PERIMETER, CIRCUMFERENCE, AND AREA

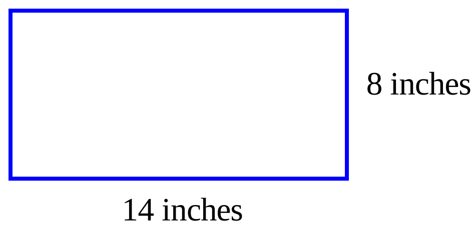
In this unit, you will determine perimeter, find circumference, and calculate area of two-dimensional shapes. Perimeter is the distance around polygons and circumference is the distance around circles. Area is the region that any two-dimensional shape covers and is measured in square units. You will extend your knowledge of area by finding the area of circle sectors and the area of composite shapes.

Perimeter

Perimeter of Rectangles

To find the **perimeter** of a rectangle, calculate the sum of the lengths of all of the sides.

Example 1: Find the perimeter of the rectangle that has a length of 14 inches and a width of 8 inches.



Solution 1: Add the lengths of all the sides.

14 + 8 +14 + 8 = **44 inches.**

A formula for this method is $P = L + W + L + W$.

Solution 2: There is another way to find the perimeter of a rectangle in which the work is shortened somewhat.

Since, by definition, the opposite sides of a rectangle have equal length, then the perimeter may be found by doubling the length and doubling the width, and then calculating the sum.

$$P = (2 \times 14) + (2 \times 8)$$
$$P = 28 + 16$$
$$P = 44 \text{ inches}$$

A formula for this method is $P = (2 \times L) + (2 \times W)$.

This formula may be simplified to $P = 2L + 2W$.

Solution 3: There is a third way to find the perimeter of the rectangle.

Add the length and width first, and then double the sum.

$P = 2 \times (14 + 8)$
$P = 2 \times 22$
$P = 44$ inches

A formula for this method is $P = 2 \times (L + W)$.

This formula may be simplified to $P = 2(L + W)$

Three Formulas for Finding the Perimeter of a Rectangle:

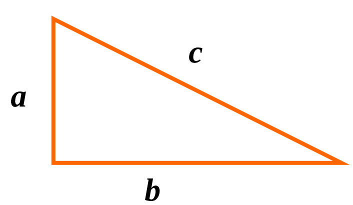
$$P = L + W + L + W$$
or
$$P = 2L + 2W$$
or
$$P = 2(L + W)$$

When calculating perimeter, pick the formula that works best for you!

Perimeter of Polygons

To find the **perimeter** of any polygon, find the sum of the lengths of all sides.

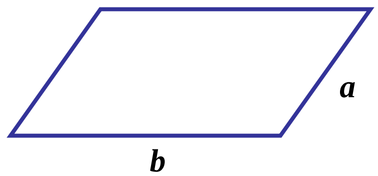
Example 2: Determine the formula for finding the **perimeter of a triangle** where the length of the sides are represented with *a*, *b*, and *c*.



Solution: Add the length of all sides.

Formula for Perimeter of a Triangle: $P = a + b + c$

Example 3: Derive a formula for finding the **perimeter of a parallelogram** where the length of the longer side is represent by b and the length of the shorter side is represented by a.



Solution: Add the length of all four sides.

$P = b + a + b + a$

$P = b + b + a + a$ Commutative Property

$P = 2b + 2a$ Collect Like Terms

Formula for Perimeter of a Parallelogram: $P = 2a + 2b$

Example 4: Derive a formula for finding the **perimeter of a square** where the length of one side is represented by s.

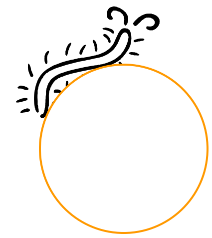
Solution: Add the length of all four sides.

$P = s + s + s + s$

$P = 4s$ Collect Like Terms

Formula for Perimeter of a Square: $P = 4s$

Circumference



The circumference of a circle is the distance around a circle. The ratio of the circumference of a circle and its diameter is equal to "pi" (π). We will use this concept to develop two formulas for determining the circumference of a circle.

Pi (π) is the ratio of the circumference of a circle to its diameter $\left(\frac{C}{d}\right)$.

π is approximately equal to 3.14.*

$*\pi =$ 3.14159265358979323846264338327950288419716939937510…

$\pi = \frac{C}{d}$	
$\frac{\pi}{1} = \frac{C}{d}$	-make a proportion
$1 \times C = \pi \times d$	-cross multiply
$C = \pi d$	-simplify
$d = 2r$	-1 diameter = 2 radii
$C = \pi d$	-determined above
$C = \pi(2r)$	-substitute $2r$ into $C = \pi d$
$C = (\pi 2)r$	-associative property
$C = (2\pi)r$	-commutative property
$C = 2\pi r$	-simplify

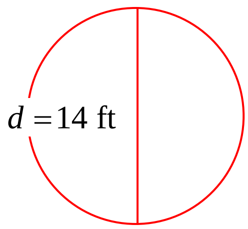
Formula for Circumference of a Circle:

$$C = \pi d$$

or

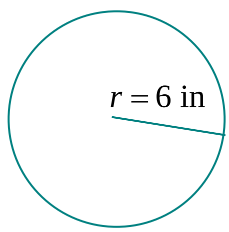
$$C = 2\pi r$$

Example 1: Find the circumference of a circle with a diameter of 14 feet.



$$C = \pi d$$
$$C = 3.14(14)$$
$$C = 43.96 \text{ ft}$$

Example 2: Find the circumference of a circle with a radius of 6 inches.



$$C = 2\pi r$$
$$C = 2(3.14)(6)$$
$$C = 37.68 \text{ in}$$

Example 3: If the circumference of a flower garden is 40 feet, what is the length of its radius, to the nearest tenth of a foot?

$C = 2\pi r$	Circumference Formula
$40 = 2\pi r$	Substitute C = 40 feet
$40 = 2(3.14)r$	Substitute 3.14 for "pi".
$40 = 6.28r$	Simplify
$r \approx 6.4 \text{ ft}$	Divide and Round

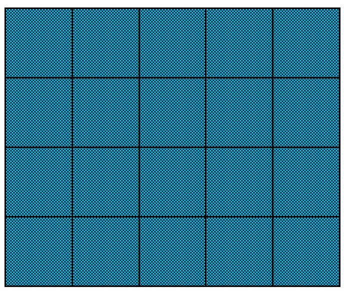
Area

Area of a Rectangle

The **area of a rectangle** is the product of its length and width. Area is a measurement of coverage and is measured in **square units**.

Example 1: Find the area of a rectangle that has a length of 5 units and a width of 4 units.

Length = 5 units



Width = 4 units

$$A = l \times w$$

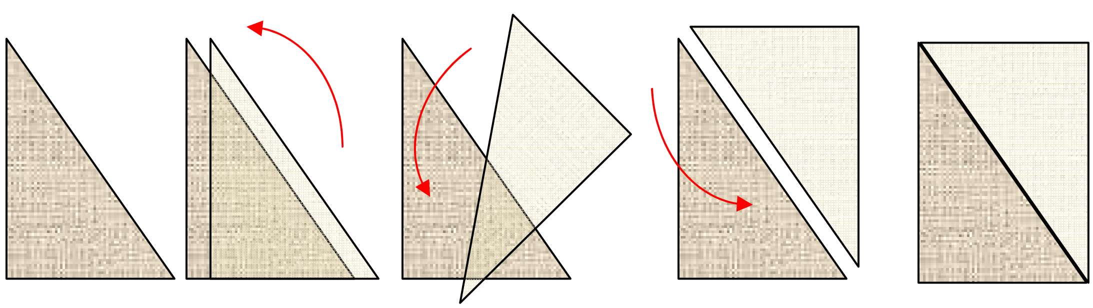
$$A = 5 \times 4$$

$$A = 20 \text{ square units}$$

Sometimes the length is called the base and the width is called the height. The formula can then be written as $A = bh$.

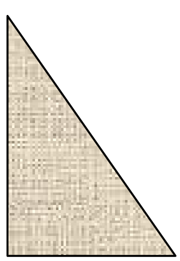
Area of a Triangle

A **triangle's area** is equal to half of the area of a rectangle with the same base and height.



Example 2: Find the area of a triangle with a base of 5 units and a height of 6 units.

Height = 6 units



Base = 5 units

$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times 5 \times 6$$

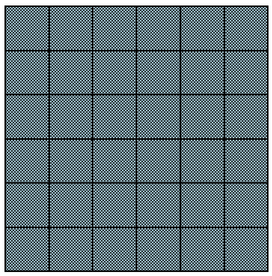
$$A = \frac{1}{2} \times 30$$

$$A = 15 \text{ square units}$$

Area of a Square

The **area of a square** is the product of its length and width. Since squares have sides of equal length, the area of a square is the product of its length (side) and its width (side).

Example 3: Find the area of a square with a side that measures 6 units.



Side = 6 units

$A = l \times w$

$A = s \times s$ -side $\times$ side

$A = s^2$ -$s \times s = s^2$

$A = 6^2$ -substitute

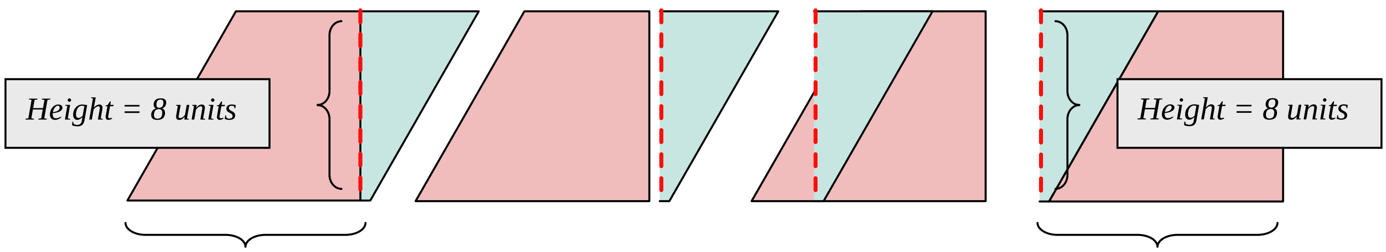
$A = 36$ square units

Area of a Parallelogram

The formula for the area of a parallelogram is the same as the area of a rectangle with one stipulation; that is, the height of the parallelogram is the measurement of a length between the bases and is perpendicular to the base or an extension of the base.

The area of a parallelogram can be rearranged into the shape of a rectangle when the parallelogram is cut along a perpendicular height from the top to the base. Thus, the formula for the area of a rectangle ($A = bh$) may be used to find the area of a parallelogram.

Example 4: Find the area of a parallelogram with a base of 10 units and a height of 8 units.

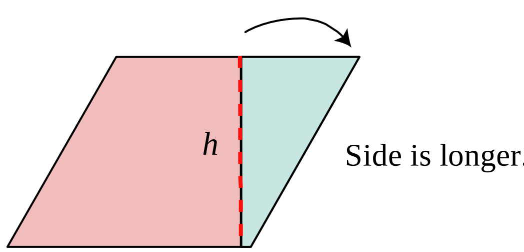


$$A = b \times h$$

$$A = 10 \times 8$$

$$A = 80 \text{ square units}$$

Why can't we use the length of the side of a parallelogram as the height of the parallelogram?



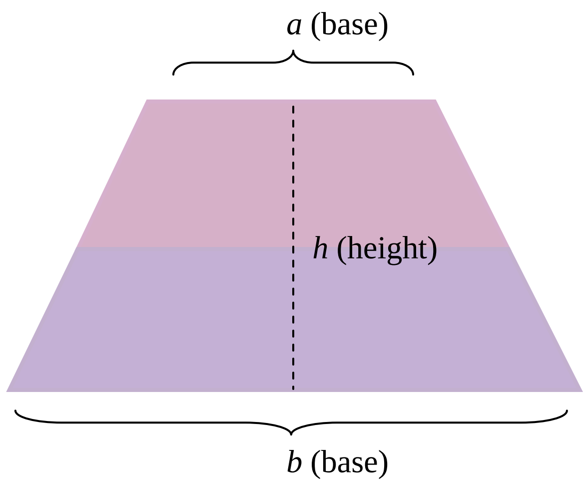
Answer: The height of a parallelogram is shorter than the length of the side.

When determining a parallelogram's area, be sure to measure the height of a parallelogram, not its side.

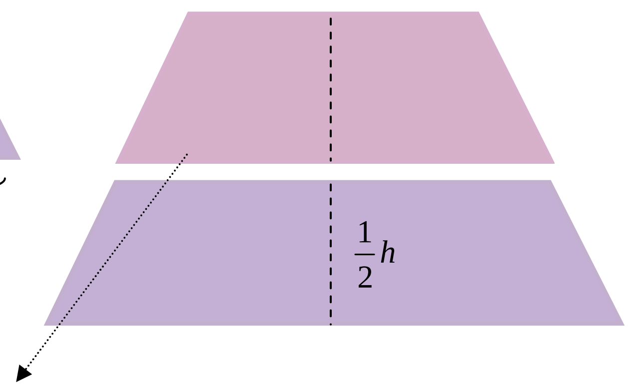
Note: Rectangles and squares are parallelograms where the sides and height are the same measure.

Area of Trapezoid

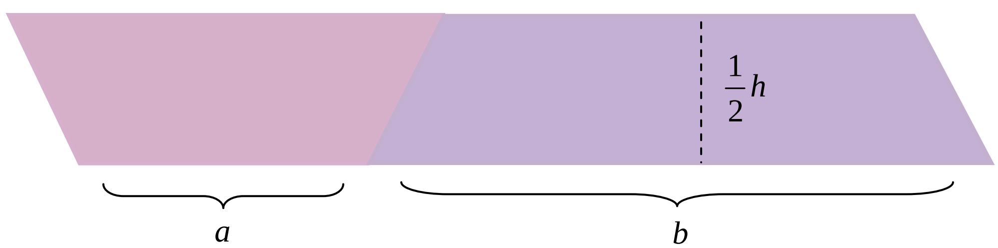
The formula for the area of a trapezoid can be developed by slicing and rearranging the trapezoid's area into a parallelogram.



Make a horizontal cut of the trapezoid's area through the halfway point of the height.



Flip the top area and slide it down beside the bottom area to make a parallelogram.



The base of the new parallelogram is equation to the sum of the bases ($a + b$) of the original trapezoid.

The height of the new parallelogram is $\frac{1}{2}$ of the height of the original trapezoid.

Use the formula for the area of a parallelogram to determine the formula for the area of a trapezoid.

$$A = \quad b \quad \times \quad h$$

$$A = \quad (a + b) \times \frac{1}{2}h \qquad\qquad \text{-substitute}$$

$$A = \quad \frac{1}{2}h \times (a + b) \qquad\qquad \text{-commutative property}$$

$$A = \frac{1}{2}h(a+b) \qquad\qquad \text{-simplify}$$

The area of a trapezoid is equal to half of the distance between the parallel sides of the trapezoid times the sum of the length of the two bases (parallel sides).

Example 5: Find the area of a trapezoid with bases of 4 feet and 10 feet and the height is 15 feet.

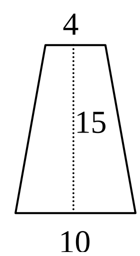
$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2} \times 15 \times (4+10)$$

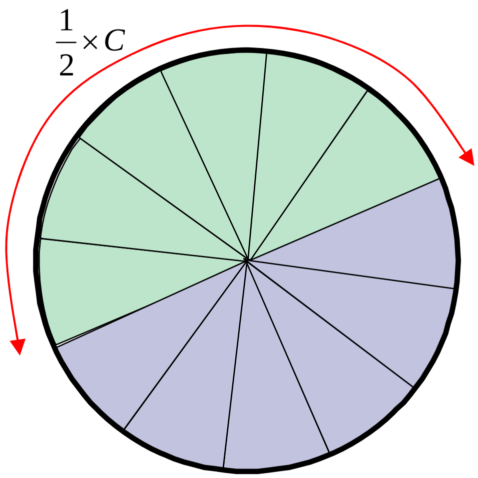
$$A = \frac{1}{2} \times 15 \times 14$$

$$A = \frac{1}{2} \times 210$$

$$A = 105 \text{ square feet}$$



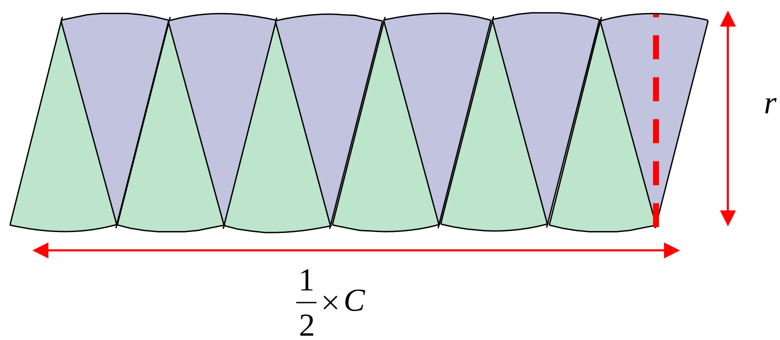
Area of a Circle



A circle's area can be rearranged into a shape that approximates a parallelogram.

The length of the parallelogram is the same length as half the circle's circumference. The height of the parallelogram is the same as the radius of the circle.

Reminder: The area of a circle is the area that the circle covers.

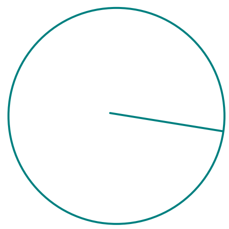


Follow along in the statements and reasons below to see how the formula for the area of a circle is developed from the formula for the area of a parallelogram.

Statement	*Reason*
$A = b \times h$	area of a parallelogram
$A = (\frac{1}{2}\times C)\times r$	base $= \frac{1}{2}\times C$ height $= r$
$A = (\frac{1}{2}\times 2\times \pi \times r)\times r$	$C = 2\times \pi \times r$ (substitute for C)
$A = 1\times(\pi \times r)\times r$	$\frac{1}{2}\times 2 = 1$
$A = (\pi \times r)\times r$	Identity Property of Multiplication ($n\times 1 = n$)
$A = \pi \times (r\times r)$	Associative Property of Multiplication (the regrouping property).
$A = \pi \times r^2$	$r\times r = r^2$

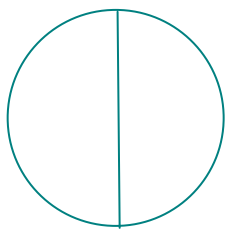
Example 6: Find the area for each of the circles.

Radius = 5 inches



$$A = \pi \times r^2$$
$$A = 3.14 \times 5^2$$
$$A = 3.14 \times 25$$
$$A = 78.5 \text{ square inches}$$

Diameter = 20 feet



$$A = \pi \times r^2$$
$$A = 3.14 \times 10^2 \qquad d = 20,\ r = 10*$$
$$A = 3.14 \times 100$$
$$A = 314 \text{ square feet}$$

*Since the diameter is given as 20 feet, we must find the radius, which is half the diameter, and then substitute into the formula. ($\frac{1}{2}$ of 20 is 10)

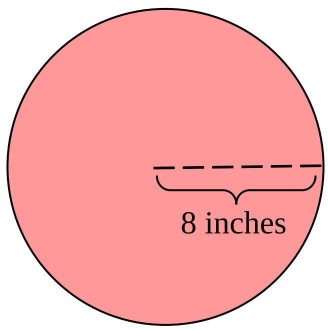
Remember, **area** is a measurement of **coverage**; thus, area calculations result in **square units**.

Area of a Circle Sector

To find the area of a sector of a circle, first determine the **area of the whole circle,** and then find the **fractional part** that represents the circle.

Example: Find the area of three-fourths of a circle with a radius of eight inches.

First, find the area of the whole circle.



$$A = \pi \times r^2$$

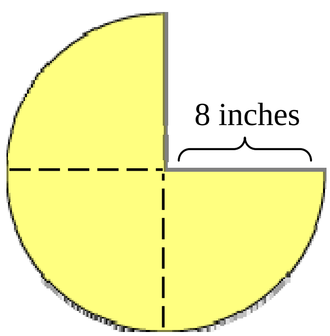
$$A = 3.14 \times 8^2$$

$$A = 3.14 \times 64$$

$$A = 200.96 \text{ sq in}$$

Then, find three-fourths of the total area.

Since the sector is 3/4 of the area of the entire circle, the area of the sector is:



$$\frac{3}{4} \text{ of } 200.96 \quad = \frac{3}{4} \times \frac{200.96}{1}$$

$$= \frac{602.88}{4}$$

$$= 150.72 \text{ sq in}$$

Area of a Composite Figure

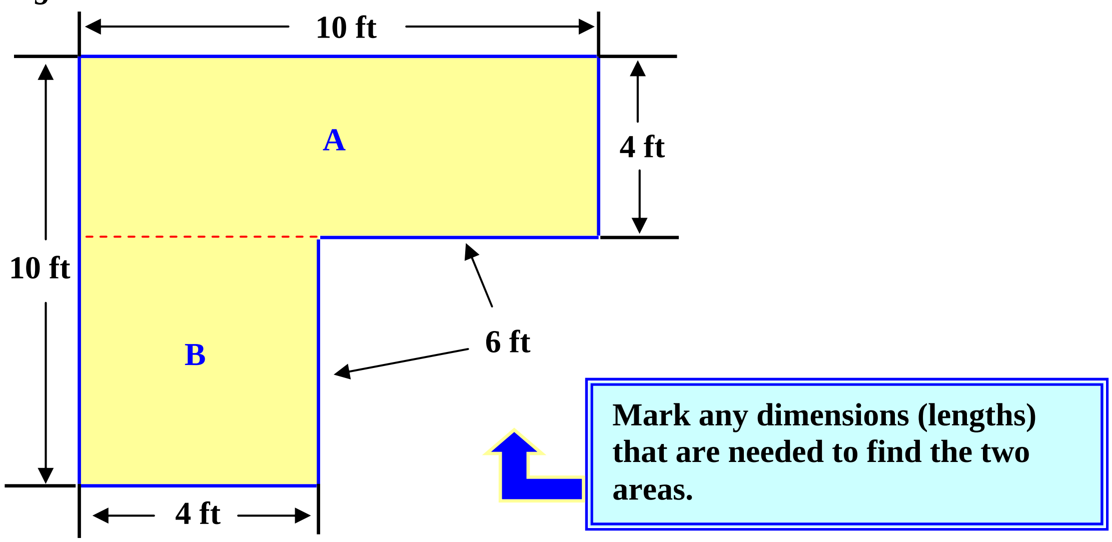
Composite figures are shapes that are made up of simpler distinct shapes.

Figure 1 is an illustration of two rectangles combined to make a composite figure.

To find the area of a composite figure, determine the simpler shapes that make up the figure. Draw lines to divide the larger figure into the smaller shapes.

The red dotted line clearly differentiates between Rectangle A and Rectangle B.

***Figure 1*:**



Example 1: Find the area of Figure 1.

- Find the area of rectangle **A**.

  A = $L \times W$ = 10 × 4 = 40 square feet

- Find the area of Rectangle **B**.

  A = $L \times W$ = 4 × 6 = 24 square feet

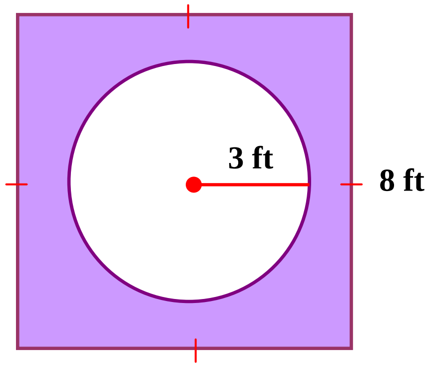
- Add the area of Rectangle **A** and Rectangle **B**.

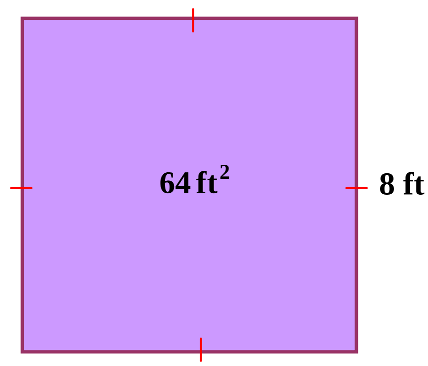
  40 + 24 = 64 sq ft

Therefore, the area of Figure 1 is 64 square feet.

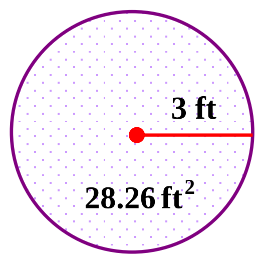
Sometimes, a shape may have **"holes"** in it, and then it is necessary to **subtract** to find the area of the shape.

Example 2: Find the area for the region shaded outside of the circle, but within the square (purple area).

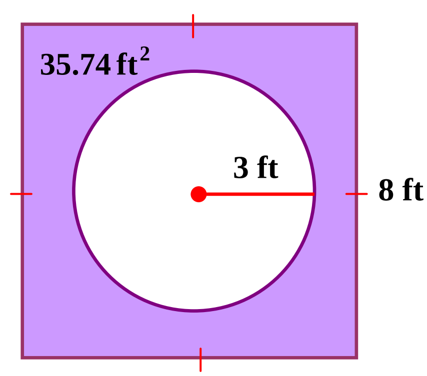




- Find the **entire area** of the **square**.
    - Area (square) = s^2
    - $A = 8^2 = 64\ \text{ft}^2$



- Find the **area** of the **circle (hole).**
    - Area (circle) $= \pi \times r^2$
    - $A = 3.14 \times 3^2 = 28.26\ \text{ft}^2$



- **Subtract** the area of the circle from the area of the square.
    - 64 sq ft – 28.26 sq ft = 35.74 sq ft

***Therefore, the area of the region outside of the circle, but within the square, (light purple region) is 35.74 square feet.**