

## **WELCOME!**

We will explore lots of exciting topics in math and examine applications of the concepts learned in real world settings. Let's begin with the basics, and then see how we apply them to actual math problems that are encountered in every day math.

## **RATIONAL AND IRRATIONAL NUMBERS**

We will begin by looking at rational and irrational numbers. Rational numbers are numbers that can be expressed as a fraction where both the numerator and the denominator are integers with the exception of 0 as a denominator (division by 0 is undefined). Some rational numbers have decimals that terminate while others have decimals that form a repeating pattern and go on forever. Irrational numbers are numbers that cannot be expressed as a fraction where both the numerator and the denominator are integers. The decimals for irrationals are decimals that go on forever and never form a repeating pattern.

## **ADDING FRACTIONS AND MIXED FRACTIONS**

Fractions are rational numbers. We will review addition of fractions and mixed fractions. Remember, to add fractions with like denominators, add just the numerators. We'll practice adding fractions and mixed fractions with like denominators. Next we'll practice adding a fraction with a whole number and also adding mixed fractions whose sum totals a whole number.

To add fractions with unlike denominators, we must find the least common denominator (LCD), and then add the numerators. When adding mixed fractions with unlike denominators, we must find the LCD, add the numerators, add the whole numbers, and then look for ways to simplify the answer as needed.

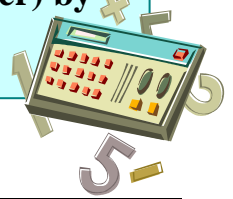
Write fraction answers using the form in these examples.

## Rational and Irrational Numbers

**Calculators** help us understand the difference between **rational** and **irrational** numbers.

**Rational numbers** are numbers that can be written as fractions and when expressed as decimals are either a repeating or terminal decimal. **Irrational numbers** are numbers that have decimals that go on forever, but never develop a repeating pattern.

To express a fraction as a decimal, divide the numerator (top number) by the denominator (bottom number).



Definition	Type of Decimal	Calculator Buttons
<p><b>Rational numbers</b> are numbers that <b>can</b> be expressed as a fraction where both the numerator and the denominator are integers (with the exception of 0 as a denominator*).</p>	<p><b>Repeating</b>  <math>\frac{5}{11} = 0.454545\dots</math></p> <p><b>Terminating</b>  <math>\frac{7}{8} = 0.875</math></p>	<p>5 ÷ 1 1 =</p> <p>7 ÷ 8 =</p>
<p><b>Irrational numbers</b> are numbers that <b>cannot</b> be expressed as a fraction where both the numerator and the denominator are integers.</p>	<p><b>Do not repeat or terminate</b></p> <p>First 12 digits of <math>\pi</math> (<i>pi</i>) = 3.14159265359</p> <p>First 7 digits of <math>\sqrt{2} = 1.414213</math></p>	<p>Not all calculators use the same symbols, so the “pi” and “square root” buttons may look a little different.</p> <p>pi</p> <p>√</p>

*\* Note: Since division by zero is undefined, the denominator of a fraction cannot equal 0.*

Using a calculator, we can explore irrational or rational numbers.

$$\frac{2}{3} = 2 \div 3 = 0.6666666 \text{ - repeating decimal - } \mathbf{rational}$$

$$\sqrt{50} = \text{square root } (\sqrt{\quad}) \text{ of } 50 = 7.0710678 \text{ - non-repeating decimal - } \mathbf{irrational}$$

$$\frac{4}{5} = 4 \div 5 = 0.8 \text{ - terminating decimal - } \mathbf{rational}$$

$$\sqrt{64} = \mathbf{8 \text{ or } 8.0} \text{ - terminating decimal - rational}$$

## Addition of Fractions

To add fractions, identify the numerators and the denominators, and then make the adjustments necessary to complete the computation.

$$\begin{array}{c} \text{numerator} \longrightarrow 3 \\ \hline 4 \longleftarrow \text{denominator} \end{array}$$

When adding fractions and the denominators are the same, just add the numerators. If the denominators are different, write the fractions as equivalent fractions with the same denominator, and then add the numerators.

## Same Denominators

*Example 1:* Add  $\frac{4}{9} + \frac{2}{9}$ .

$$\left[ \begin{array}{l} \text{Add the numerators} \\ \text{of the fractions.} \end{array} \right] \begin{array}{r} \frac{4}{9} \\ + \frac{2}{9} \\ \hline \frac{6}{9} = \frac{2}{3} \end{array}$$

\*To simplify (reduce) the answer, divide the numerator and denominator by three.

$$\frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

*Click on the log below to play a game.*



Example 2: Add  $2\frac{5}{8} + 9\frac{7}{8}$ .

Add the **whole numbers** and add the **numerators** of the fractions.

$$\begin{array}{r} 2\frac{5}{8} \\ +9\frac{7}{8} \\ \hline 11\frac{12}{8} = 12\frac{1}{2} \end{array}$$

\*Let's take a closer look at how to convert an improper fraction to a mixed number in simplest form. To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator.

$$\frac{12}{8} = 8 \overline{)12} = 1\frac{4}{8} = 1\frac{1}{2} \quad \text{therefore} \quad 11\frac{12}{8} = 11 + 1\frac{1}{2} = 12\frac{1}{2}$$

\*To simplify (reduce)  $\frac{4}{8}$ , divide the numerator and denominator by four.

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Example 3: Add  $\frac{3}{4} + 8$ .

$$\begin{array}{r} \frac{3}{4} \\ +8 \\ \hline 8\frac{3}{4} \end{array}$$

Just bring down the 8  
since  $8 + 0 = 8$  and  
bring down the  $\frac{3}{4}$   
since  $\frac{3}{4} + 0 = \frac{3}{4}$ .

Example 4: Add  $5\frac{1}{6} + 3\frac{5}{6}$ .

Add the whole numbers and add the numerators of the fractions.

$$\begin{array}{r} 5\frac{1}{6} \\ +3\frac{5}{6} \\ \hline 8\frac{6}{6} = 9 \end{array}$$

\*To simplify the answer, rewrite  $\frac{6}{6}$  as one, and then combine the one with the eight.

$$8\frac{6}{6} = 8 + 1 = 9$$

## Different Denominators

In the previous four examples, the denominators were the same. Now let's take a look at adding fractions with different denominators.

*Example 5:* Add  $7/8 + 5/16$ .

First, find the least common denominator (LCD) for 8 and 16.

List the multiples of 16:  $16 = \{16, 32, 48, 64, 80, \dots\}$

List the multiples of 8:  $8 = \{8, 16, 24, 32, 40, \dots\}$

\*A good “rule of thumb” is to list the multiples of the larger denominator first because the LCD will occur sooner in the multiples of the larger number.

The “least” multiple that is common to both sets is 16; that is, the **LCD = 16**.

$$\begin{array}{r} \frac{7}{8} = \frac{14}{16} \\ + \frac{5}{16} = \frac{5}{16} \\ \hline \frac{19}{16} = 1\frac{3}{16} \end{array} \quad \left( \begin{array}{l} \frac{7}{8} \times \frac{2}{2} = \frac{14}{16} \\ \text{or say 8 divides into 16,} \\ \text{two times, } 7 \times 2 = 14. \end{array} \right)$$

\*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number.

$$\frac{19}{16} = 16 \overline{)19} = 1\frac{3}{16}$$

*Example 6:* Add  $3\frac{7}{9} + 2\frac{5}{6}$ .

First, find the least common denominator (LCD) for 9 and 6.

List the multiples of 9:  $9 = \{9, 18, 27, 36, 45, \dots\}$

List the multiples of 6:  $6 = \{6, 12, 18, 24, 30, \dots\}$

The “least” multiple that is common to both sets is 18; that is, the **LCD = 18**.

When changing the fraction part of the mixed number to the same denominator, just rewrite the whole number with the fraction.

$$\begin{array}{r} 3\frac{7}{9} = 3\frac{14}{18} \\ + 2\frac{5}{6} = 2\frac{15}{18} \\ \hline 5\frac{29}{18} = 6\frac{11}{18} \end{array} \quad \left( \begin{array}{l} \frac{7}{9} \times \frac{2}{2} = \frac{14}{18} \\ \frac{5}{6} \times \frac{3}{3} = \frac{15}{18} \end{array} \right)$$

\*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number. Finally, combine the two whole numbers.

$$\frac{29}{18} = 18 \overline{)29} \begin{array}{r} 1 \\ 18 \\ \hline 11 \end{array} = 1\frac{11}{18}$$

$$5\frac{29}{18} = 5 + 1\frac{11}{18} = 6\frac{11}{18}$$