SYSTEMS OF EQUATIONS: ELIMINATION

In this unit, you will be introduced to another way of solving systems of equations using elimination. You will also learn about consistent, inconsistent, dependent, and independent systems. The unit will conclude with using systems to solve real world problems.

The Elimination Method

Another way to algebraically solve a system of equations is by eliminating a variable. This process involves adding or subtracting the equations, depending on whether the terms are opposites (then you add) or the same (then you subtract).

Example 1: $3x - 2y = 1$
$-3x + 4y = 7$

Step 1: Ask yourself if there are any terms that are the same or opposites. In this case the $3x$ and $-3x$ are opposites.

Step 2: Use the addition property of equality to combine the two equations into one.

$$\begin{array}{rr} & 3x - 2y = 1 \\ (+) & -3x + 4y = 7 \\ \hline & 2y = 8 \end{array}$$

Step 3: Solve the resulting equation for y and substitute this value into one of the original equations for y and solve for x.

$2y = 8$

$y = 4$

$3x - 2y = 1$

$3x - 2(4) = 1$

$3x - 8 = 1$

$3x = 9$

$x = 3$

The solution to this system is (3, 4)

Example 2: $2x + 3y = 5$
$2x + y = 3$

In this example the x terms are the same. To eliminate the x's, subtract the two equations to combine them into one.

$$\begin{array}{rr} & 2x + 3y = 5 \\ (-) & 2x + y = 3 \\ \hline \\ & 2y = 2 \\ \\ & y = 1 \end{array}$$

Substitute 1 for y into either of the original equations and solve for x.

$2x + 3y = 5$

$2x + 3(1) = 5$

$2x + 3 = 5$

$2x = 2$

$x = 1$

The solution to this system of equations is (1, 1).

Sometimes it is necessary to multiply one or both equations by a number to produce the same coefficient or opposites. You make the choice. If you are more comfortable adding, then produce opposites; if you are more comfortable subtracting, then make them the same.

Example 3: $3x - y = 8$
$x + 2y = -2$

Multiply the first equation by 2 to produce opposites for y.

$$\begin{array}{ll} 2(3x - y = 8) \qquad & 6x - 2y = 16 \\ x + 2y = -2 & x + 2y = -2 \end{array}$$

Add the two equations to eliminate y.

$$\begin{array}{rr} & 6x - 2y = 16 \\ (+) & x + 2y = -2 \\ \hline \end{array}$$

$$7x = 14$$

$$x = 2$$

Substitute 2 for x in either of the original equations and solve for y.

$$3x - y = 8$$

$$3(2) - y = 8$$

$$6 - y = 8$$

$$-y = 2$$

$$y = -2$$

The solution to this system is (2, –2)

Example 4: $2x - 7y = 20$
$5x + 8y = -1$

Multiply the first equation by 5 and the second equation by 2 to produce the same coefficient on the x term.

$$5(2x - 7y = 20) \qquad 10x - 35y = 100$$
$$2(5x + 8y = -1) \qquad 10x + 16y = -2$$

Subtract the two new equations to eliminate x.

$$\begin{array}{rr} & 10x - 35y = 100 \\ (-) & 10x + 16y = -2 \\ \hline \end{array}$$

$$-51y = 102$$

$$y = -2$$

Substitute –2 for y in either of the original equations.

$$2x - 7y = 20$$

$2x - 7(-2) = 20$

$2x + 14 = 20$

$2x = 6$

$x = 3$

The solution to this system is $(3, -2)$

Choosing a Method for Solving Systems

Example	Suggested Method	Reason
$5x + y = 32$ $y = 2$	Substitution	The value of y is known and can be substituted into the first equation.
$4x - 3y = -13$ $7x + 3y = -9$	Elimination	$3y$ and $-3y$ are opposites and can easily be eliminated using addition.
$8x + 3y = -7$ $4x + y = 4$	Elimination	y can be eliminated by multiplying the second equation by -3.

Consistent and Inconsistent Systems

In the previous unit you learned that a system of equations may have a unique solution (one ordered pair (x, y)), many solutions, or no solution. In this unit we are going to expand on this and say that if a system has **one or many solutions**, the system is called ***consistent***. If a system has **no solution**, it is ***inconsistent***.

To determine if a system is consistent or inconsistent you need to solve it algebraically, substitution or elimination, or graphically. For our purposes we will solve all systems algebraically.

Example 1: Solve the system of equations shown below.

$$2x + y = -2$$
$$4x + y = -4$$

Solve the system algebraically by subtracting the systems.

$$\begin{array}{rr} & 2x + y = -2 \\ (-) & 4x + y = -4 \\ \hline & -2x = 2 \end{array}$$

$$x = -1$$

$$2(-1) + y = -2$$

$$-2 + y = -2$$

$$y = 0$$

The solution is $(-1, 0)$, so the system has one unique solution and is **consistent**.

Example 2: Solve the system of equations shown below.

$$4x - y = 6$$
$$y - 4x = 4$$

Solve the system by using substitution. Solve the second equation for y and substitute this value into the first equation.

$$y = 4 + 4x$$

$$4x - (4 + 4x) = 6$$

$$4x - 4 - 4x = 6$$

$$-4 \neq 6$$

Since -4 does not equal 6, there is no ordered pair that satisfies the system; therefore, the system is **inconsistent**.

Independent and Dependent Systems

The consistent systems that you just learned about can be categorized as ***independent*** or ***dependent***.

If one unique ***ordered pair* (*x, y*) *satisfies both equations, then the system is an independent*** system (one solution).

If **every ordered** pair is a solution of both equations, then the system is a ***dependent*** system (many solutions).

To determine if a system is independent or dependent, solve the system algebraically or graphically.

Example 1: Solve the system of equations shown below.

$4x - y = 5$
$6x + 4y = -9$

Step 1: Multiply the first equation by 4 so the "*y*" values are opposites, and then add the two equations.

$$\begin{array}{lr} 4(4x - y = 5) & 16x - 4y = 20 \\ 6x + 4y = -9 & (+)\quad 6x + 4y = -9 \\ \hline & 22x = 11 \end{array}$$

$$x = \frac{1}{2}$$

Step 2: Substitute $\frac{1}{2}$ for x in either of the original equations and solve for y.

$4x - y = 5$

$4\left(\frac{1}{2}\right) - y = 5$

$2 - y = 5$

$-y = 3$

$y = -3$

The solution is $(\frac{1}{2}, -3)$. Since the system has one solution, this means the system is **independent**.

Example 2: Solve the system of equations shown below.

$$2x - y = 9$$
$$-2x + y = -9$$

Step 1: Add the two equations together to eliminate x or y.

$$\begin{array}{rr} & 2x - y = 9 \\ (+) & -2x + y = -9 \\ \hline & 0 = 0 \end{array}$$

Since this solution produces a true statement that 0 = 0, the solution is many solutions and means that the system is **dependent**.

Applications

Systems of equations can be used for many real-world problems when more than one variable is unknown. The following examples demonstrate this process.

Example 1: The Jets scored 4 more points than the Vets. The total of their scores was 38. How many points did each team score?

Step 1: Define a variable for each unknown.

j = Jet's score v = Vet's score

Step 2: Determine what is known and represent this information using equations.

We know the Jets scored 4 more than the Vets so,

$$j = v + 4$$

We also know that their combined score was 38 so,

$$j + v = 38$$

Step 3: Use the two equations just found to determine each team score by using substitution or elimination.

$$j = v + 4$$
$$j + v = 38$$

Since you know the value of j, $\boldsymbol{j = v + 4}$, substitute this into the second equation for j and solve for v.

$$(v + 4) + v = 38$$

$$2v + 4 = 38$$

$$2v = 34$$

$$v = 17$$

The Vets scored 17 points.

Use this information to substitute v in either of the original equations to determine the number of points the Jets scored.

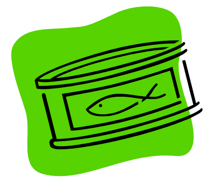
$j = 17 + 4$

$j = 21$

The Jets scored 21 points.

The answer to the problem is the Vets scored 17 points and the Jets scored 21 points.

Example 2: Four cans of tuna and 2 boxes of rice cost $7.40. Six cans of tuna and 2 boxes of rice cost $9.70. Find the cost of each item.



Step 1: Define variables for each unknown.

t = one can of tuna r = one box of rice

Step 2: Use the given information to write the equations for the system.

Since we know 4 cans of tuna and 2 boxes of rice cost $7.40, the equation would be:

$4t + 2r = 7.40$

We also know that 6 cans of tuna and 2 boxes of rice cost $9.70. The equation would be:

$6t + 2r = 9.70$

Our two equations would be:

$4t + 2r = 7.40$
$6t + 2r = 9.70$

Step 3: Use elimination with subtraction to eliminate r.

$$\begin{array}{r} 4t + 2r = 7.40 \\ (-)\ \underline{6t + 2r = 9.70} \\ -2t = -2.30 \end{array}$$

$t = 1.15$

This tells us that each can of tuna costs $1.15.

Step 4: Now substitute this value back into one of the original equations to find out how much each box of rice costs.

$$4(1.15) + 2r = 7.40$$

$$4.60 + 2r = 7.40$$

$$2r = 2.80$$

$$r = 1.40$$

Each box or rice costs $1.40.

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