factorization)

To find the **Greatest Common Factor (GCF)** of two numbers, you can apply prime factorization by finding all the common factors in each prime factorization, and then multiplying them.

Example: Find the GCF of 36 and 54.

The steps for finding the prime factorization of 36:

$$36 = 2 \times 18$$
$$36 = 2 \times 2 \times 9$$
$$36 = 2 \times 2 \times 3 \times 3$$

The steps for finding the prime factorization of 54:

$$54 = 2 \times 27$$
$$54 = 2 \times 3 \times 9$$
$$54 = 2 \times 3 \times 3 \times 3$$

The GCF is the product of the common prime factors (those factors appearing in both factorizations).

36 is $2 \times 2 \times 3 \times 3$
54 is $2 \times 3 \times 3 \times 3$

There is a two, a three, and a second three that is found in both prime factorizations.

Therefore, the GCF is $2 \times 3 \times 3 = 18$.

Now let's show the GCF using the distributive property of multiplication. You will know when to distribute when you see an equation like this…

$$\mathbf{5\ (12 + 3)}$$

The word distribute means to give out. The number on the outside of the parentheses is the number multiplied by each number on the inside of the parentheses. So we are going to distribute the 5 to the 13 then to the 3. So the next step would look like this…

$$\mathbf{5 \times 12 + 5 \times 3}$$

Now just simplify the expression.

5 × 12 = 60

5 × 3 = 15

60 + 15 = 75

The answer is 75.

We can also take a normal addition or subtraction problem and set it up as a distributive problem to show the GCF of the two numbers.

32 + 48

First find the GCF of 32 and 48.

The steps for finding the prime factorization of 32:

32	=	2	×	16						
32	=	2	×	2	×	8				
32	=	2	×	2	×	2	×	4		
32	=	2	×	2	×	2	×	2	×	2

The steps for finding the prime factorization of 48:

48	=	2	×	24						
48	=	2	×	2	×	12				
48	=	2	×	2	×	2	×	6		
48	=	2	×	2	×	2	×	2	×	3

The GCF is the product of the common prime factors (those factors appearing in both factorizations).

32 is 2 × 2 × 2 × 2 × 2

48 is 2 × 2 × 2 × 2 × 3

Therefore the GCF is **2** × **2** × **2** × **2** = **16**

So the number 16 goes on the outside of the parentheses, it is the GCF of both numbers.

16 goes into 32, two times. 16 goes into 48, three times. Inside the parentheses you should have 2 + 3.

$$\mathbf{16\ (2 + 3)}$$

We can check if this is correct by distributing like in the first example.

$$\mathbf{16\ (2 + 3)}$$

$$\mathbf{16 \times 2 + 16 \times 3}$$

$$\mathbf{16 \times 2 = 32}$$

$$\mathbf{16 \times 3 = 48}$$

$$\mathbf{32 + 48}$$

Notice this is the original expression we started with. That means we were correct.

Least Common Multiple (Prime Factorization)

To find the **Least Common Multiple (LCM)** of two numbers, you can apply prime factorization by finding all the prime factors in each prime factorization, and then multiplying the *highest occurrence of each **different*** factor.

Example: Find the LCM of 18 and 24.

The steps for finding the prime factorization of 18:

$$18 = 2 \times 9$$
$$18 = 2 \times 3 \times 3$$
$$18 = 2 \times 3^2$$

The steps for finding the prime factorization of 24:

$$24 = 2 \times 12$$
$$24 = 2 \times 2 \times 6$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$24 = 2^3 \times 3$$

The LCM is the product of the highest occurrence of each prime factor.
When determining the LCM, you multiply each number you see in each prime factorization the most times it occurs in any one factorization. (Example: If both prime factorizations have one 2, you include one 2. If one prime factorization has one 3 and the other has three 3's, you include three 3's.)

18 is 2×3^2
24 is $2^3 \times 3$

The highest occurrence of "two" is 2^3 and the highest occurrence of "three" is 3^2.

Therefore, the LCM is $2^3 \times 3^2 = 72$.