EXPONENTS

When simplifying algebraic expressions that involve exponents, there are a few rules (properties) to apply that will shorten the process. You will be introduced to these rules in this unit, and then apply them to simplify various algebraic expressions. You will also use these rules to compute with monomials.

Exponent Properties

Exponential Property Summary

Multiplying and Dividing Monomials

Exponent Properties

Recall that exponents tell us how many times we use the base as a factor. For example, $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$. An expression written with exponents is said to be in **exponential form**.

When simplifying algebraic expressions involving exponents, there are a few rules (properties) that apply to allow you to completely simplify the expression. Those properties are listed below with examples.

#1) Product Property

When multiplying like bases, **add** the exponents.

$$a^m \cdot a^n = a^{m+n}$$

$$x^7 \cdot x^3 = x^{7+3} = x^{10}$$

#2) Power of a Power

When a base to a power is raised to another power, **multiply** the exponents.

$$\left(a^m\right)^n = a^{mn}$$

$$\left(x^4\right)^3 = x^{4 \times 3} = x^{12}$$

#3) Power of a Product

When a product is raised to a power, **both** or all bases are raised to the power.

$$\left(ab\right)^n = a^n b^n$$

$$\left(2x\right)^3 = 2^3 \cdot x^3 = 8x^3$$

#4) Power of a Quotient

When a quotient is raised to a power, **both** the numerator and denominator are raised to the power.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

#5) Quotient Property

When dividing like bases, **subtract** the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{x^8}{x^3} = x^{8-3} = x^5$$

***Exponential expressions are not considered completely simplified unless the exponents are positive; for this, the next property is especially important.**

#6) Negative Exponent Property

When a base has a negative exponent, use the **reciprocal** of the base to produce a positive exponent.

$$a^{-m} = \frac{1}{a^m} \quad \text{or} \quad \left(\frac{1}{a}\right)^{-m} = a^m$$

$$x^{-3} = \frac{1}{x^3} \quad \text{or} \quad \left(\frac{1}{x}\right)^{-3} = x^3$$

Exponent Property Summary

Exponent Property Summary		
Product of Powers $a^m \cdot a^n = a^{m+n}$	**Quotient of Powers** $\frac{a^m}{a^n} = a^{m-n}$	**Power of a Power** $(a^m)^n = a^{m \times n}$
Powers of a Product $(ab)^n = a^n\ b^n$	**Power of a Fraction** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	**Negative Exponents** $a^{-m} = \frac{1}{a^m}$ *or* $\frac{1}{a^{-m}} = a^m$
	Zero Exponent $a^0 = 1$	

Multiplying and Dividing Monomials

In this section, the exponent properties will be applied to compute with **monomials**. A monomial can be a number, a variable, or products of numbers and variables. For example, 3, x, and $3xy$ are all considered monomials.

To multiply or divide monomials, apply the properties of exponents as shown in the examples that follow.

Example 1: Simplify: $(3x^2)(4x^6)$

Step 1: Arrange the terms so that the numbers are together and the bases of x are together.

$$(3\cdot 4)(x^2\cdot x^6)$$

Step 2: Multiply the numbers and use the exponent properties to simplify the exponents.

$$12x^{2+6} = 12x^8$$

Example 2: Simplify: $\dfrac{15k^3m^2}{12km}$

Step 1: Simplify the rational number $\dfrac{15}{12}$.

$$\frac{5k^3m^2}{4km}$$

Step 2: Use the properties of exponents to simplify.

*Note: $4km$ can be write as $4k^1m^1$

$$\frac{5}{4}k^{3-1}m^{2-1} = \frac{5}{4}k^2m$$