SCALE DRAWINGS, SIMILAR FIGURES, RIGHT TRIANGLE TRIGONOMETRY

In this unit, you will review ratio and proportion and solve problems related to scale drawings and similar figures. You will then investigate right triangle trigonometry. You will examine how trigonometry is derived from the angles and sides of right triangles and then apply "trig" functions to solve problems.

Proportions

Applications of Proportions

Scale Drawings and Map Distances

Similar Polygons

Trigonometric Ratios

Table of Trigonometric Ratios

Map of Ohio

Proportions

A **ratio** is a comparison of two quantities and is often written as a fraction. For example, Emily is on the basketball team and during her first game she made 14 out of 20 shots.

You can make a ratio out of shots made and shots attempted.

$$\frac{\text{shots made}}{\text{shots attempted}} = \frac{14}{20} = \frac{7}{10}$$

A **proportion** is any statement that two ratios are equal. For example, if Rachel is on the same basketball team as Emily, and she made 21 out of 30 baskets, Rachel's ratio can be written as:

$$\frac{\text{shots made}}{\text{shots attempted}} = \frac{21}{30} = \frac{7}{10}$$

It can now be said that Emily's ratio $\dfrac{14}{20}$ and Rachel's ratio $\dfrac{21}{30}$ are equal. To prove this we will use the "cross products property".

> For all real numbers a, b, c, and d, where $b \neq 0$ and $d \neq 0$.
>
> $$\frac{a}{b} = \frac{c}{d} \text{ if } ad = bc$$

Let's take a look at Emily and Rachel's ratios and use the cross product property to make sure they are equal.

$$\frac{14}{20} = \frac{21}{30}$$

$$14 \times 30 = 20 \times 21$$

$$420 = 420 \quad \text{true}$$

Use cross products to determine if the two ratios are proportional.

Example 1: $\dfrac{4}{5} = \dfrac{7}{8}$

$4 \times 8 = 5 \times 7$

$32 = 35 \quad$ false

Example 2: $\dfrac{33}{22} = \dfrac{24}{16}$

$33 \times 16 = 22 \times 24$

$528 = 528 \quad$ true

Cross products are also used to solve for a missing value in a proportion. Follow along with the examples below.

Example 3: $\dfrac{x}{5} = \dfrac{2}{10}$

$x \cdot 10 = 5 \cdot 2$

$10x = 10$

$\dfrac{10x}{10} = \dfrac{10}{10}$

$x = 1$

Example 4: $\dfrac{y}{2.4} = \dfrac{7}{8}$

$y \cdot 8 = 7 \cdot 2.4$

$8y = 16.8$

$\dfrac{8y}{8} = \dfrac{16.8}{8}$

$y = 2.1$

Sometimes there will be more than one term in either the numerator or denominator, and in this case you must make sure that you distribute when you multiply.

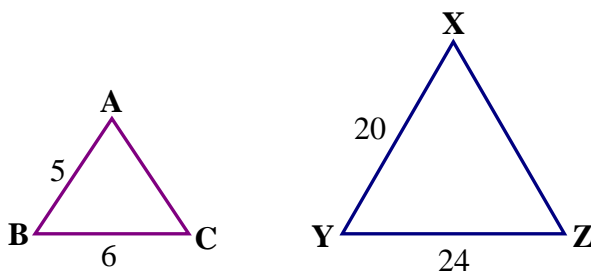*Example 5*: Solve $\dfrac{x}{3} = \dfrac{2x-4}{5}$

$x \cdot 5 = 3(2x - 4)$

$5x = 6x - 12$

$-x = -12$

$x = 12$

*Similar polygons have corresponding sides that are proportional. The $\sim$ refers to similar polygons. Let's take a look at how this works.

Example 6: Given $\triangle ABC$ and $\triangle XYZ$, prove that they are similar by using proportions.

If the two triangles above are similar, the ratios of the lengths of the sides will be proportional:

$$\frac{AB}{XY} = \frac{BC}{YZ}$$

$$\frac{5}{20} = \frac{6}{24}$$

$$5 \times 24 = 20 \times 6$$

$$120 \ = \ 120$$

Since this is true, we have proven these two triangles are similar.

Applications of Proportions

Cindy surveyed the ninth grade class and found that 11 out of 25 ninth-grade students have a video game console. There are 500 students in the class. Based on the data Cindy collected predict how many ninth grade students may have a videogame console.
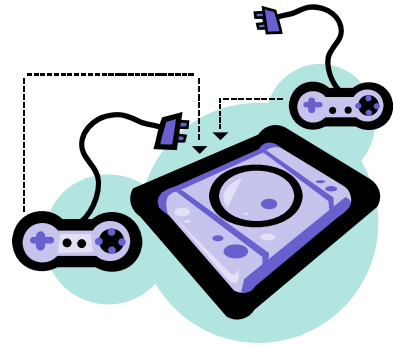
To solve, first find the comparison ratio.

$$\frac{11\,(\text{have a videogame console})}{25\,(\text{all students surveyed})}$$

Then, set up a proportion so that both numerators represent the same type of quantity and both denominators represent the same type of quantity.

Setting up the problem in a word ratio first can be very helpful.

For this problem, the word ratio would be: $\dfrac{\text{have a videogame console}}{\text{all students in the group}}$.

We know the following:

Twenty-five (25) students were surveyed which represents the total surveyed.

Eleven (11) students have a video game console in the sample group.

There are 500 students in the all in the class.

Let *n* represent the number of students predicted to have a videogame console.

$$\frac{\text{have a video game console}}{\text{all students in the group}} = \frac{11}{25} = \frac{n}{500}$$

Thus, the proportion to solve is: $\dfrac{11}{25} = \dfrac{n}{500}$.

$$\frac{11}{25} = \frac{n}{500} \qquad \xrightarrow[\text{multiply}]{\text{cross}} \qquad \frac{11}{25} = \frac{n}{500}$$

$$25 \times n = 11 \times 500 \qquad \text{Simplify.}$$
$$25n = 5500 \qquad \text{Divide.}$$
$$n = 220$$

**Cindy predicted that about 220 ninth grade students have a video game console.**

Scale Drawings and Map Distances

Scale Drawings

A **scale drawing** or a **scale model** is used to represent an object that is too large or too small to be drawn or built at actual size. Examples are blueprints, maps, models of vehicles, and models of animal anatomy.

A **scale** is determined by the ratio of a given length in a drawing or a model to its corresponding actual length.

The blueprint for a house is given below superimposed on a grid.
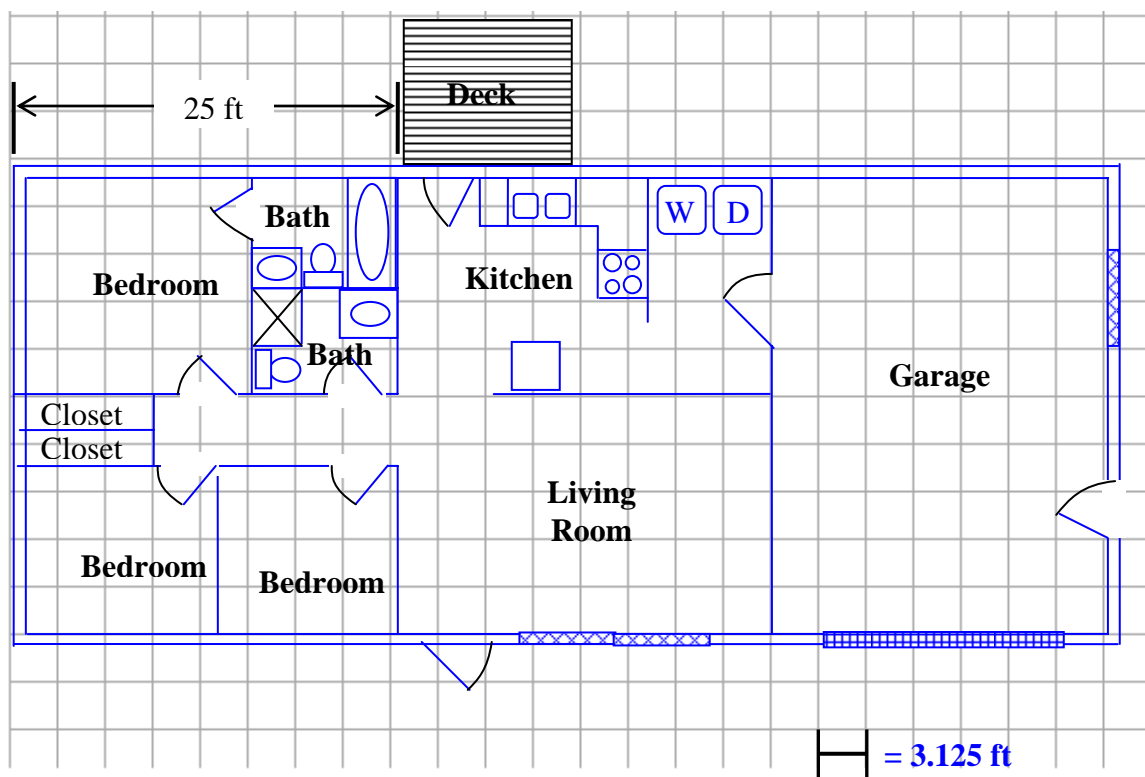
In order to determine the scale, you can complete the following.

How many units wide is the largest bedroom including the bath? 8 units

The actual width of the master bedroom is 25 feet. Write a ratio comparing the drawing width to the actual width. 8 units : 25 feet

Simplify the ratio above and compare it to the scale shown at the bottom of the drawing.

8 units : 25 units means 8 to 25

$$\frac{8}{25}$$ Write 8 to 25 as a fraction.

$$\frac{8 \div 8}{25 \div 8}$$ Reduce by 8 so that the numerator is one.

$$\frac{1}{3.125}$$ Simplify

1 : 3.125   Write the fraction as a ratio.

According to the blueprint, the length of one unit equals 3.125 feet. Therefore, the ratio and the scale indicate that **the length of one unit on the drawing is equal to 3.125 feet in reality**.

Distances on a scale drawing are proportional to distances in real-life.
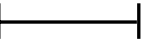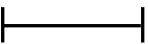
*Note: A general rule for scale models is that scales and scale factors are written so that the drawing length occurs first in the ratio.

Map Distances

The following map was generated at *http://maps.yahoo.com*. Point A represents Quaker City, OH. Point B represents Mason, OH.

Key: ├———————┤ = **50 miles**
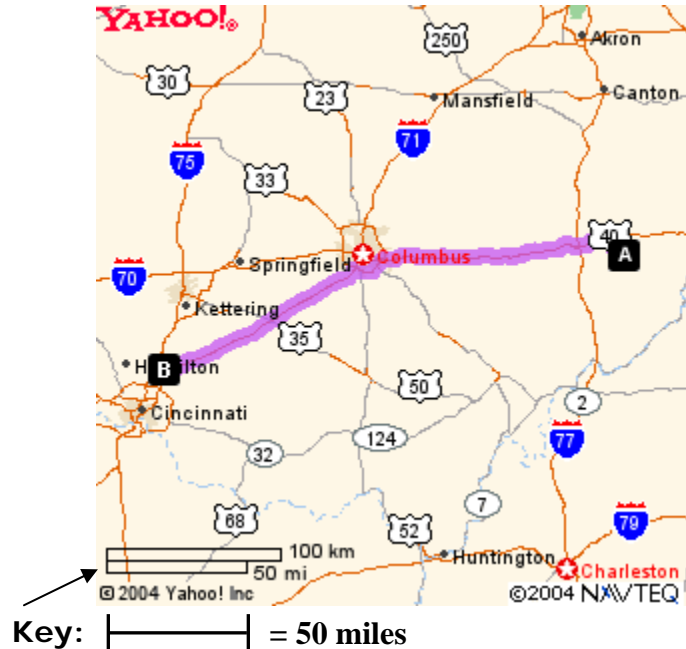
On paper mark the length of one bar in the key that represents 50 miles. Then, hold the paper along the highlighted path on the map from Quaker City, OH, to Mason, OH, and count how many lengths of the bar there are between the two communitites traveling along the marked route.

It takes about 3 of ├———————┤ plus another $\frac{3}{4}$ of ├———————┤ to get from point A to point B (3.75).

Let x represent the actual distance to point B, write a proportion, and then solve.

$$\frac{1}{50} = \frac{3.75}{x}$$ Proportion Comparing: $\dfrac{\text{Map Distance}}{\text{Actual Distance}}$

$(1)x = 50(3.75)$ Cross Multiply

$x = 187.5$ Simplify

The actual distance from point A to point B is about 187.5 miles.

Similar Polygons

Similar figures: figures that have the same shape but different sizes. The symbol ~ means similar.

Definition of similar polygons

Two polygons are similar, if and only if, their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Look at the similar quadrilaterals given below. The congruent angles are identified by curved hash marks. Each congruent angle has the same number of hash marks drawn within the angle.

$$MNOP \sim WXYZ$$
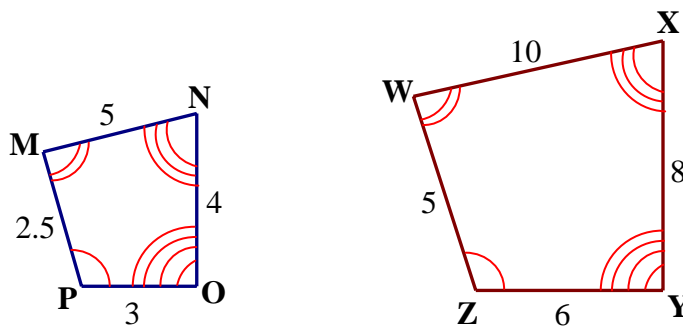
From the diagram we can conclude the following:

$$\angle M \cong \angle W, \angle P \cong \angle Z, \angle O \cong \angle Y, \text{ and } \angle N \cong \angle X$$

$$\frac{MP}{WZ} = \frac{PO}{ZY} = \frac{ON}{YX} = \frac{NM}{XW}$$

$$\frac{2.5}{5} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \Rightarrow \frac{1}{2}$$

The **scale factor** of quadrilateral *MNOP* to quadrilateral *WXYZ* is $\frac{1}{2}$ or we can say that the scale factor of *WXYZ* to *MNOP* is 2. The scale factor is the ratio of the lengths of two corresponding sides of similar polygons and can be written going from the small figure to the large figure or vice versa.

Example: Determine the scale factor and the value of x and y in the similar polygons: $ABCDE \sim MNOPQ$.
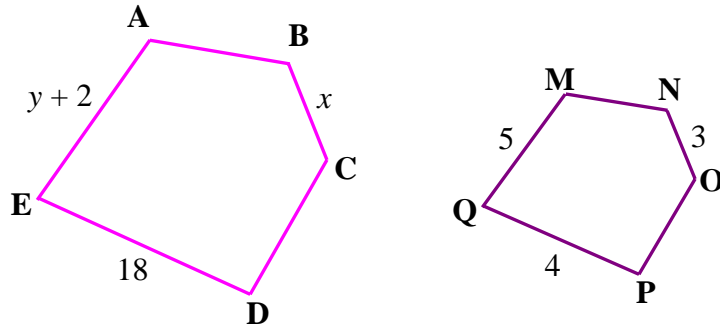
a.) Find two corresponding sides that have measures and determine the scale factor. ED and QP are corresponding sides and they both have measures so use those two sides.

$$\frac{ED}{QP} = \frac{18}{4} = \frac{9}{2}$$

b.) Find the value of x and y by using the scale factor as one ratio and the corresponding sides of $y + 2$ and x to find the value of x and y.

To find y:

$$\frac{y+2}{5} = \frac{9}{2}$$

$$2(y+2) = 5 \cdot 9$$

$$2y + 4 = 45$$

$$2y = 41$$

$$y = 20.5$$

To find x:

$$\frac{x}{3} = \frac{9}{2}$$

$$2x = 3 \cdot 9$$

$$2x = 27$$

$$x = 13.5$$

*At this point we need to mention that not all polygons of different types are similar. For example, all rectangles **are not** similar.

Trigonometric Ratios

trigonometric ratio - A trigonometric ratio is a ratio between the sides of a right triangle.
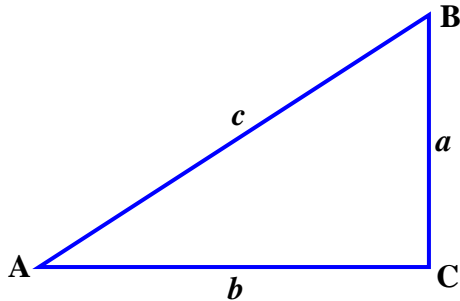
The "trig" (short for trigonometry) ratios, *sine*, *cosine*, and *tangent* are based on properties of right triangles. These three ratios are the most common trig ratios.

$$\text{sine } \angle x = \frac{\text{side opposite } \angle x}{\text{hypotenuse}} = \sin x \text{ (abbreviated)}$$
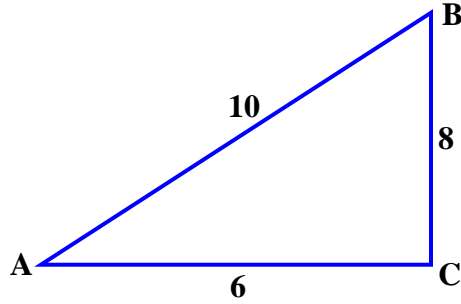
$$\text{cosine } \angle x = \frac{\text{side adjacent } \angle x}{\text{hypotenuse}} = \cos x \text{ (abbreviated)}$$

$$\text{tangent } \angle x = \frac{\text{side opposite } \angle x}{\text{side adjacent } \angle x} = \tan x \text{ (abbreviated)}$$
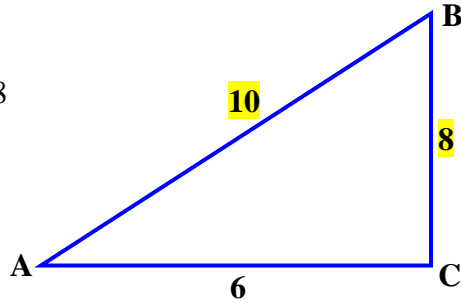
In the figure below, $\sin A = \dfrac{a}{c}$, $\cos A = \dfrac{b}{c}$, and $\tan A = \dfrac{a}{b}$.
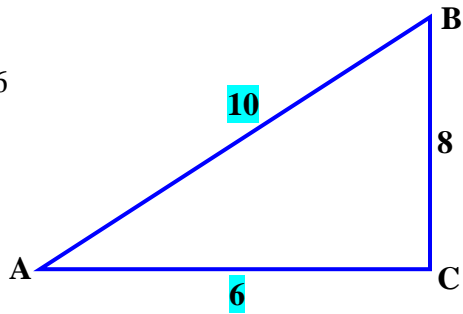
Example 1: Find the sin, cos, and tan of $\angle A$ in the triangle below.

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{8}{10} = 0.8$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{6}{10} = 0.6$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{8}{6} \approx 1.3$$

Example 2: Find the trigonometric ratios of sin, cos, and tan for the same triangle, but for $\angle B$.
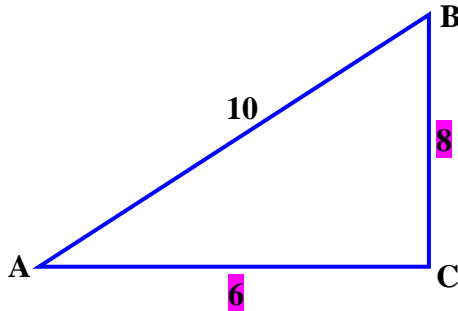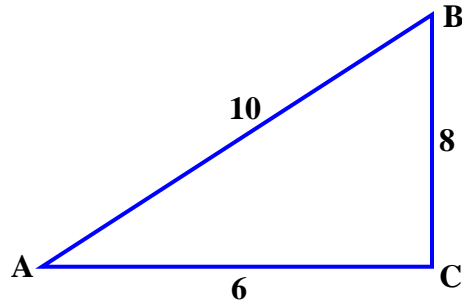

$$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = 0.6$$
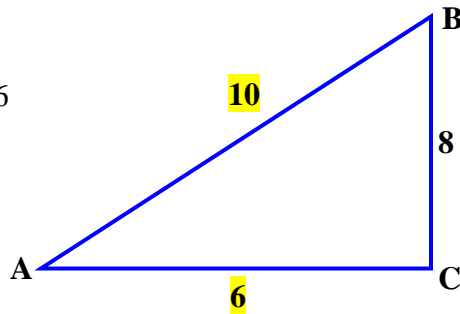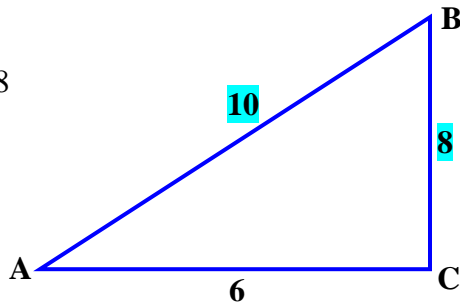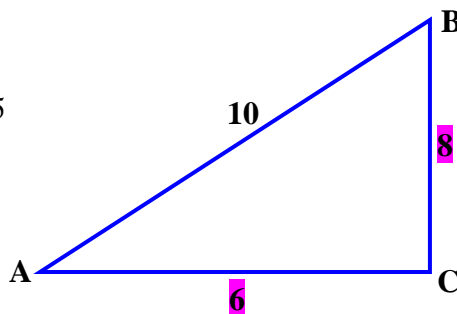

$$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = 0.8$$

$$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = 0.75$$

You may use a table of trigonometric ratios to find the sin, cos, and tan if you know the angle measure. You may also use a scientific calculator or a graphing calculator to find the ratios.

Example 3: Use the "Table of Trigonometric Ratios" to evaluate. Go back to the overview page for a link to the "Table of Trigonometric Ratios". (The table includes other trig ratios that you will study in a more advanced math class.) Once you have determined the ratios, check using the computer's scientific caclulator.

   a. $\sin 35°$

      Go to the table and go down to the 35-degree angle row, and then move over to the right to the *sin* column and find the value, 0.573576. (It is highlighted.)

      $\sin 35° \approx 0.573576$

   b. $\cos 58°$

      Since the table only goes down to 45 degrees, you must go to the bottom of the table and read up the right side to find the 58-degree row, and then move left across to the *cos* column and find the value, 0.529919.

      $\cos 58° \approx 0.529919$

   *NOTE: The headings for the columns will be at the bottom of the table for angles greater than 45 degrees. These headings are different than the headings at the top of the table, so pay close attention to them.

   c. $\tan 61°$

      Again, since the table only goes down to 45 degrees, you must go to the bottom of the table and read up the right side to find 61-degree row, and then move across to the left to the *tan* column and find the value, 1.804048.

      $\tan 61° \approx 1.804048$

   The trig values of these three angles are highlighted in the unit link's table.

We will now find the same values using the computer's scientific calculator. Make sure you open the calculator program (usually find in the Accessories Menu) and display it in scientific view. (View/Scientific) Point and click in the angle measure and then select the correct trig button.

a. $\sin 35°$ 35 | sin | 0.57357643635104609610803191282616

b. $\cos 58°$ 58 | cos | 0.52991926423320495404678115181609

c. $\tan 61°$ 61 | tan | 1.8040477552714239373811784748237

*To copy the results from the computer's calculator, select Edit/Copy, and then select Edit/Paste to paste the value into the document of your choice.

**You may also use a scientific or graphing calculator to determine the trig values.

Example 4: An airplane is taking off from an airport and has traveled 1000 ft climbing steadily at the same angle. The plane is 290 feet from the ground. At what angle has the plane been ascending for its flight?
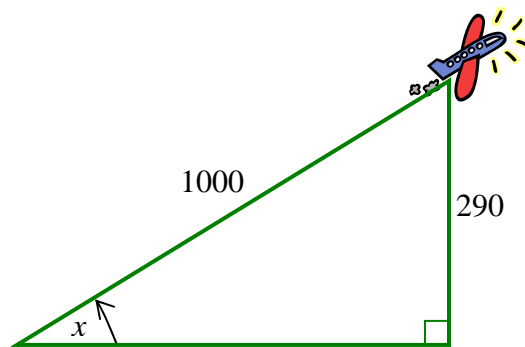
To solve this problem, determine which trig function would be most useful. In this problem, we will use $\sin x$.

$\sin x = \dfrac{290}{1000}$ $\dfrac{\text{opposite}}{\text{adjacent}}$

$\sin x = 0.29$ $290 \div 1000$

Using the table of trig values...

> Look up the value 0.29 in the *sin* column of table of trig ratios, and then look to the left to find the angle which is closest to the value.

> The closest angle to 0.29 is 17°.

Using the computer's scientific calculator....

> Select the checkbox before INV for inverse function, point and click in the decimal value, and then click on the [sin] button.

> The angle is approximately 16.86°.

Using a graphing calculator....

> Make sure the calculator is in degree mode, and then press on [2nd] [sin] 0.29.

> The angle is approximately 16.85795602°.

Example 5: Find the measurement of angle J in right △JKL with vertices of J(2,7), K(2,1) and L(10,1). Angle K is a right angle.
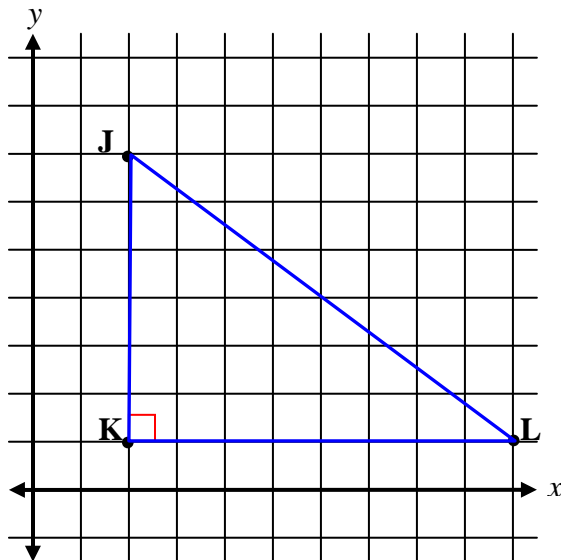
We can find the measure of angle J by determining sin, cos, or tan of angle J.

We will find cos J.

We can determine the length of the hypotenuse and the adjacent side by using the distance formula.

$$JL = \sqrt{(2-10)^2 + (7-1)^2}$$

$$JL = \sqrt{(-8)^2 + (6)^2}$$

$$JL = \sqrt{100}$$

$$JL = 10$$

$$JK = \sqrt{(2-2)^2 + (7-1)^2}$$

$$JK = \sqrt{0+36}$$

$$JK = \sqrt{36}$$

$$JK = 6$$

$$\cos \angle J = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{JK}{JL}$$

$$\cos \angle J = \frac{6}{10}$$

$$\cos \angle J = 0.6$$

$$\angle J \approx 53.1°$$

Table of Trigonometric Ratios

Deg	Sin	Cos	Tan	Csc	Sec	Cot	Deg
0	0	1	0	Infinity	1	Infinity	90
1	0.017452	0.999848	0.017455	57.298688	1.000152	57.289962	89
2	0.034899	0.999391	0.034921	28.653708	1.00061	28.636253	88
3	0.052336	0.99863	0.052408	19.107323	1.001372	19.081137	87
4	0.069756	0.997564	0.069927	14.335587	1.002442	14.300666	86
5	0.087156	0.996195	0.087489	11.473713	1.00382	11.430052	85
6	0.104528	0.994522	0.105104	9.566772	1.005508	9.514364	84
7	0.121869	0.992546	0.122785	8.205509	1.00751	8.144346	83
8	0.139173	0.990268	0.140541	7.185297	1.009828	7.11537	82
9	0.156434	0.987688	0.158384	6.392453	1.012465	6.313752	81
10	0.173648	0.984808	0.176327	5.75877	1.015427	5.671282	80
11	0.190809	0.981627	0.19438	5.240843	1.018717	5.144554	79
12	0.207912	0.978148	0.212557	4.809734	1.022341	4.70463	78
13	0.224951	0.97437	0.230868	4.445411	1.026304	4.331476	77
14	0.241922	0.970296	0.249328	4.133565	1.030614	4.010781	76
15	0.258819	0.965926	0.267949	3.863703	1.035276	3.732051	75
16	0.275637	0.961262	0.286745	3.627955	1.040299	3.487414	74
17	0.292372	0.956305	0.305731	3.420304	1.045692	3.270853	73
18	0.309017	0.951057	0.32492	3.236068	1.051462	3.077684	72
19	0.325568	0.945519	0.344328	3.071553	1.057621	2.904211	71
20	0.34202	0.939693	0.36397	2.923804	1.064178	2.747477	70
21	0.358368	0.93358	0.383864	2.790428	1.071145	2.605089	69
22	0.374607	0.927184	0.404026	2.669467	1.078535	2.475087	68
23	0.390731	0.920505	0.424475	2.559305	1.08636	2.355852	67
24	0.406737	0.913545	0.445229	2.458593	1.094636	2.246037	66
25	0.422618	0.906308	0.466308	2.366202	1.103378	2.144507	65
26	0.438371	0.898794	0.487733	2.281172	1.112602	2.050304	64
27	0.45399	0.891007	0.509525	2.202689	1.122326	1.962611	63
28	0.469472	0.882948	0.531709	2.130054	1.13257	1.880726	62
29	0.48481	0.87462	0.554309	2.062665	1.143354	1.804048	61
30	0.5	0.866025	0.57735	2	1.154701	1.732051	60
31	0.515038	0.857167	0.600861	1.941604	1.166633	1.664279	59
32	0.529919	0.848048	0.624869	1.88708	1.179178	1.600335	58
33	0.544639	0.838671	0.649408	1.836078	1.192363	1.539865	57
34	0.559193	0.829038	0.674509	1.788292	1.206218	1.482561	56
35	0.573576	0.819152	0.700208	1.743447	1.220775	1.428148	55
36	0.587785	0.809017	0.726543	1.701302	1.236068	1.376382	54
37	0.601815	0.798636	0.753554	1.66164	1.252136	1.327045	53
38	0.615661	0.788011	0.781286	1.624269	1.269018	1.279942	52
39	0.62932	0.777146	0.809784	1.589016	1.28676	1.234897	51
40	0.642788	0.766044	0.8391	1.555724	1.305407	1.191754	50
41	0.656059	0.75471	0.869287	1.524253	1.325013	1.150368	49
42	0.669131	0.743145	0.900404	1.494477	1.345633	1.110613	48
43	0.681998	0.731354	0.932515	1.466279	1.367327	1.072369	47
44	0.694658	0.71934	0.965689	1.439557	1.390164	1.03553	46
45	0.707107	0.707107	1	1.414214	1.414214	1	45
Deg	Cos	Sin	Cot	Sec	Csc	Tan	Deg

sin 17°

tan 61°

sin 35°

cos 58°