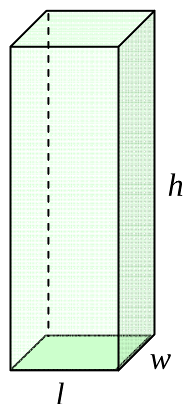
VOLUME

In this unit, you will examine the volume of several solids. You will develop and apply formulas for volume of cubes, prisms, cones, cylinders, pyramids, and spheres.

Volume of Prisms

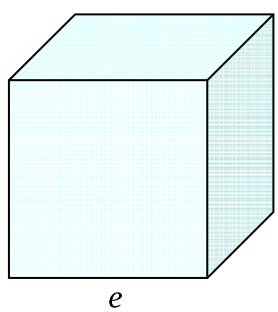
To find the volume of many 3-D solids, we will begin with the basic formula, $V = Bh$. In this formula, B represents the area of the base and h represents the height.

Volume of Rectangular Prisms and Cubes



To calculate the volume of a **rectangular prism**, multiply the area of the base times the height.

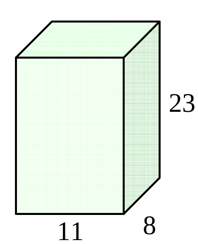
$$
\begin{aligned}
V &= B \times h \\
V &= \text{Area of Base} \times \text{Height} \\
V &= (\text{length} \times \text{width}) \times \text{height} \\
V &= l \times w \times h
\end{aligned}
$$



To calculate the volume of a **cube**, multiply the edge times itself three times.

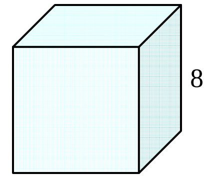
$$
\begin{aligned}
V &= B \times h \\
V &= \text{Area of Base} \times \text{height} \\
V &= (\text{length} \times \text{width}) \times \text{height} \\
V &= (e \times e) \times e \\
V &= e^3
\end{aligned}
$$

Example 1: Calculate the volume of a rectangular prism with a length of 11 feet, a width of 8 feet, and a height of 23 feet.



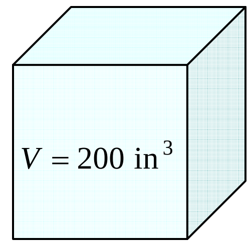
$$
\begin{aligned}
V &= l \times w \times h \\
V &= 11 \times 8 \times 23 \\
V &= 2024 \text{ cubic feet}
\end{aligned}
$$

Example 2: Calculate the volume of a cube with an edge length of 8 feet.



$$\begin{aligned} V &= e^3 \\ V &= 8^3 \\ V &= 8 \times 8 \times 8 \\ V &= 512 \text{ cubic feet} \end{aligned}$$

Example 3: If the volume of a cube is 200 cubic inches, what is the length of one edge?



$V = e^3$	Cube's volume formula
$200 = e^3$	Substitution
$\sqrt[3]{200} = \sqrt[3]{e^3}$	Find the third root of both sides of the equation.
$\sqrt[3]{200} = e$	$\sqrt[3]{e^3} = \sqrt[3]{e \cdot e \cdot e} = e$

To find the cube root (third root) of 200, we will explore two methods.

Method 1: Use approximation.

List the first ten perfect cubes:

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$	$10^3 = 1000$

Find the two perfect cubes that 200 falls between.

$125 < 200 < 216$

$5^3 < 200 < 6^3$

First approximate to nearest tenth.

200 is closer to 216...

Try 5.7 as the root ... $5.7 \times 5.7 \times 5.7 = 185.193$, which is too low.

Try 5.8 as the root ... $5.8 \times 5.8 \times 5.8 = 195.112$, which is too low, but closer.

Try 5.9 as the root ... $5.9 \times 5.9 \times 5.9 = 205.379$, which is too high.

However, we have determined that the cube root is between 5.8 and 5.9

Now approximate to nearest hundredth.

Try 5.85 as the root ... $5.85 \times 5.85 \times 5.85 = 200.201625$, which is too high.

Try 5.84 as the root ... $5.84 \times 5.84 \times 5.84 = 199.176704$, which is too low.

We will use 5.85 since 200.2 (2 tenths away) is closer to 200 than 199.1 (9 tenths away).

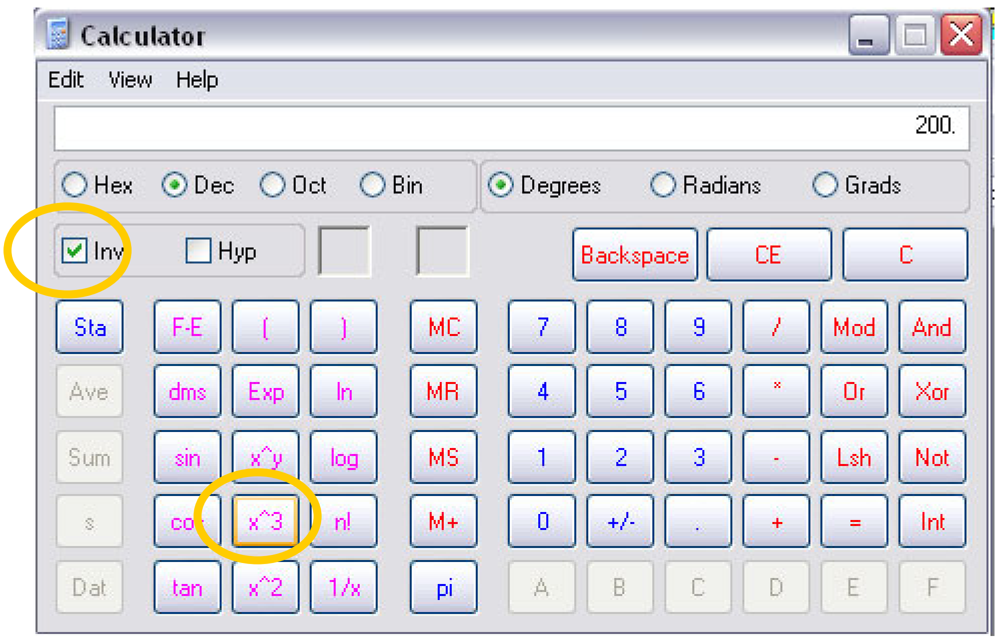
Therefore, $\sqrt[3]{200} \approx 5.85$

The length of the edge of a cube that has a volume of 200 cubic inches is approximately 5.85 inches.

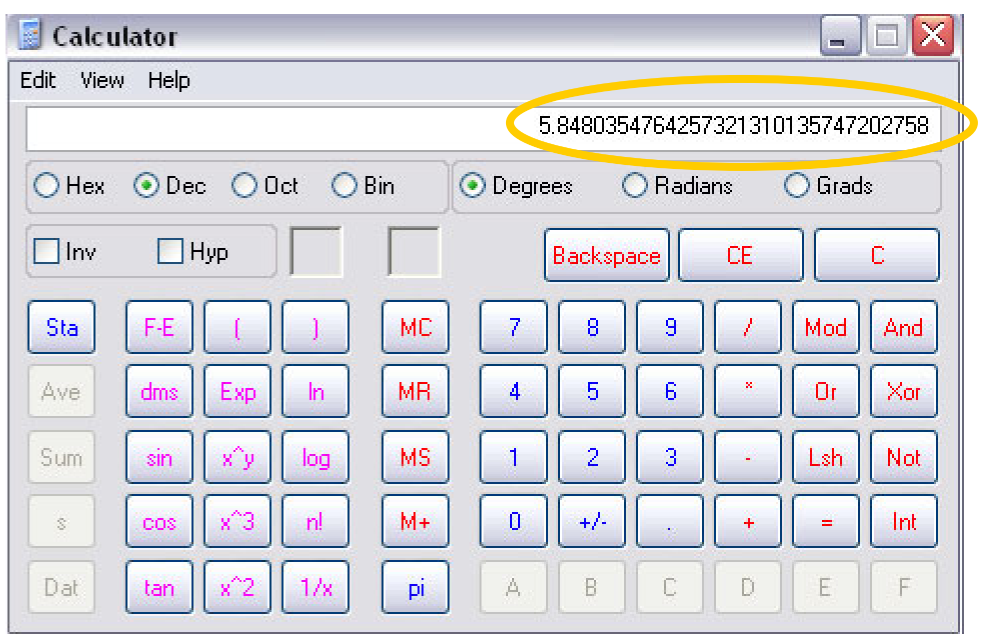
Method 2: Use a scientific calculator.

There are many types of scientific or graphing calculators. You will need to consult the manual of your personal calculator to find out how to calculate cube roots. The explanation below is based on the calculator that is found on most computers in the accessories group. (Start / Programs / Accessories / Calculator).

1) Type 200 in the white textbox.
2) Select “Inv” which will display a check in the checkbox after selected.
3) Click on x^3.



4) The answer will be displayed in the white textbox.

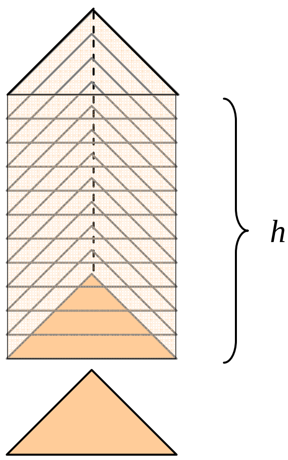


Again, we see that $\sqrt[3]{200} \approx 5.85$ when rounded to nearest hundredth.

Volume of Other Types of Prisms

The **volume of a prism** is the amount the prism can hold measured in cubic units. To calculate the volume of a prism, multiply the area of the base times the height.

Triangular Prism

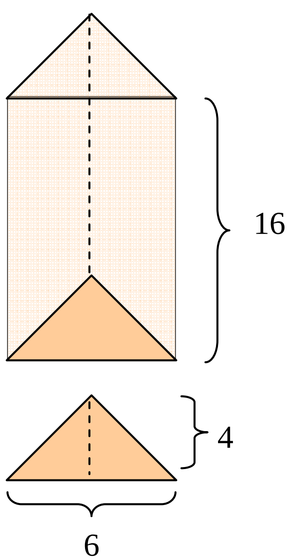


Area of Base

$$V = B \times h$$

$$V = (\frac{1}{2}bh) \times h$$

Example 3: Find volume of a triangular prism that has a height of 16 inches, and the dimensions of the triangular base are 6 inches by 4 inches.



Find the area of the triangular base first.

$$A = b \times h$$

$$A = \frac{1}{2} \times 6 \times 4$$

$$A = \frac{1}{2} \times 24$$

$$A = 12 \text{ square inches}$$

Let ***B*** represent the area of the triangular base and ***h*** represent the height of the prism**.**

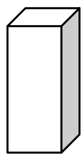
$$V = B \times h$$

$$V = 12 \times 16$$

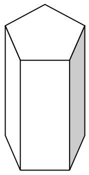
$$V = 192 \text{ cubic inches}$$

Reminder: Volume is measured in cubit units.

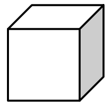
There are many kinds of prisms. They are each named by the shape of their bases. The formula, $\boldsymbol{V = B \times h}$ may be used to determine their volumes; however, each base area will be calculated differently. Here are some examples.



Rectangular Prism



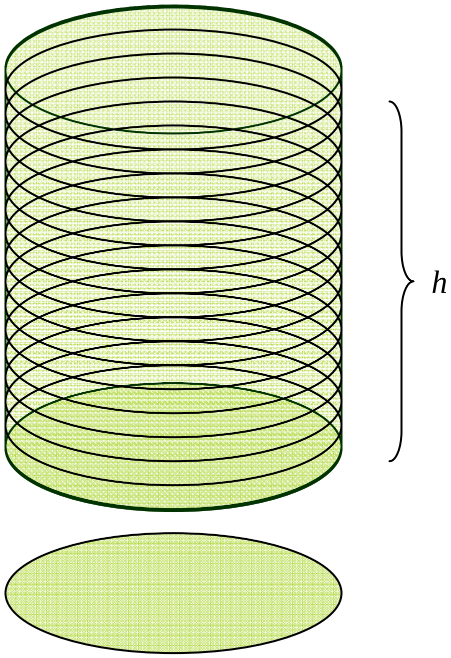
Pentagonal Prism



Cube

Volume of Cylinders

The volume of a cylinder is the amount a cylinder can hold measured in cubic units. To calculate the volume of a cylinder, multiply the area of its base times its height



$$V = \text{Area of Base} \times \text{Height}$$
$$V = (\pi \times r^2) \times h$$
$$V = \pi r^2 h$$

Area of Base

The base of a cylinder is a circle.

Example 1: Find volume of a cylinder with a radius of 5 inches and a height of 9 inches.

$$V = \pi \times r^2 \times h$$
$$V = 3.14 \times 5^2 \times 9$$
$$V = 3.14 \times 25 \times 9$$
$$V = 706.5 \text{ cubic inches}$$

Reminder: Volume is measured in cubit units.

Example 2: Find the radius of a tank that has a volume of 58.5 cubic feet and a height of 5.5 feet. Round the answer to the nearest tenth.

$V = \pi r^2 h$	Formula for Volume of a Cylinder
$58.5 = (3.14)r^2(5.5)$	Substitution ($V = 58.5,\ h = 5.5,\ \pi \approx 3.14$)
$58.5 = 17.27r^2$	Simplify
$\frac{58.5}{17.27} = r^2$	Divide both sides by 17.27
$3.39 = r^2$	Simplify to the nearest hundredth.
$\sqrt{3.39} = \sqrt{r^2}$	Take the square root of both sides.
$1.8 \approx r$	Simplify to the nearest tenth.

The radius of the tank is approximately 1.8 feet.

Example 3: Find the diameter of the tank in the previous problem.

Since the diameter of a circle is twice the radius, we can use the following formula: $d = 2r$.

$d = 2r$	Formula to calculate diameter when given radius.
$d = 2(1.8)$	Substitution ($r \approx 1.8$)
$d \approx 3.6$	Simplify

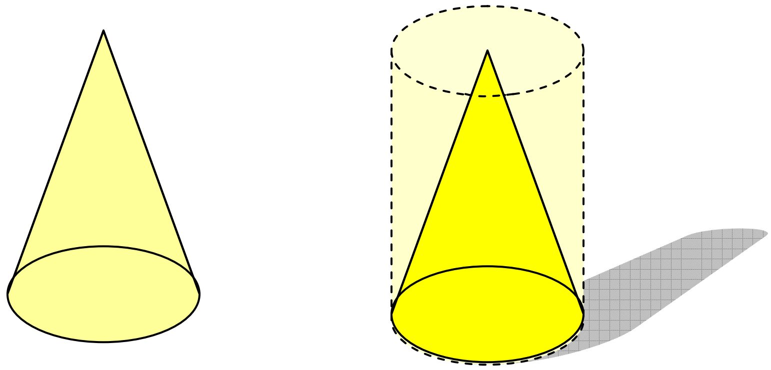
The diameter of the tank is approximately 3.6 feet.

*Note: Since the radius is an approximate measurement, then the diameter will also be an approximate measurement.

Volume of Cones and Pyramids

Volume of a Cone

A cone's volume can be found similar to finding the volume of a cylinder. The volume of a cone is equal to 1/3 the volume of a cylinder with the same base area and height.



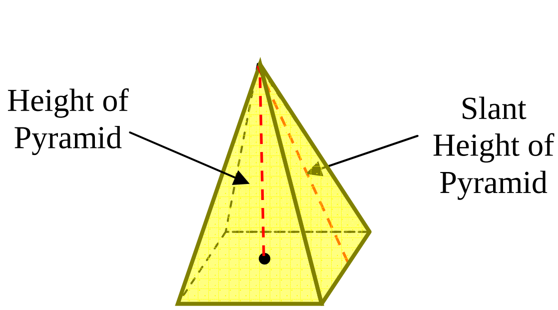
$$V = \frac{1}{3}\,\pi\, r^2 \times h$$

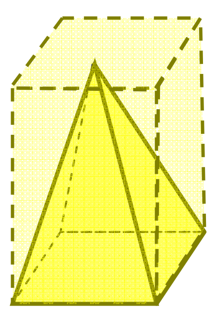
Example 1: Find volume of a cone with a radius of 5 inches and a height of 9 inches.

$$V = \frac{1}{3}\pi\, r^2 h$$
$$V = \frac{1}{3}\times 3.14\times 5^2\times 9$$
$$V = \frac{1}{3}\times 3.14\times 25\times 9$$
$$V = 235.5 \text{ cubic inches}$$

Volume of a Pyramid

The volume of a pyramid is found in a similar manner as the volume of a cone. The volume of a pyramid is 1/3 the volume of a rectangular prism with the same base area and height. Be sure to use the height of the pyramid, not the slant height, and use the area of the base of the pyramid.





$$V = \frac{1}{3}Bh$$

Example 2: Find the volume of a right square pyramid with an edge that measures 8 feet and the height is 12 feet.

$$V = \frac{1}{3}Bh$$

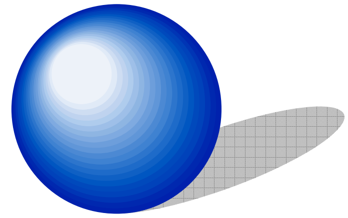
$$V = \frac{1}{3} \times (8 \times 8) \times 12$$

$$V = \frac{1}{3} \times 64 \times 12$$

$$V = 256 \text{ cubic inches}$$

Volume of Spheres

The volume of a sphere is given by the formula:



$$V = \frac{4}{3}\pi r^3$$

To find the volume of a sphere insert the radius and calculate.

Example 1: Find the volume of a sphere with a radius of 7 cm rounded to the nearest thousandth of a centimeter.

$$V = \frac{4}{3}\pi (7)^3$$

$$V = \frac{4}{3} \times 3.14 \times 343$$

$$V \approx 1436.027 \text{ cu cm}$$

Volume of Composite Solids

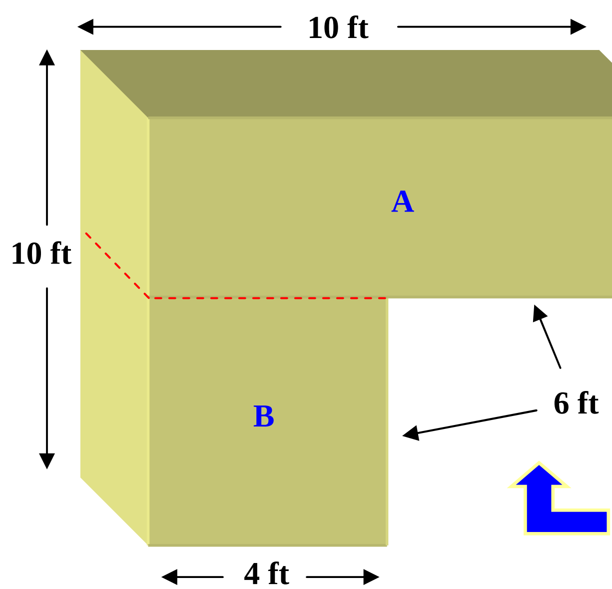
Composite solids are solids that are made up of simpler distinct solids

Figure 1 is an illustration of two rectangular prisms combined to make a composite solid.

To find the volume of a composite solid, determine the simpler solids that make up the solid. Draw lines to divide the larger solid into the smaller solids.

The red dotted line clearly differentiates between rectangular prism A and rectangular prism B.

***Figure 1*:**



Mark any dimensions (lengths) that are needed to find the two volumes.

Example 1: Find the Volume of Figure 1.

- Find the volume of rectangular prism **A**.

  $V = lwh = 10(3)(4) = 120$ cubic feet

- Find the Volume of rectangular prism **B**.

  $V = lwh = 4(3)(6) = 72$ cubic feet

- Add the volumes of rectangular prism **A** and rectangular prism **B**.

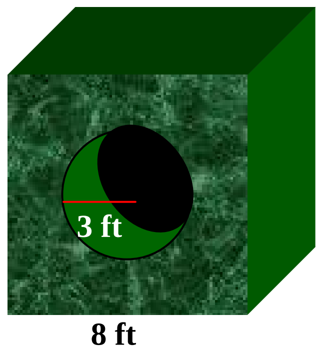
  $V(\text{solid}) = V(\text{rectangular prism A}) + V(\text{rectangular prism B})$

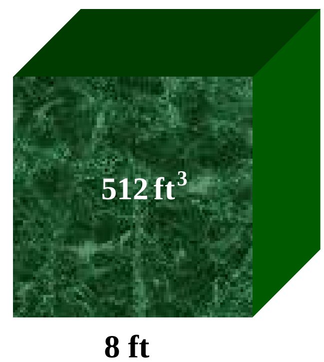
$$V(\text{solid}) = 120 + 72 = 192 \text{ ft}^3$$

Therefore, the volume of Figure 1 is 192cubic feet.

Sometimes, solids may have **"holes"** in them, and then it is necessary to **subtract** to find the volume of the solid

Example 2: What is the volume of a block that is an 8-foot cube and has a round hole cut out of it that has a radius of 3 feet?

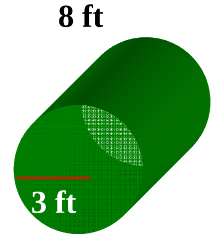




- Find the **entire volume** of the block (**cube**).

$$V = s^3$$

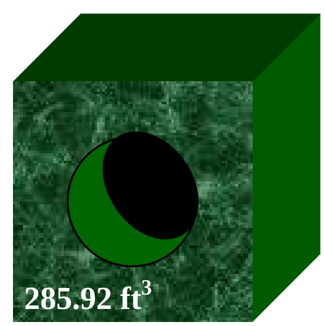
$$V = 8^3 = 512 \text{ ft}^3$$



- Find the **volume** of the hole **(cylinder).**

$$V = \pi r^2 h$$

$$V = (3.14)(3^2)(8) = 226.08 \text{ ft}^3$$



- **Subtract** the volume of the cylinder from the volume of the cube.

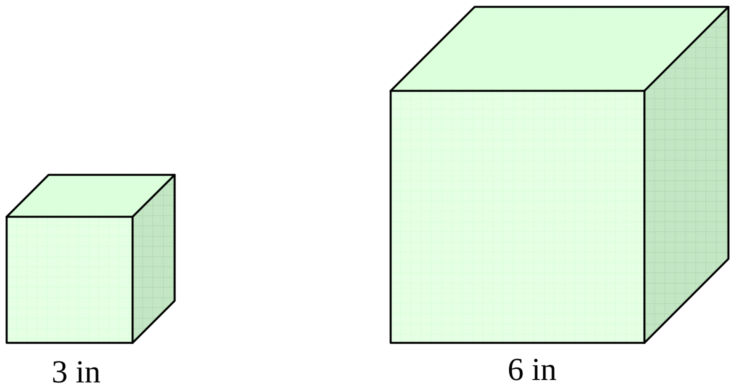
$$V(\text{solid}) = V(\text{cube}) - V(\text{hole})$$

$$V(\text{solid}) = 512 - 226.08 = 285.92 \text{ ft}^3$$

Therefore, the volume of the block (with the hold cut out) is 285.92 cubic feet.

Changing Dimensions

Study the cubes and notice that the edge of the larger cube is double the length of the smaller cube. How many times larger is the volume of the larger cube than the volume of the smaller cube?



$V = e^3$	$V = e^3$
$V = 3^3$	$V = 6^3$
$V = 27$ cu in	$V = 216$ cu in

To compare the difference in volume, divide 216 by 27 to get 8. The volume of the bigger cube is **8 times** larger than the volume of the smaller cube when the dimensions of the smaller cube are doubled.

Note: $8 = 2 \times 2 \times 2$ or 2^3.