RELATIONS AND FUNCTIONS

In this unit, you will be introduced to relations and functions. You will determine when a relation is a function. You will also explore various methods used to graph linear equations and verify that a point lies on a line for a given equation. You will examine equations that contain one variable. In addition, you will identify the graphs of functions and write equations as functions.

Relations and Functions

A **relation** is a set of ordered pairs and can be symbolized using a table, graph, or set of ordered pairs.

There are two main parts of a relation. The first is the **domain,** which is the set of first coordinates or the *x*-values. The second part is the **range**, which is the set of second coordinates or the *y*-values.

Example 1: Given the relation {(–1, 5), (4, 2) (2, 3), (1, 5)}, what is the domain and the range?

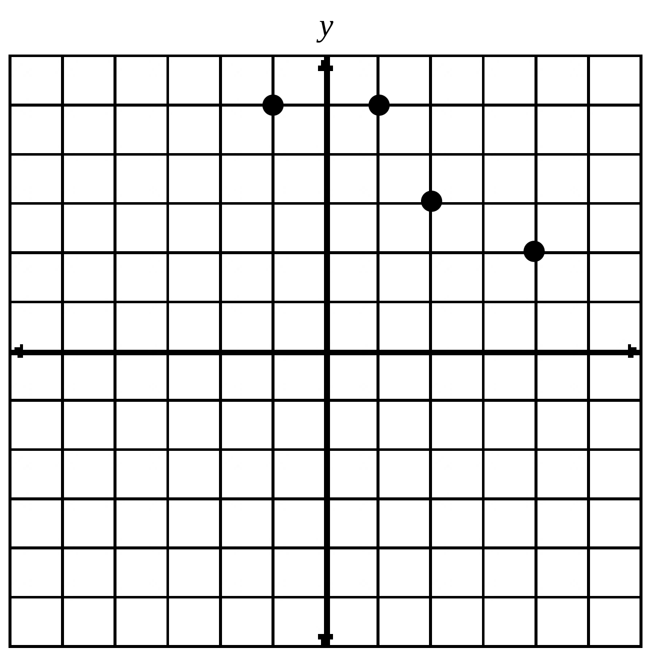
The domain = {–1, 4, 2, 1}, the *x*-values.

The range = {5, 2, 3}, the *y*-values

*Note: The 5 is only listed once in the range because once a number is shown in a set, it does not have to be shown again.

This relation can also be displayed as a table or in as graph as shown below.

x	y
–1	5
4	2
2	3
1	5



A special type of a relation is called a function. A function is a **relation** in which all *x*-values are different.

Example 2: Determine if the relation is a function: {(9, 7), (7, 4), (–2, 0), (–5, 0)}

First, state the domain.

D = {9, 7, –2, –5}

Next, state the range.

R = {7, 4, 0}

Check to see if all *x*-values are different.

Yes, all *x*-values are different, so this relation represents a function.

Graphing Linear Functions

Did you know?

Did you know that a cricket is nature's thermometer?

The equation $t = c + 39$ can be used to find the temperature "*t*", if the number of chirps "*c*" a cricket makes in 15 seconds is known.

Background: Crickets are sensitive to changes in air temperature, and chirp at faster rates as the temperature rises. It is possible to use the chirps of the male snowy tree cricket, common throughout the United States, to gauge temperature using the formula shown above. The temperature is a function of the number of chirps made by a cricket in 15 seconds.

A linear function (equation) can have an infinite number of solutions which can be represented by ordered pairs and a straight line.

To determine a solution to an equation in two variables, substitute the given coordinate into the equation and solve for the other coordinate.

Example 1: Complete each ordered pair so that it is a solution to $3x + 2y = 1$.

a) (1, ?)

b) $\left(\frac{5}{3}, ?\right)$

c) (–3, ?)

a) Substitute 1 for *x* and solve for *y*.

$$3(1) + 2y = 1$$
$$3 + 2y = 1$$
$$2y = -2$$
$$y = -1$$
$$(1, -1)$$

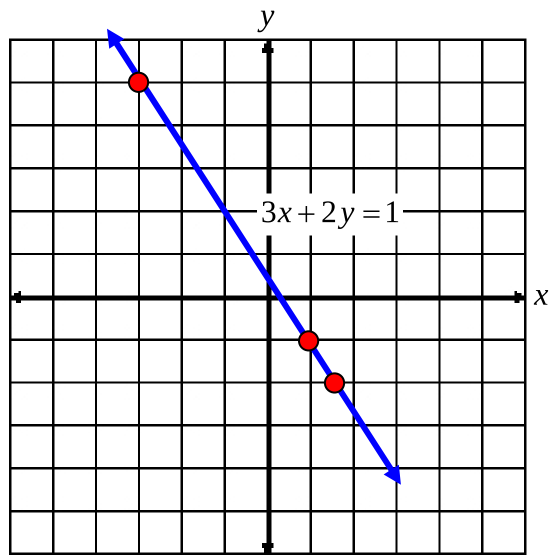
b) Substitute $\frac{5}{3}$ for *x* and solve for *y*.

$$3\left(\frac{5}{3}\right) + 2y = 1$$
$$5 + 2y = 1$$
$$2y = -4$$
$$y = -2$$
$$\left(\frac{5}{3}, -2\right)$$

c) Substitute –3 for *x* and solve for *y*.

$$3(-3) + 2y = 1$$
$$-9 + 2y = 1$$
$$2y = 10$$
$$y = 5$$
$$(-3, 5)$$

Now, graph the three ordered pairs.



*Note: Any point on the line satisfies the equation.

For example: It appears that (–1, 2) is part of the line. So, check to see that if $x = -1$ produces $y = 2$ in the equation.

$3x + 2y = 1$	
$3(-1) + 2y = 1$	Substitute -1 in for x.
$-3 + 2y = 1$	Simplify
$2y = 4$	Add 3 to both sides of the equation.
$y = 2$	Divide by 2.

Thus, point (–1, 2) is part of the line and satisfies the equation $3x + 2y = 1$.

Solving an equation means to replace the variable(s) so that a true sentence results. The solutions of an equation with two variables are ordered pairs.

Graphing Solutions to Equations

1. Choose at least three convenient numbers to replace for "x".

2. Substitute "x" with the numbers you have chosen and solve for "y".

3. Write the solutions as ordered pairs.

4. Graph the ordered pairs on a coordinate plane and connect them using a straight edge.

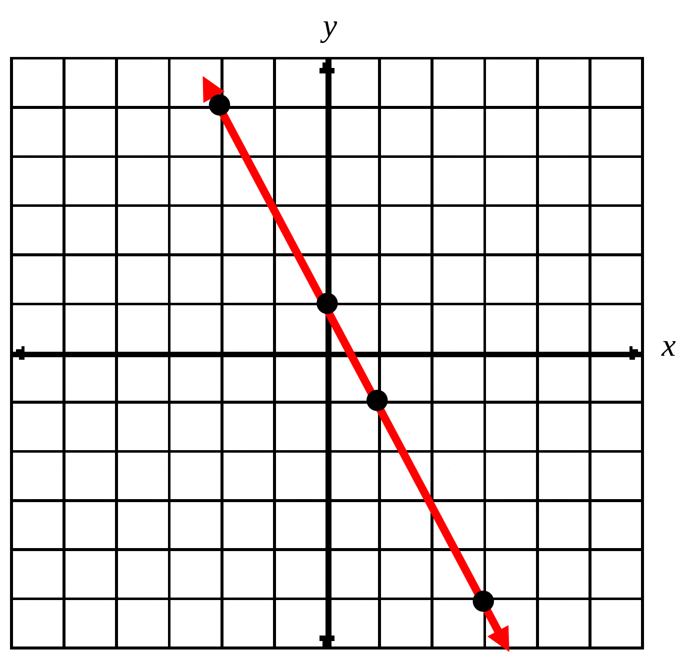
Example 2: Find solutions to the equation $y = -2x + 1$.

We will choose four values for x: –2, 0, 1, and 3.

Substitute the chosen x values into the equation to solve for the corresponding y value.

x	$-2x + 1$	y	(x, y)
–2	$-2(-2) + 1$ $4 + 1$	5	(–2, 5)
0	$-2(0) + 1$ $0 + 1$	1	(0, 1)
1	$-2(1) + 1$ $-2 + 1$	–1	(1, –1)
3	$-2(3) + 1$ $-6 + 1$	–5	(3, –5)

The solutions are (–2, 5), (0, 1), (1, –1), and (3, –5). Now graph each of these points on a coordinate plane and connect them using a straight edge.



*An equation such as $y = -2x + 1$ is called a linear equation because its graph is a straight line.

Properties of Linear Equations

*Must have at least one variable and no more than two variables.

*The exponents of the variables must be one (1).

Example 3: Determine if each equation is linear.

a) $4x - y = 15$ — This is a linear equation because there are no more than two variables and the exponent of each variable is one (1). This equation may be written as $4x^1 - y^1 = 15$.

b) $3y = 2x^2$ — This is NOT a linear equation because the exponent of the x is two (2).

c) $x = 10$ — This is a linear equation because there is at least one variable and the exponent of the variable is one (1).

d) $\frac{3}{x} + 4 = y$ — This is NOT a linear equation. When a variable is in the denominator, the exponent is negative, and therefore, not equal to one (1). This equation may be written as $3x^{-1} + 4 = y$.

Equations Containing One Variable

Equations containing one variable are special lines. They are either vertical or horizontal lines. In this section we will discuss how to graph each type.

Example 1: Graph $x = -2$ on a coordinate plane.

a.) Make a table and list four values of x that equal –2. Since the equation states $x = -2$, all x-values will be –2.

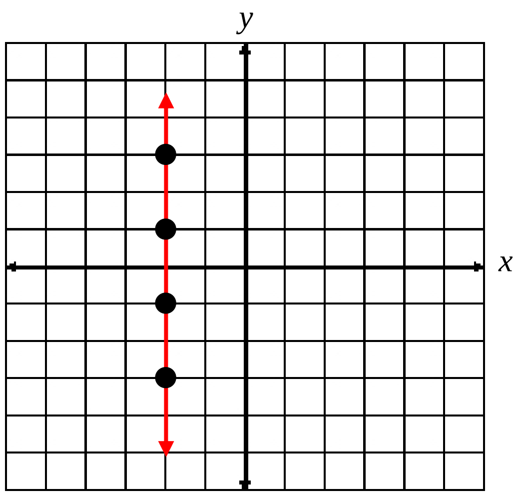
x	***y***
–2	
–2	
–2	
–2	

b.) Choose any number for y.

x	***y***
–2	–3
–2	–1
–2	1
–2	3

c.) Plot each of the four points and connect them with a straight edge.

The points determined in the previous table are:(–2, –3), (–2, –1), (–2, 1), and (–2, 3).



$x = -2$

This graph shows that a **vertical** line will be in the form of ***x* = *a***, where "*a*" represents a real number.

Example 2: Graph $y = 4$ on a coordinate plane.

a) Make a table and list four values of y that equal 4. Since the equation states $y = 4$, all y-values will be 4.

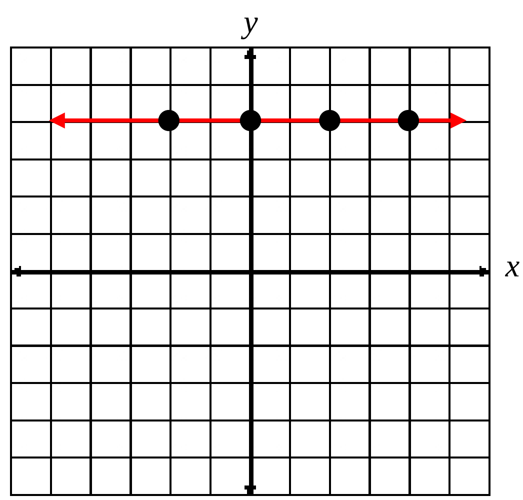
x	***y***
	4
	4
	4
	4

b) Choose any number for x.

x	***y***
–2	4
0	4
2	4
4	4

c) Plot each of the four points and connect them with a straight edge.

The points determined in the previous table are:(–2, 4), (0, 4), (2, 4), and (4, 4).



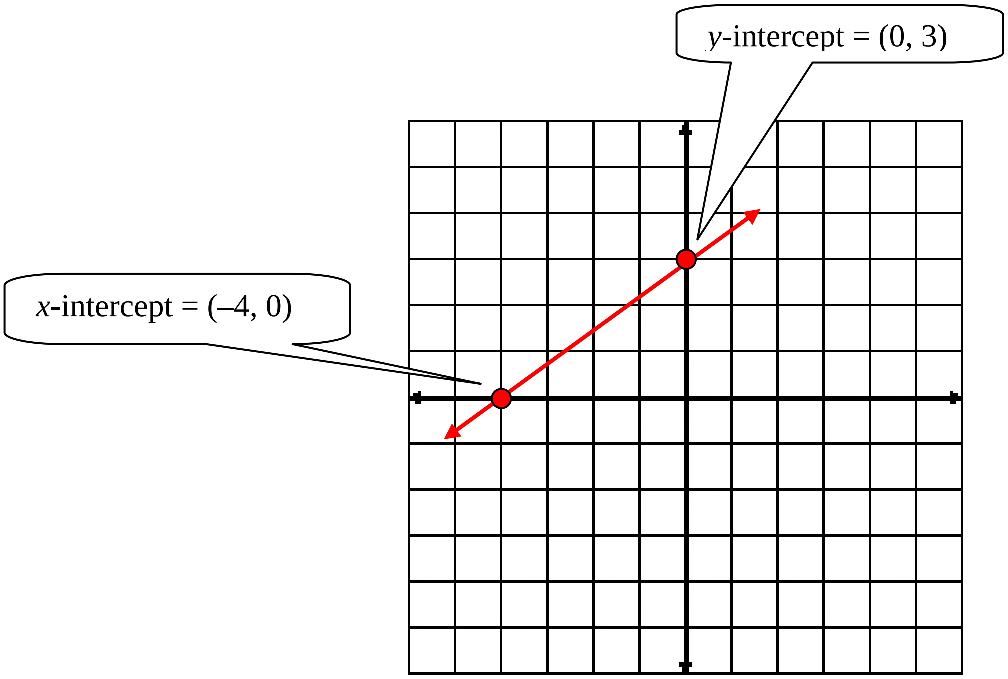
$y = 4$

This graph shows that a **horizontal** line will be in the form of $\boldsymbol{y = b}$ where "b" represents a real number.

Intercepts

The graph of a linear function may cross the *x*-axis, the *y*-axis, or both axes. If the function crosses the *x*-axis, the point of intersection is called the *x*-intercept. If the function crosses the *y*-axis, the point of intersection is called the *y*-intercept.

Study the illustration below.



Graphing a Line Using the *x*-intercept and the *y*-intercept

a) Find the *x*-intercept:
   -replace "*y*" in the equation with 0 and solve for *x*.

b) Find the *y*-intercept:
   -replace "*x*" in the equation with 0 and solve for *y*.

c) Plot both points on a coordinate plane and connect them with a straight edge.

Example: Graph $y = \frac{1}{2}x + 3$ by determining the *x*-intercept and the *y*-intercept.

a) Replace *y* with 0 and solve for *x*.

$0 = \frac{1}{2}x + 3$	Substitute zero for *y*.
$-3 = \frac{1}{2}x$	Subtract 3 from both sides of the equation.
$2(-3) = \frac{1}{2}x(2)$	Multiply both sides of the equation by 2.
$-6 = x$	Simplify

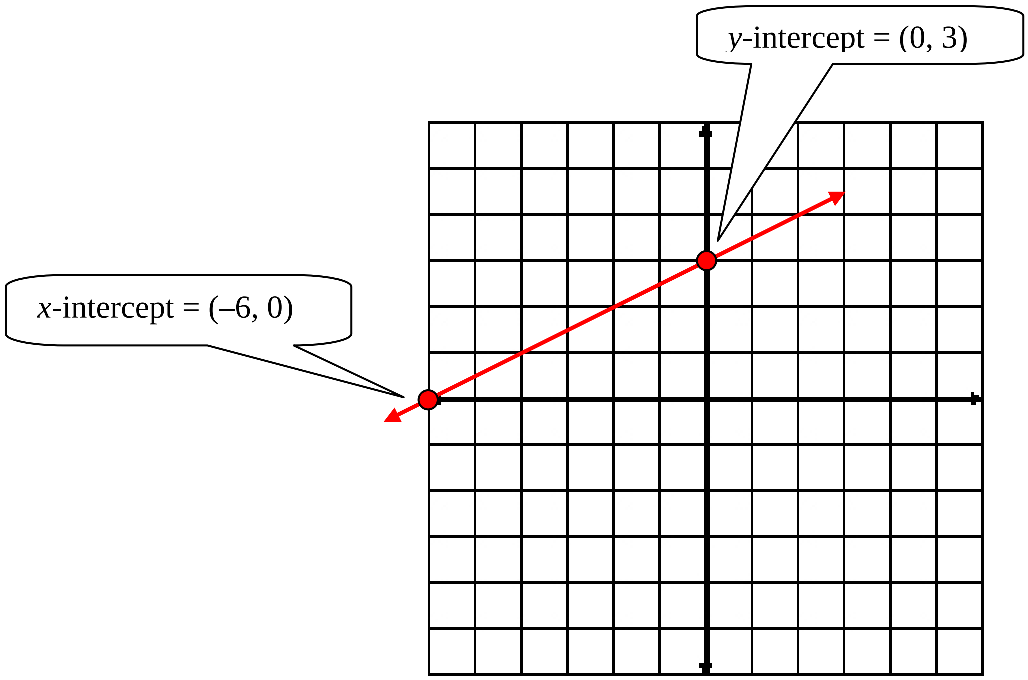
The *x*-intercept is located at (–6, 0).

b) Replace *x* with 0 and solve for *y*.

$y = \frac{1}{2}(0) + 3$	Substitute 0 for *x*.
$y = 3$	Simplify

The *y*-intercept is located at (0, 3).

c) Plot the points and connect.



$$y = \frac{1}{2}x + 3$$

Equations as Functions

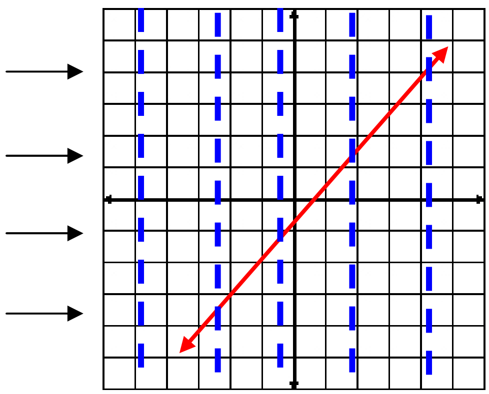
In this unit you will learn how to determine if an equation or a graph of an equation is a function.

Let's first determine if the graph of an equation is a function. There is a test that can be used on a graph to determine if it is a function. This test is called the **vertical line** test.

1) Hold a ruler, a pencil, or something straight vertically against the graph of a relation.

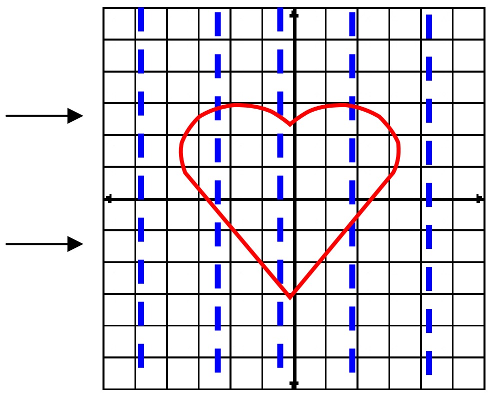
2) If, when passing the vertical line through the graph, you cross only one point of the graph at a time, the relation is a function.

Example 1: Use the vertical line test to determine if the relation (graphed line) is a function.



Notice that, as the vertical lines (blue dashed lines) are passed through the graph of the relation, only one point is crossed by each vertical line. This relation is a function.

Example 2: Use the vertical line test to determine if the relation (graphed heart) is a function.



Notice that, as the vertical lines (blue dashed lines) are passed through the graph of the relation, two points are crossed by each vertical line. This relation is a NOT a function.

Now let's take a look at how to determine if a given equation is a function.

Example 3: a. Determine four ordered pairs for $y = 2x + 3$ if the domain is $\{-2, -1, 0, 1\}$.

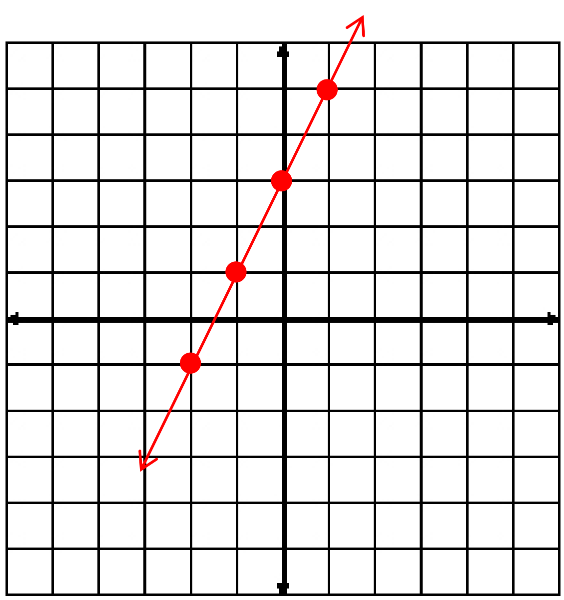
b. Graph the equation.

c. Determine if the graph is a function by using the vertical line test.

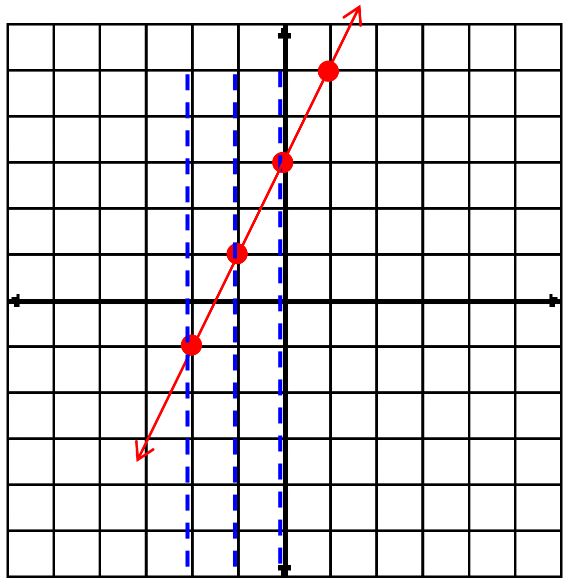
For part (a), make a table of values showing the domain, equation, corresponding range values, and the ordered pairs that are found after replacing x with the values of the domain and solving for y.

x	$2x + 3$	y	(x, y)
–2	2(–2) + 3	–1	(–2, –1)
–1	2(–1) + 3	1	(–1, 1)
0	2(0) + 3	3	(0, 3)
1	2(1) + 3	5	(1, 5)

For part (b), graph the ordered pairs that were found above:
{(–2, –1), (–1, 1), (0, 3), (1, 5)}.



For part (c), use the vertical line test to determine how many points the vertical line goes through at one time.



Notice that when a vertical line is passed through the graph of the relation, the line only crosses one point at a time; therefore, we can say that this relation is a function.

Expressing Equations in Function Notation

Equations that represent functions may be written in functional notation, $f(x)$. Other examples of representations of functional notation are $g(x)$ and $h(x)$. This functional notation is read "f of x".

The function of x represents the range of y-values that correspond to the values of x in the domain.

To determine a value of the function ($f(x)$), substitute the given x-value into the equation and evaluate.

Example 4: If $f(x) = 2x - 1$, find each of the following:

a.) $h(3)$

$h(3) = 2(3) - 1$ Substitute 3 in for *x*.

$h(3) = 5$ Simplify

b.) $h(-4)$

$h(-4) = 2(-4) - 1$ Substitute -4 in for *x*.

$h(-4) = -9$ Simplify

c.) $2h(1)$

$2h(1) = 2[2(1) - 1]$ Substitute 1 in for *x* and multiply the whole expression by 2.

$2h(1) = 2(1)$ Simplify inside brackets.

$2h(1) = 2$ Simplify

Graph Paper