POLYNOMIALS

In this unit, you will be introduced to polynomials. You will examine three basic types of polynomials that are used frequently in algebra: monomials, binomials, and trinomials. You will also perform various computations using polynomials.

Polynomials

In the previous unit, you learned about monomials. In this unit we are going to combine monomials with addition and subtraction and identify them with other names.

You have learned that a number, 3, a variable, *x*, or a product of a number and variable(s), 5mn, are called monomials. **At this point it should be mentioned that to be considered a monomial, a variable cannot have a negative exponent or appear in the denominator of a rational number.**

For example, the following are **not** monomials: x^{-4} and $\frac{3}{m^2}$.

In this unit you will be working with expressions like the following:

$5m - 2$ $\qquad$ $4x^2 + 7x - 3$ $\qquad$ $-5a^2b^3 + \frac{1}{3}ab$

Each of these is a sum or difference of monomials called a **polynomial.**

Each monomial within a polynomial is called a term. A polynomial with exactly **two** terms is called a **binomial** and a polynomial with exactly **three** terms is called a **trinomial**.

Example 1: $y^2 + y$ is called a **binomial** because it has two (2) terms.

Example 2: $\frac{1}{x} + 2x^2 + \frac{1}{3}x^4$ is **not** a polynomial because $\frac{1}{x}$ is not a monomial.

Example 3: $3x^2 + 6x + 5$ is a **trinomial** because it has three (3) terms.

Recall that the numerical factor of a monomial is called the coefficient. In the term, $-2x^3y^4$, –2 is the coefficient.

Example 4: Identify the terms and give the coefficient of each.

$4x^3 + 5x^2 - 3$

The terms are $4x^3$, $5x^2$, and –3.

The coefficient of $4x^3$ is 4, the coefficient of $5x^2$ is 5, and the coefficient of –3 is –3.

To simplify polynomials, we collect like terms. Recall that terms like $3xy^2$ and $7xy^2$, whose variable factors are exactly the same, are called like terms. To simplify like terms, we combine the coefficients.

Example 5: $3m^3 - 5m^3 = (3-5)m^3$

$= -2m^3$

Example 6: $7x^5y^4 - 6y^3 - 2x^5y^4 + 3y^3$

$7x^5y^4 - 2x^5y^4 - 6y^3 + 3y^3$

$(7-2)x^5y^4 + (-6+3)y^3$

$5x^5y^4 + (-3y^3)$

$5x^5y^4 - 3y^3$

In future units it will be necessary to determine the degree of a term or polynomial; so, at this point we will talk about the degree of terms and polynomials. The **degree** of a term is found by adding all the exponents of the variables. The degree of a polynomial is then determined by the highest degree of all its terms.

Example 7: Find the degree of the monomial $7x^4y^3z$

-add the exponents 4 + 3 + 1

-the degree of this monomial is **8**

***If the exponent is not shown, you must assume that it is 1.**

Example 8: Find the degree of the polynomial $8a^4b^2 - 4ab + 7$

-degree of $8a^4b^2$ is 4 + 2 = **6**

-degree of $4ab$ is 1 + 1 = **2**

-degree of 7 is 0 (because there are no variables)

Therefore the degree of $8a^4b^2 - 4ab + 7$ is **6**.

Remember: The degree of a polynomial is then determined by the highest degree of all its terms.

Polynomials are generally written in either descending order (largest to smallest) in terms of a variable or ascending order (smallest to largest).

For example: $-3x^3y + 4x^2y^2 + 5xy - 9y$ has been written in **descending** order for the variable *x* because the exponents of *x* are aligned largest to smallest.

$-7x + 9x^2y + 3x^3y^2 - 14x^4$ has been written in ascending order for the variable *x*.

Adding Polynomials

We are going to add and subtract polynomials in the same way that we simplified expressions, by combining like terms. Follow along with the examples below.

Example 1: Find the sum: $(3x^2 - 15) + (5x^2 + 7x)$

This sum equals the single polynomial:

$$3x^2 - 15 + 5x^2 + 7x$$

Collect like terms:

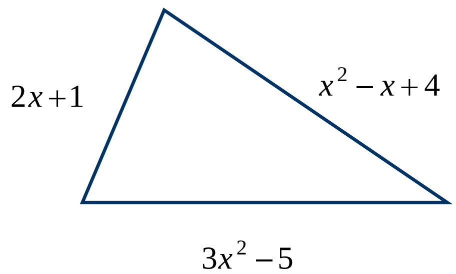
$$(3+5)x^2 - 15 + 7x$$

$$8x^2 - 15 + 7x$$

Write the polynomial in descending order of x:

$$8x^2 + 7x - 15$$

Example 2: Given the following triangle with sides of given length, find its perimeter by adding all of the sides.



$$P=(2x+1)+(x^2-x+4)+(3x^2-5)$$

The sum equals the single polynomial:

$$P=2x+1+x^2-x+4+3x^2-5$$

Collect like terms:

$$P=2x-x+1+4-5+x^2+3x^2$$

$$=(2-1)x+(1+4-5)+(1+3)x^2$$

$$=x+0+4x^2$$

Write in descending order of *x*:

$$P=4x^2+x$$

The perimeter of the given triangle is $4x^2+x$.

Subtracting Polynomials

Subtracting polynomials is done with the same process as adding in the fact that you will combine like terms. Be very careful when subtracting a quantity to make sure you subtract each term within the quantity.

*It is very helpful to change all the signs of the quantity you are subtracting, and then combine like terms. Follow along with the example below.

Example 1: Find the difference: $(5x^4+3x^2)-(6x^4-2x^2)$

a) Change all the signs in the second quantity:

$5x^4+3x^2-6x^4+2x^2$

b) Combine like terms:

$5x^4-6x^4+3x^2+2x^2$

$(5-6)x^4+(3+2)x^2$

$-x^4+5x^2$

Example 2: Find the difference: $(-7m^3+2m+4)-(-2m^3-4)$

a) Change all the signs in the second quantity:

$-7m^3+2m+4+2m^3+4$

b) Combine like terms:

$(-7+2)m^3+2m+(4+4)$

$-5m^3+2m+8$

Multiplying and Dividing Monomials and Polynomials

To multiply or divide a polynomial by a monomial, apply the distributive property and the properties of exponents.

Example 1: Find the product: $(3x^2)(x^3+2x-4)$

Step1: Distribute the $3x^2$ over each term of the polynomial.

$$(3x^2)x^3+(3x^2)2x-(3x^2)4$$

Step 2: Multiply the numbers and apply the exponent properties to simplify the exponents.

$$3\cdot x^{2+3} + 3\cdot 2\cdot x^{2+1} - 3\cdot 4\cdot x^2$$

Step 3: Simplify the exponents.

$$3x^5+6x^3-12x^2$$

Example 2: Find the quotient: $\dfrac{8x^6-6x^4+4x}{-2x}$

Step 1: Rewrite the quotient into a sum of three terms.

$$\frac{8x^6}{-2x}+\frac{-6x^4}{-2x}+\frac{+4x}{-2x}$$

Step 2: Apply the properties of exponents to each term.

$$\frac{8}{-2}x^{6-1}+\frac{-6}{-2}x^{4-1}+\frac{+4}{-2}x^{1-1}$$

Step 3: Simplify the coefficients and exponents of each term.

$$-4x^5+3x^3+-2x^0$$

Step 4: Rewrite $+-2x^0$ with one sign applying the definition of subtraction. Recall that any number to the zero power equals one. Simplify.

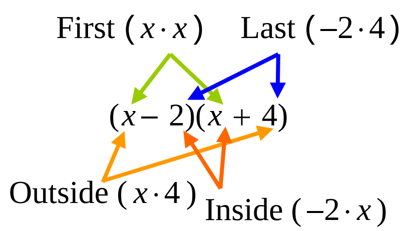
$$-4x^5+3x^3-2\cdot 1 \quad = \quad -4x^5+3x^3-2$$

Multiplying Binomials

The distributive property can be used to multiply two binomials; however, there is a popular mnemonic (memory) method for multiplying binomials called **FOIL**.

- Multiply the **F**irst terms of each binomial.
- Multiply the **O**utside terms of the binomials.
- Multiply the **I**nside terms of the binomials.
- Multiply the **L**ast terms of each binomial.

Example 1: Multiply $(x-2)(x+4)$.



F		O		I		L
x^2	$+$	$4x$	$-$	$2x$	$-$	8
x^2	$+$		$2x$		$-$	8

Example 2: Multiply $(2x+3)(6x-1)$.

$(2x+3)(6x-1)$	
$(2x \times 6x)+(2x \times -1)+(3 \times 6x)+(3 \times -1)$	FOIL Method
$12x^2-2x+18x-3$	Combine Like Terms
$12x^2+16x-3$	