SURFACE AREA

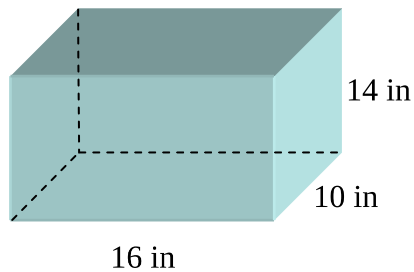
In this unit, you will examine the surface area of several solids. First, you will examine the nets of solids (two-dimensional representations of solids) and calculate the surface area by examining the areas of each face of the solid. You will then develop and apply formulas for the surface area of cubes, prisms, cylinders, pyramids, cones, and spheres. Finally, you will examine how to draw and interpret three-dimensional figures sketched on a two-dimensional surface.

Surface Area and Nets

surface area – Surface area is the sum of all the areas of a solid's outer surfaces.

net - A net is a two-dimensional representation of a solid. The surface area of a solid is equal to the area of its net.

Example: Find the surface area of rectangular prism that measures 16 inches by 10 inches by 14 inches.



Method 1:

Use the formula $A = lw$ to find the areas of the surfaces.

Front and Back:	$(16 \times 14) \times 2 = 448$
Top and Bottom:	$(16 \times 10) \times 2 = 320$
Two Sides:	$(10 \times 14) \times 2 = 280$

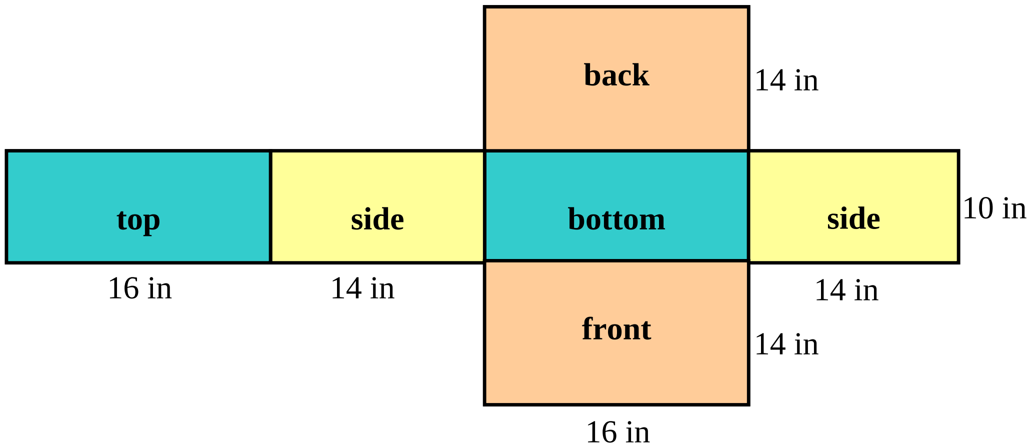
Add to find the total surface area: $448 + 320 + 280 = 1048$

$SA = 1048 \text{ in}^2$

The surface area of a 16 by 10 by 14 inch rectangular prism is 1048 square inches.

Method 2:

Draw a net for the rectangular prism and label the dimensions of each face. Find the area of each face, and then add to find the total surface area.



Side:	$14 \times 10 = 140$
Bottom:	$16 \times 10 = 160$
Side:	$14 \times 10 = 140$
Top:	$16 \times 10 = 160$
Front:	$16 \times 14 = 224$
Back:	$16 \times 14 = 224$
Total:	$140 + 160 + 140 + 160 + 224 + 224 = 1048$

The surface area of the rectangular prism is 1048 in^2.

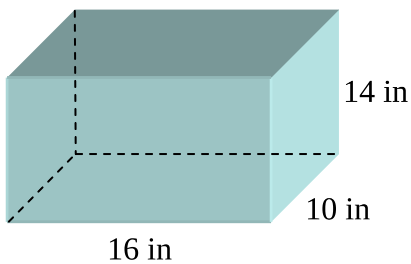
Surface Area of Prisms and Cylinders

Surface Area of a Rectangular Prism

The **surface area** of a solid is the sum of the areas of all surfaces of a figure.

A **net** is a two-dimensional representation of a solid. The surface area of a solid is equal to the area of its net.

Example 1: Find the surface area of a $16\text{ inch} \times 10\text{ inch} \times 14\text{ inch}$ (rectangular prism).



Method 1: Use the formula $A = l \times w$ to find the areas of the surfaces.

Front and Back (2 faces): $16 \times 14 \times 2 = 448$

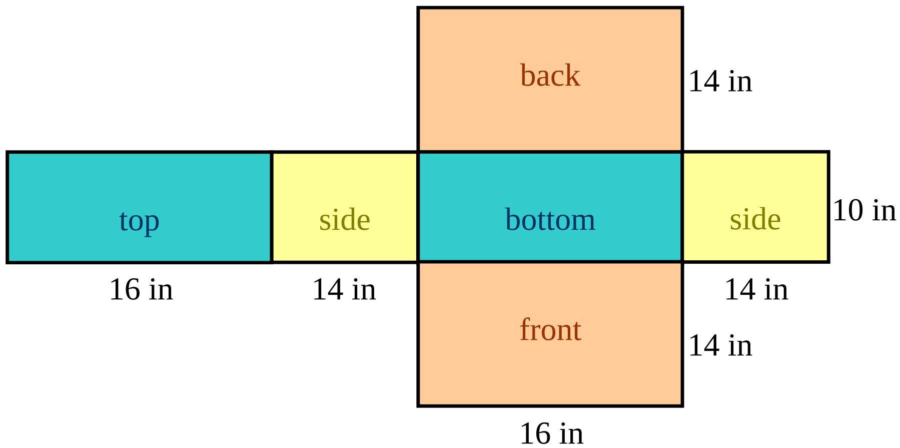
Top and Bottom (2 faces): $16 \times 10 \times 2 = 320$

Sides (2 faces): $10 \times 14 \times 2 = 280$

Add to find the total surface area: $448 + 320 + 280 = 1048$.

The surface area of a $16\text{ inch} \times 10\text{ inch} \times 14\text{ inch}$ rectangular prism is 1048 in^2.

Method 2: Draw a net for the prism and label the dimensions for each face. Find the area of each face, and then add to find the total surface area.



Back:	$16 \times 14 = 224$
Bottom:	$16 \times 10 = 160$
Front:	$16 \times 14 = 224$
Top:	$16 \times 10 = 160$
Side:	$10 \times 14 = 140$
Side:	$10 \times 14 = 140$

Total: $160 + 140 + 160 + 140 + 224 + 224 = 1048$

The surface area of the box is 1048 in^2.

Surface Area of a Prism

A **prism** is any figure that has two parallel and congruent bases in the shape of polygons and the other faces are all parallelograms. The parallelograms are called **lateral faces** of a prism and connect the bases.

The **lateral area** of a prism is the sum of the areas of the lateral faces.

The **surface area** of a prism is the sum of the lateral area plus the areas of the bases.

Example 2: Find the surface area of a triangular prism with a height of 15 inches and a base which is a right triangle. The dimensions of the right triangle are a base measuring 8 inches and a height measuring 6 inches.

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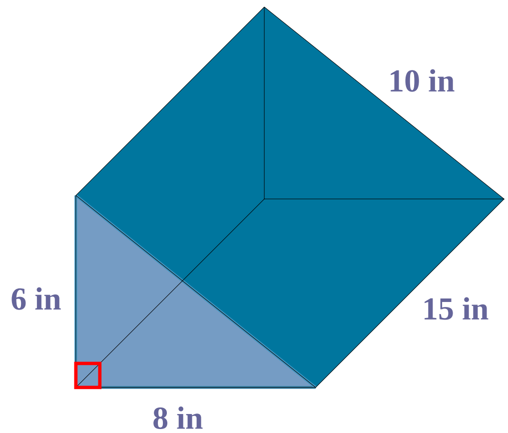
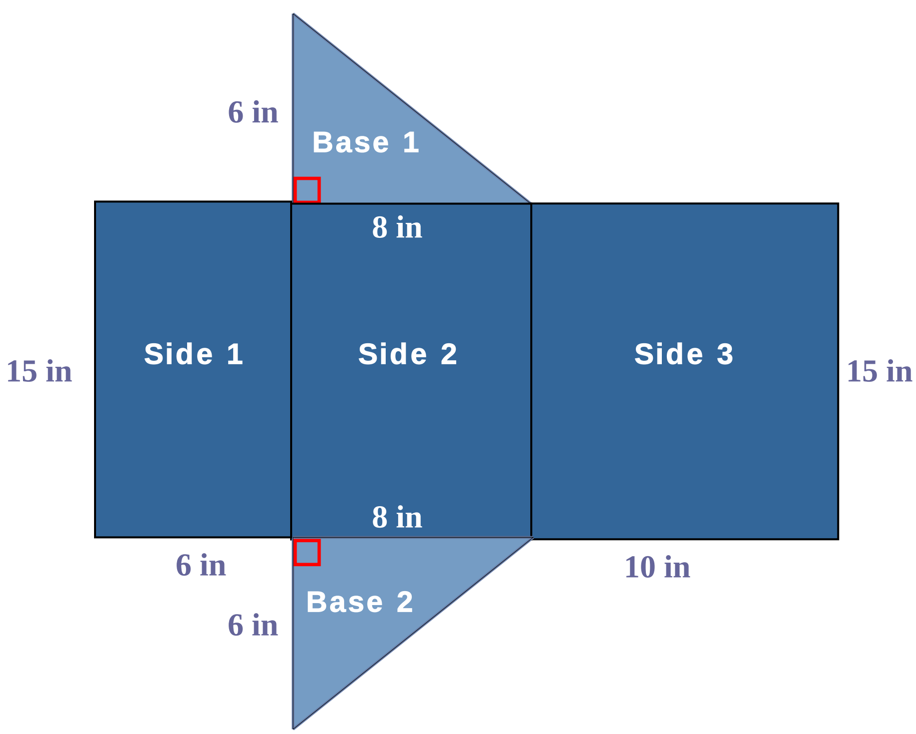


This prism is called a **triangular prism** because the bases of the prism are shaped like triangles. Therefore,

Step 1: Draw a net for the triangular prism and the label the dimensions for each face.



Step 2: Find the area of the triangular bases.

Use $A = \frac{1}{2}bh$ to find the area of the bases.

$A(\text{Base 1}) = \frac{1}{2} \cdot 8 \cdot 6 = 24$

$A(\text{Base 2}) = \frac{1}{2} \cdot 8 \cdot 6 = 24$

Step 3: Find the area of each of the rectangular lateral faces.

Use $A = bh$ to find the area of the sides.

$A(\text{Side 1}) = 6 \times 15 = 90$

$A(\text{Side 2}) = 8 \times 15 = 120$

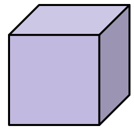
$A(\text{Side 3}) = 10 \times 15 = 150$

Step 4: Find the sum of all the areas.

Add: 24 + 24 + 90 + 120 + 150 = 408.

The surface area of the triangular prism is 408 in^2.

Surface Area of a Cube

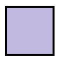


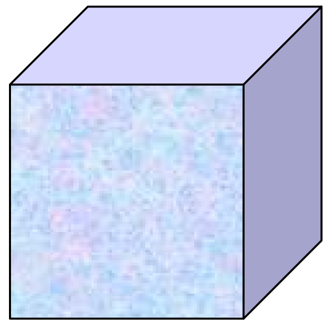
The surface area of a cube is the total area of all of the square faces measured in square units.

A **cube** is a special rectangular prism because the lengths of all of its edges are the same and all of its faces have the same area.

Area is measured in square units.

Square Unit





Let's develop the formula to compute the surface area of a cube.

Step 1: One face of a cube is a square. Its area is found by multiplying the length (e) and the width (e). (Note: e represents the length of one edge of the cube.)

$$A(\text{one face}) = e \times e = e^2$$

Step 2: A cube has six (6) faces and they all have the same area.

$$A(\text{one face}) = e^2$$

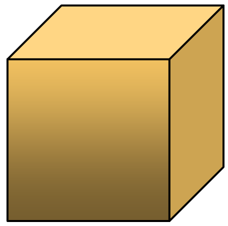
$$\text{Surface Area } (SA) = A(\text{six faces}) = 6 \times e^2$$

$$SA = 6e^2$$

To find the surface area of a cube, multiply the area of one face (e^2) times 6.

Example 3: Find the surface area of a cube with an edge that measures 7 inches.

$$SA = 6e^2$$
$$SA = 6 \times 7^2$$
$$SA = 6 \times 49$$
$$SA = 294$$



7 in

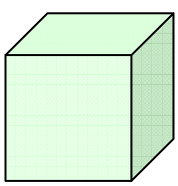
Area of one face is 7 × 7 or 49 sq in.

Area of six faces is 49 × 6 or 294 square inches.

The surface area of cube with an edge that measures 7 inches is 294 square inches.

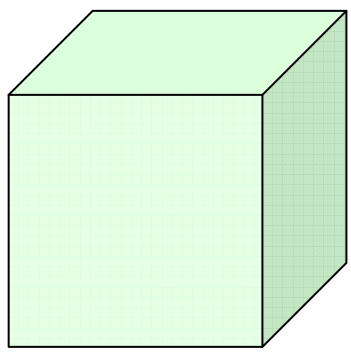
Changing Dimensions

Find the surface area of the two cubes. Notice that the edge of the larger cube is twice as long as the edge of the smaller cube.



3 in

$$SA = 6e^2$$
$$SA = 6 \times 3^2$$
$$SA = 54 \text{ sq in}$$

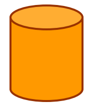


6 in

$$SA = 6e^2$$
$$SA = 6 \times 6^2$$
$$SA = 216 \text{ sq in}$$

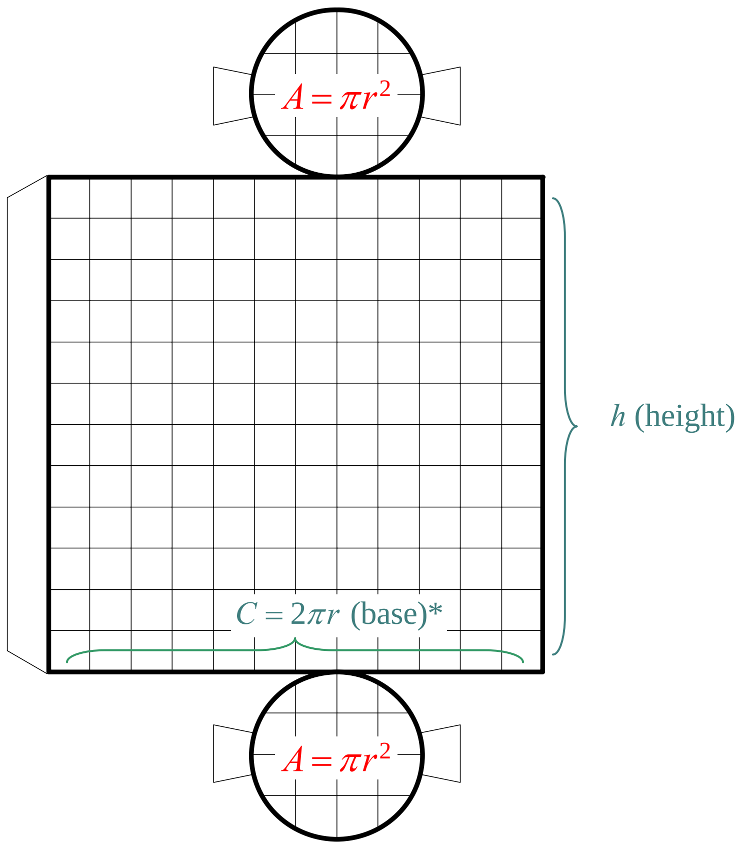
To compare the two surface areas, divide 216 by 54, to get four (4). The surface area of the larger cube is **four times** the surface area of the smaller cube.

When the **length, width, and height** of a cube are **doubled**, the **surface area is four times greater.**

Surface Area of a Cylinder 

The surface area of a cylinder is determined by adding the lateral area to the area of the two circular bases. The lateral area, the body of the cylinder, is rectangular when laid flat.

Let's examine how the formula is derived for finding the surface area of a cylinder.



*The base of the rectangular area is equal to the circumference of either of the circular bases.

To calculate the surface area of a cylinder, calculate the area of the three parts of the cylinder: the top, the bottom, and the body.

Top: Circle	**Bottom: Circle**	**Body**
$A = \pi \times r^2$	$A = \pi \times r^2$	$A = bh$ $A = C \times h$ $A = 2\pi r \times h$ $A = 2\pi rh$

Therefore, $SA = \pi r^2 + \pi r^2 + 2\pi rh$.

This formula simplifies to $SA = 2\pi r^2 + 2\pi rh$

Thus, the surface area of a cylinder can be found by using the formula:

$$SA = 2\pi r^2 + 2\pi rh$$

The lateral area of a cylinder (the area of the body) is found using $LA = 2\pi rh$.

Example 4: Find the surface area of a cylinder with a radius that measures 2 inches and a height that measures 3 inches.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2(3.14)(2^2) + 2(3.14)(2)(3)$$
$$SA = 2(3.14)(4) + 2(3.14)(2)(3)$$
$$SA = 25.12 + 37.68$$
$$SA = 62.8$$

The surface area of the cylinder is 62.8 square inches.

Example 5: Find the lateral area of the cylinder above.

$$LA = 2\pi rh$$
$$LA = 2(3.14)(2)(3)$$
$$LA = 37.68$$

The lateral area (area of the body) of the cylinder is 37.68 square inches

Surface Area of Pyramids and Cones

Surface Area of a Pyramid

A **pyramid** is a figure which has a base that is a polygon and triangles for its sides that all meet at a common vertex. A pyramid is named for the shape of its base. The lateral faces of the pyramid are the triangular sides that intersect at the vertex.

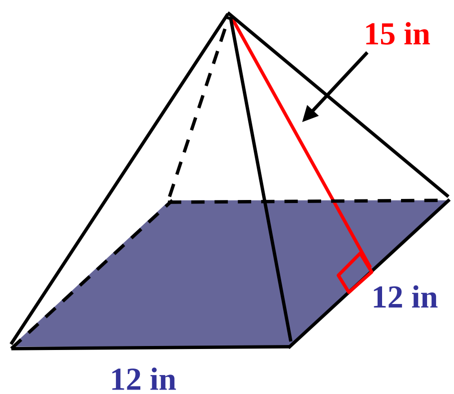
The **height of the pyramid** is the perpendicular distance between the base and the vertex.

A **regular pyramid** has a regular polygon (all sides congruent) for its base and all of the lateral faces of the pyramid are congruent isosceles triangles.

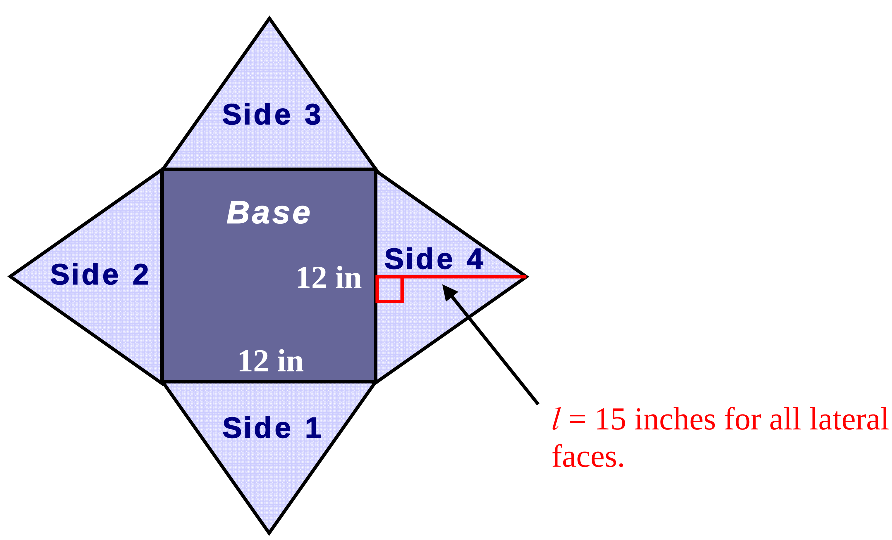
The **slant height of a pyramid** is the height of any of the triangular lateral faces. It is measured along the lateral surface. Unless other specified, the pyramids we study will be regular pyramids.

The **surface area** of any pyramid is the sum of the areas of the base and the lateral area.

Example 1: Find the surface area of a regular pyramid with a square base measuring 12 inches on one side and a slant height of 15 inches.



Step 1: Draw a net for the regular pyramid and the label the dimensions for each face.



Step 2: Find the area of the square base. Use formula $A = l \times w$ to find the area of the base.

$$A = l \times w$$

$$A(\text{Square Base}) = 12 \times 12$$

$$A(\text{Square Base}) = 144 \text{ in}^2$$

Step 3: Find the area of the triangular surfaces.

$$A = \frac{1}{2}bh$$

$$A(\text{Side 1}) = \frac{1}{2} \cdot 12 \cdot 15 = 90$$

$$A(\text{Side 2}) = \frac{1}{2} \cdot 12 \cdot 15 = 90$$

$$A(\text{Side 3}) = \frac{1}{2} \cdot 12 \cdot 15 = 90$$

$$A(\text{Side 4}) = \frac{1}{2} \cdot 12 \cdot 15 = 90$$

Step 4:

Add to find the total surface area: 144 + 90 + 90 + 90 + 90 = 504.

The surface area of the square pyramid is 504 in^2.

Surface Area of a Cone

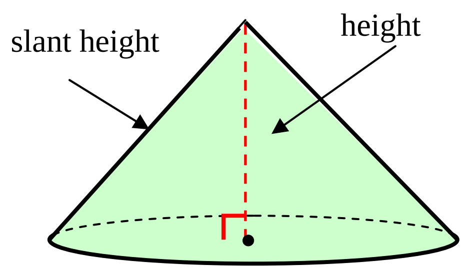
cone - A cone is a three-dimensional figure that has a circular base and one vertex. The lateral face is a circle sector.

base – The base of a cone is a circle.

height – The height of a cone is a segment that has an endpoint at the vertex and is perpendicular to the base.

slant height – The slant height of a right cone is the length of any segment that joins the vertex to the edge of the base.

Cone



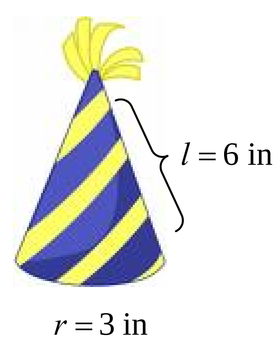
lateral surface area – The lateral surface area of a cone is the area of the curved surface.

To find the lateral surface of a cone, use the following formula:

$$LA = \pi r l$$

*Note: The development of this formula is left to study in a more advanced mathematics course.

Example 2: Find the lateral surface area of a party hat that has a radius of three inches and a slant height of six inches.

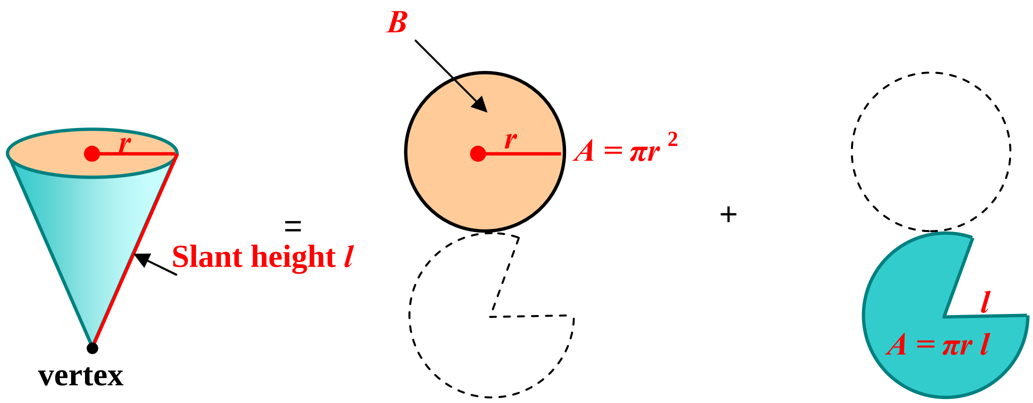


$LA = \pi r l$	Formula for Lateral Area of a Cone
$LA = \pi(3)(6)$	Substitution ($r = 3,\ l = 6$)
$LA = 18\pi$	Simplify
$LA = 56.52$	Simplify

The lateral area of the party hat (cone) is 56.52 square inches.

surface area – The surface area of a cone is the sum of the lateral area and the base area.

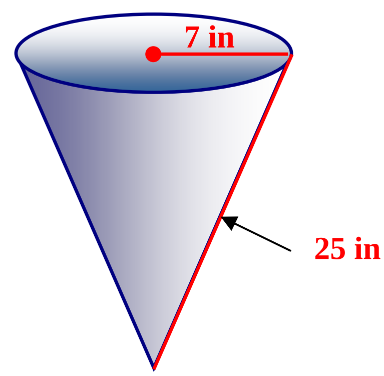
Surface area	=	Base Area	+	Lateral Area
SA	=	B	+	πrl



The surface area of a cone is the sum of the area of its base and its lateral area.

$$SA = \pi r^2 + \pi rl$$

Example 3: Find the surface area of a cone with a slant height of 25 inches and a radius of 7 inches. Round the answer to the nearest square inch.



$SA = \pi r^2 + \pi rl$	Formula for Surface Area of a Cone
$SA = \pi(7)^2 + \pi(7)(25)$	Substitution
$SA = 49\pi + 175\pi$	Simplify
$SA = 224\pi$	Simplify
$SA = 703.36$	Simplify

The surface area of the cone is approximately 703 square inches.

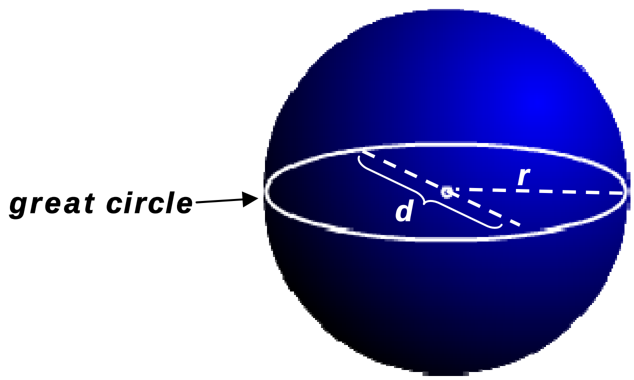
Surface Area of Spheres

A **sphere** is a 3-dimensional figure with all points equidistant from a fixed point called its center.

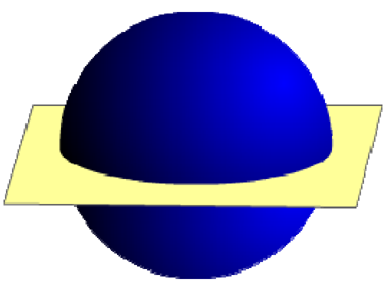
The **center of a sphere** is the fixed point from which all points on a sphere are a given distance.

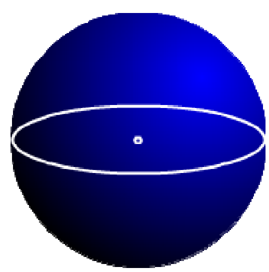
A **radius of a sphere** is a segment that has one endpoint on the sphere and the other at the center of the sphere.

A **diameter of a sphere** is a chord that passes through the center of the sphere.

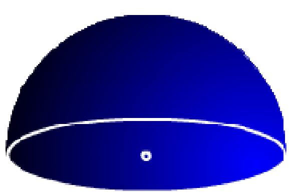


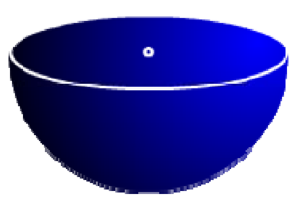
A **great circle** is the circle formed when a circle is sliced such that the slice contains the center of the sphere. The equator is the Earth's great circle.



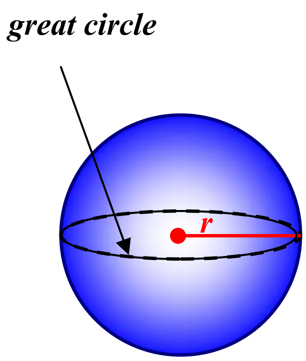
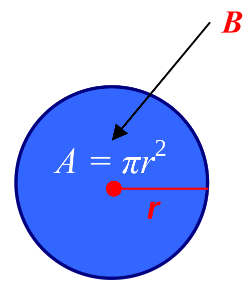


A **hemisphere** is half a sphere. A great circle divides a sphere into two congruent hemispheres.





When examining the surface area of a sphere, it takes four areas of its great circle to cover the sphere.

Surface Area	=	4	$\times$	Base Area
	=	4	$\times$	πr^2
	=	4	$\times$	

The surface area of a sphere is four times the area of its great circle.

$$SA = 4\pi r^2$$

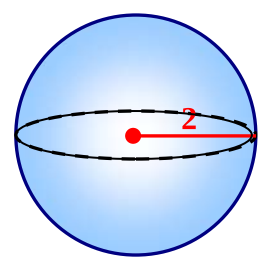
Example 1: Find the surface area of a sphere with a radius of 2 inches. Round the answer to the nearest whole square inch.

$SA = 4\pi r^2$

$SA = 4\pi(2^2)$ -Substitution

$SA = 16\pi$ -Simplify

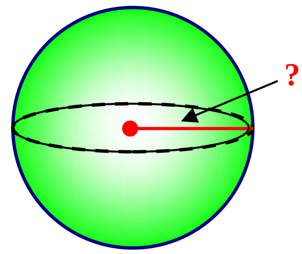
$SA = 50.24$ -Simplify



The surface area of the sphere is approximately 50 square inches.

Add the following example.

Example 2: The surface area for the sphere is 700 square centimeters. What is the radius? Round the answer to the nearest tenth.

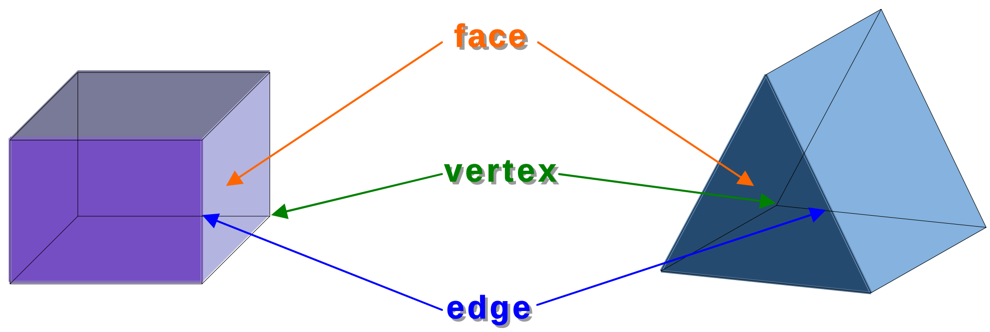


$SA = 4\pi r^2$	
$700 = 4\pi r^2$	-Substitution
$700 = 12.56r^2$	-Simplify
$\frac{700}{12.56} = r^2$	-Divide both sides by 12.56
$55.73 = r^2$	-Simplify to the nearest hundredth.
$\sqrt{55.73} = \sqrt{r^2}$	-Take the square root of both sides.
$7.5 = r$	-Simplify to the nearest tenth.

The radius of the sphere is approximately 7.5 inches.

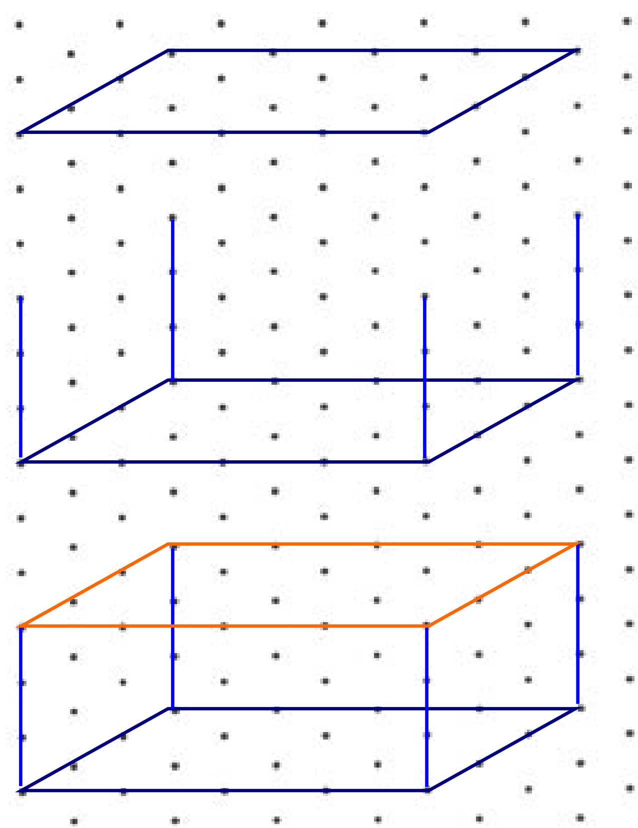
Drawing Three-Dimensional Figures

Three-dimensional figures have *faces, edges,* and *vertices*. A **face** is a flat surface. An **edge** is where two faces meet. A **vertex** is where three edges meet.



Using Isometric Dot Paper to Sketch Solids

Isometric dot paper will be used to draw various three-dimensional figures.



Example 1: Use isometric dot paper to sketch a rectangular prism that is four units long, three units wide, and three units tall.

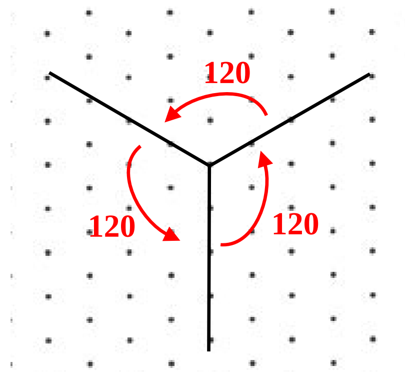
Step 1: Draw the edges of the bottom face. *(4 units by 3 units, parallelogram)*

Step 2: Draw the vertical line segments from the vertices of the base. *(3 units high)*

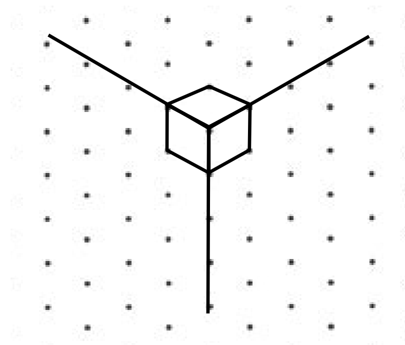
Step 3: Draw the top face by connecting the vertical lines. *(4 units by 3 units, parallelogram)*

Example 2:   Draw a unit cube using three axes that form120 degrees on isometric dot paper.

Step 1: Pick a point on the isometric dot paper. Draw a set of three axes that form 120 degrees.

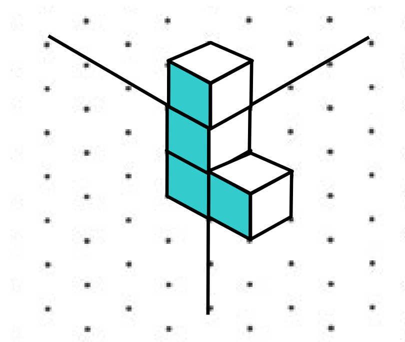


Step 2: Draw a unit cube where the three axes intersect.

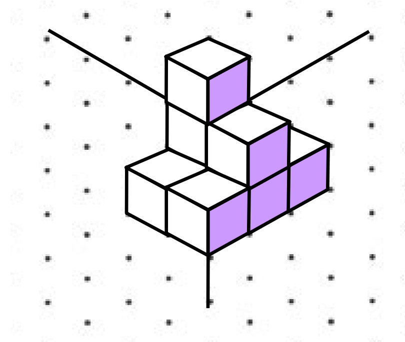


Example 3: Draw unit cubes from different points of view.

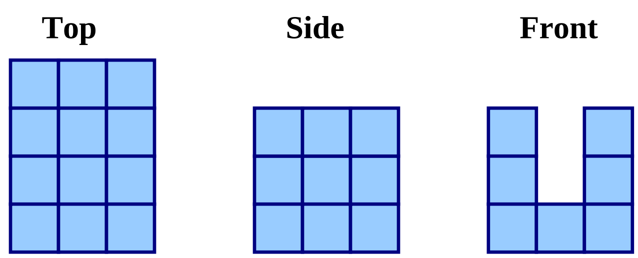
Sample 1: Unit cubes from one point of view.



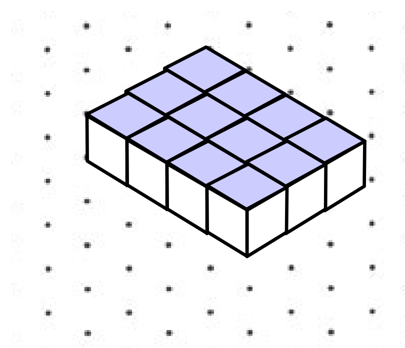
Sample 2: Unit cubes from another point of view.



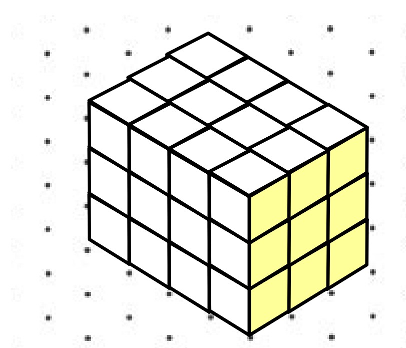
Example 4: Use the three views of the solid below to build the solid unit on isometric dot paper.



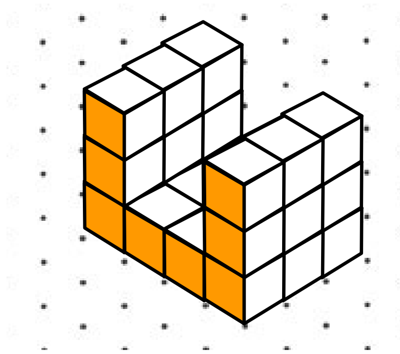
Step 1: The top view gives information about the bottom. There are 12 cubic units on the bottom.



Step 2: The side view shows that there are three layers in the solid.



Step 3: The front view shows how to form the three layers of cubes. The middle rows of cubes are missing from the top two layers.

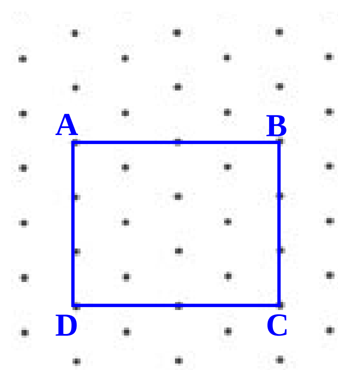


Perspective

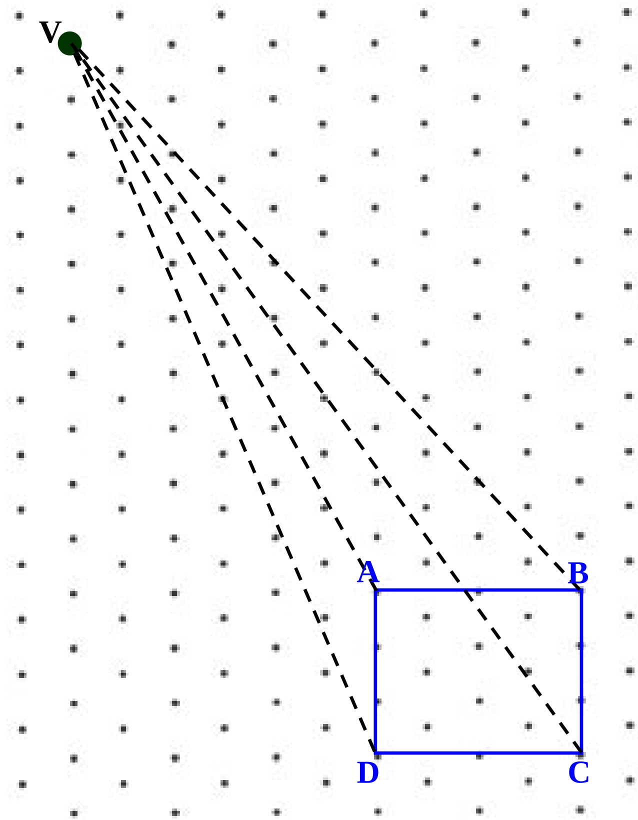
Perspective is used to make three-dimensional objects appear to have depth and distance. In a one-point perspective drawing, there is a vanishing point. The **vanishing point** is the point where lines running away from the view meet.

Example 5:   Use isometric dot paper to sketch a one-point perspective drawing of a three dimensional rectangular prism that has a front face that measures 2 units long and 3 units high .

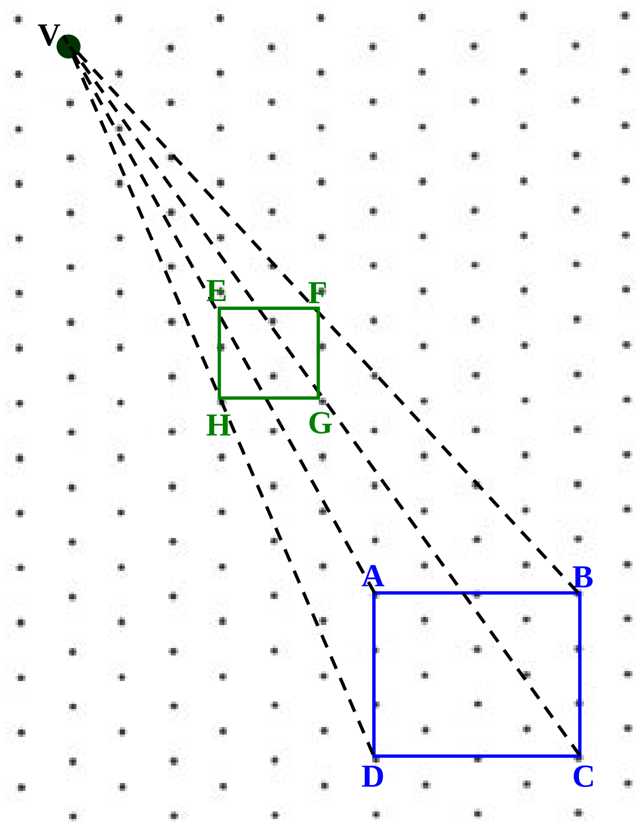
Step 1: Draw a rectangle. This is the front face. Label the vertices ABCD. (*2 units long by 3 units high)*



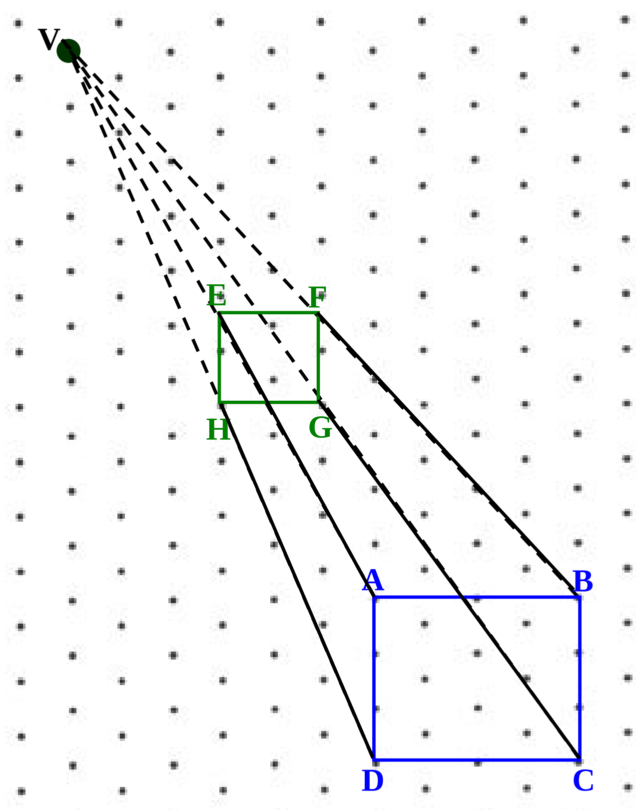
Step 2: Choose and label a vanishing point V somewhere above the rectangle and draw a dashed line from each vertex to V.



Step 3: Choose a point along the dotted segment AV. Label this point E. Draw a smaller rectangle with E as one of its vertices. Label this smaller rectangle EFGH.

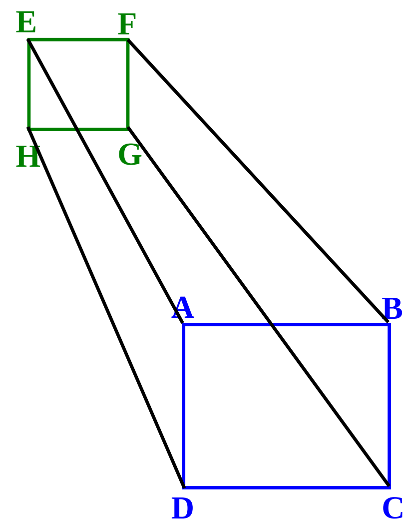


Step 4: Connect the vertices of the two rectangles along the dashed lines.



Step 5: Erase the vanishing point and all the lines connecting from the vantage point to the vertices of rectangle EFGH.

The result is a drawing of a three-dimensional rectangular prism with one-point perspective and sketched on a two-dimensional surface (paper).



Isometric Dot Paper