SLOPE AND LINEAR EQUATIONS

In this unit, you will examine the meaning of slope and the relevance of slope in linear equations. You will determine the slope of a line, investigate the slope-intercept form of a linear equation, and graph an equation using a point and the slope. You will also examine the equations and graphs of horizontal and vertical lines along with parallel and perpendicular lines.

Slope

Graphing a Line Using a Point and the Slope

Equations of Horizontal and Vertical Lines

Parallel and Perpendicular Lines

Graph Paper

Slope

The slope of a line describes the steepness of the line. The slope is the ratio of vertical rise to horizontal run $\left(\dfrac{\text{vertical rise}}{\text{horizontal run}}\right)$.

To find the slope of a line graphed on a coordinate plane:

-Identify a point on the line.

-From that point move up or down until you are directly across from the next point.

-Move left or right to the next point.

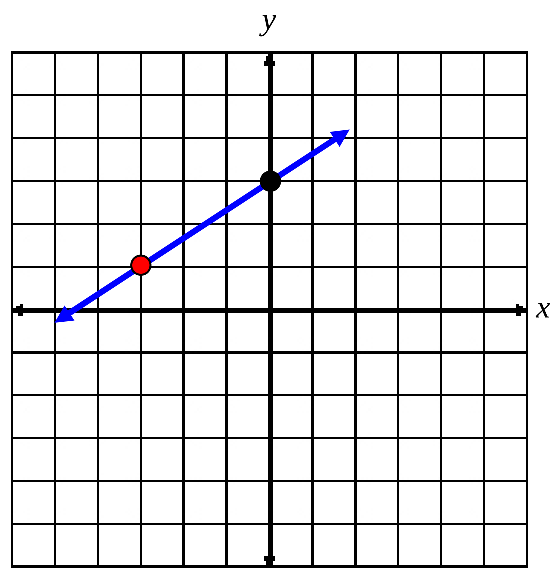
Example 1: Determine the slope of the line in the graph below.

-Start on the red point (lower point).

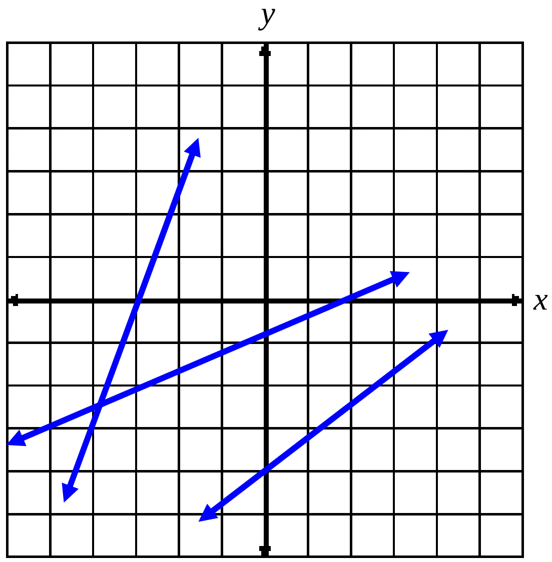
-Move straight up (vertical rise) until your pencil is in the same line as the black point, 2 units.

-Move right (horizontal run) until you reach the black point (upper point), 3 units.

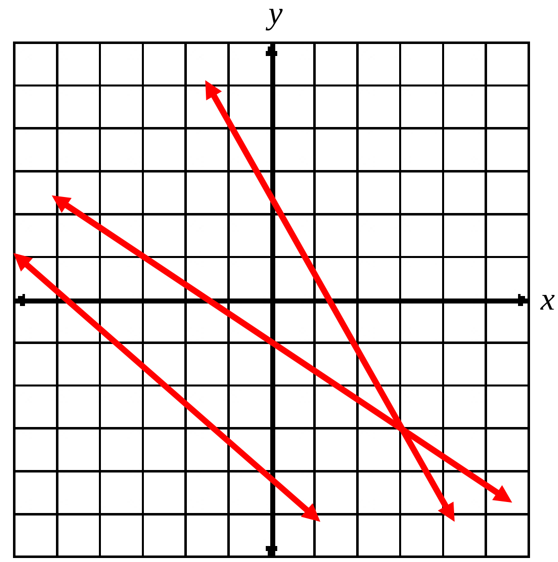
You have now determined the slope of the line to be $\frac{2}{3}$, a rise of 2 units and a run of 3 units.



On a coordinate plane, there are lines that have positive slopes and lines that have negative slopes. Below is an illustration of both.



Lines with positive slopes rise to the right.



Lines with negative slopes rise to the left.

The slope of a line describes the steepness of the line. The slope can be found by using two points on a line (x_1, y_1) and (x_2, y_2) along with the following formula:

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(\text{vertical change})}{(\text{horizontal change})}$$

Example 2: Find the slope of the line containing the points $A(-2, -6)$ and $B(3, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \downarrow \ \downarrow & \downarrow \ \downarrow \\ A(-2, -6) & \text{and } B(3,\ 5) \end{array}$$

$$m = \frac{5 - (-6)}{3 - (-2)}$$

$$m = \frac{11}{5}$$

The slope of the line is $\frac{11}{5}$.

*Note: The slope of a line is expressed as an improper fraction.

Example 3: Find the slope of the line that contains the points $A(7, -4)$ and $B(9, -1)$.

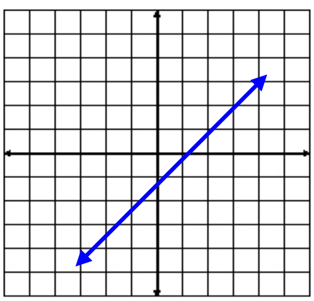
$x_1 = 7 \qquad y_1 = -4 \qquad x_2 = 9 \qquad y_2 = -1$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{9-7} = \frac{3}{2} = \frac{\text{vertical change}}{\text{horizontal change}}$$

The slope of the line is $\frac{3}{2}$.

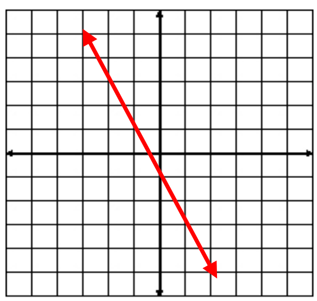
Sometimes the ***vertical*** change is called the ***rise*** and the ***horizontal*** change is called the ***run***. This $\frac{\text{rise}}{\text{run}}$ ratio will be used to graph lines at a later time.

At this time we will take a look at the different types of slopes a line could have. Study the illustrations below.



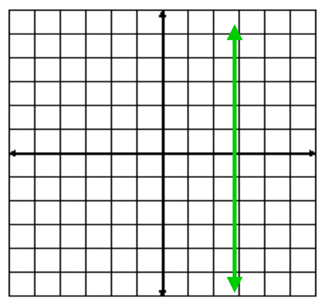
Positive slope
(rises to the right)

$\frac{\text{positive rise}}{\text{positive run}}$ or $\frac{\text{negative rise}}{\text{negative run}}$



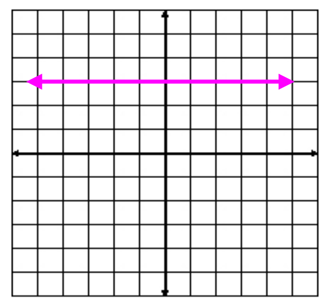
Negative slope
(rises to the left)

$\frac{\text{negative rise}}{\text{positive run}}$ or $\frac{\text{positive rise}}{\text{negative run}}$



No slope
(vertical line)

$\frac{\text{pos/neg rise}}{\text{no run}}$



0 slope
(horizontal line)

$\frac{\text{no rise}}{\text{pos/neg run}}$

Graphing a Line Using a Point and the Slope

Another way to graph a line is by using the **slope-intercept form**. The slope intercept form of a line is written as $\boldsymbol{y = mx + b}$ where $\boldsymbol{m}$ is the slope and $\boldsymbol{b}$ is the y-intercept.

Slope-Intercept Form

$$y = mx + b$$

where $\boldsymbol{m}$ represents the **slope** and $\boldsymbol{b}$ represents the $\boldsymbol{y}$**-intercept**, the point at which the graph crosses the y-axis.

The slope-intercept form of a line is where m represents the slope and b represents the y-intercept. At this time, you practice solving an equation for y and putting it in the form of $y = mx + b$. Follow along with the examples below.

Example 1: Put $3x + y = 7$ into slope-intercept form.

$3x - 3x + y = -3x + 7$ -subtract $3x$ from both sides

$y = -3x + 7$ $\longleftarrow$ -slope-intercept form

Example 2: Put $4x + 2y = 8$ into slope intercept form.

$4x - 4x + 2y = -4x + 8$ -subtract $4x$ from both sides

$2y = -4x + 8$ -divide all terms by 2

$$\frac{2y}{2} = \frac{-4x}{2} + \frac{8}{2}$$

$y = -2x + 4$ $\longleftarrow$ -slope-intercept form.

Example 3: Graph $-2x + 4y = 8$ using the slope-intercept form.

a) Solve for y: $-2x + 4y = 8$

$4y = 2x + 8$ - add $2x$ to both sides

$y = \frac{1}{2}x + 2$ - divide all terms by 4

b) Identify the y-intercept (0, 2) and plot this point.

c) Identify the slope ($m = \frac{1}{2}$).

To graph the linear equation using the slope and y-intercept, follow the steps below.

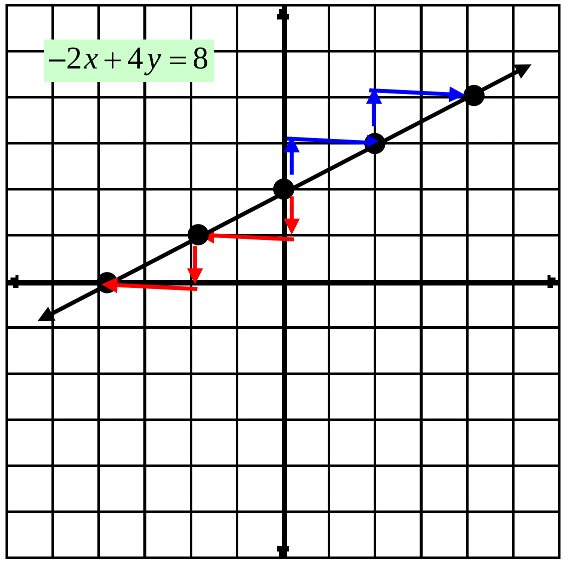
a) Solve the equation for "y" and write it in the slope-intercept form of $y = mx + b$. (This step is completed above.)

b) Identify the y- intercept "b" and plot the point on the y-axis $(0, b)$. (This step is completed above.)

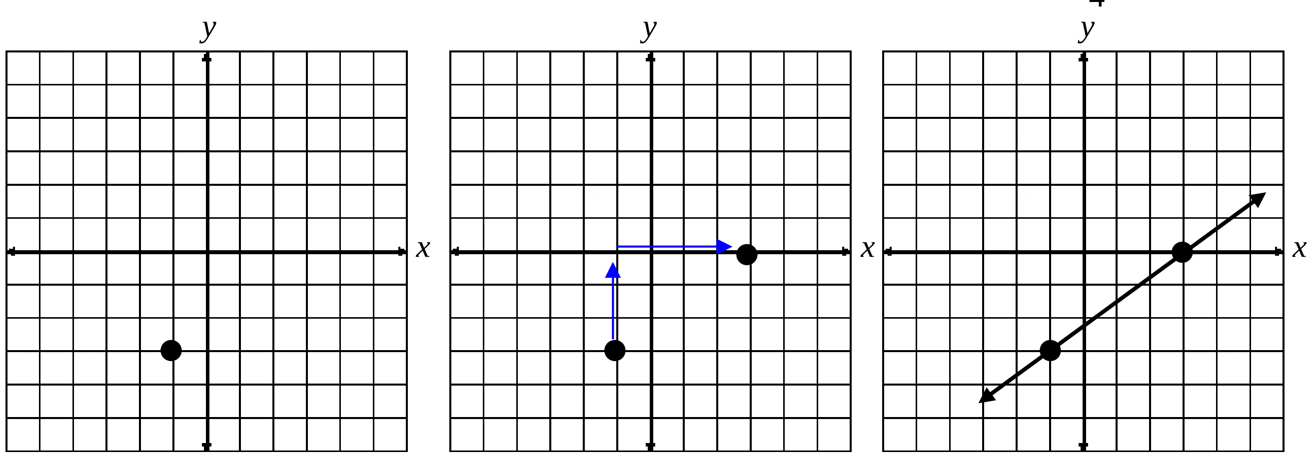
c) Use the slope ratio of $\frac{\text{rise}}{\text{run}}$ to plot more points. (Slope is identified above.)

d) From the y-intercept (0, 2) use the slope ratio to plot more points.

$\left(\frac{\text{rise}}{\text{run}} = \frac{1}{2}\right)$ (this means to move up 1 unit and move right 2 units, up 1, 2 right, etc) or you can use the complete opposite which is down 1, left 2, down 1, left 2.



Example 4: Graph the line containing the point (–1, –3) and having a slope of $m=\frac{3}{4}$.



1. Plot the point (–1, –3).

2. Use the rise (3 units) over run (4 units) ratio for slope to plot a second point.

 Remember: $m=\frac{3}{4}$

3. Draw a line through the points with a straightedge.

Example 5: Identify the slope and y-intercept.

$$y = \frac{-2}{3}x - 4$$

$$\text{slope} = \frac{-2}{3}$$

y-intercept = –4 *or* (0, –4)

To graph a line using the slope and y-intercept

1) Arrange the equation into the form $y = mx + b$.
   (This means solve the equation for y.)

2) Identify the y-intercept (b) and plot the point (0, b).

3) Use the $\frac{rise}{run}$ ratio for slope to plot more points.

4) Draw a line through the points with a straight edge.

Example 6: Graph $-3x + 2y = -6$

1) Solve the equation for y.

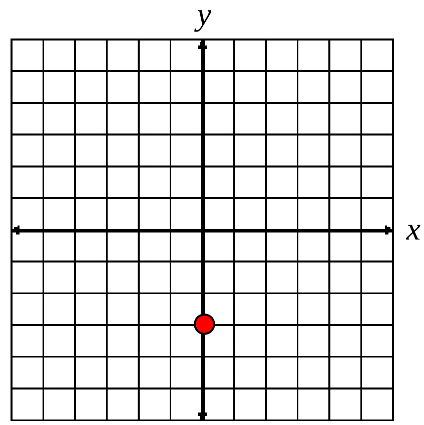
$$\begin{aligned} -3x + 2y &= -6 \\ +3x \qquad &\quad +3x \end{aligned}$$ -add $3x$ to both sides

$$2y = 3x - 6$$ -divide all terms by 2

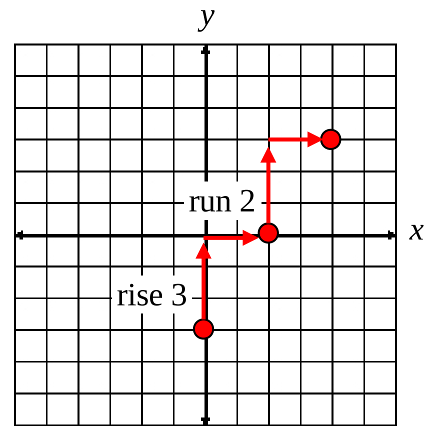
$$\frac{2y}{2} = \frac{3x-6}{2} \qquad \left(\frac{3x-6}{2} = \frac{3x}{2} - \frac{6}{2}\right)$$

$$y = \frac{3}{2}x - 3$$

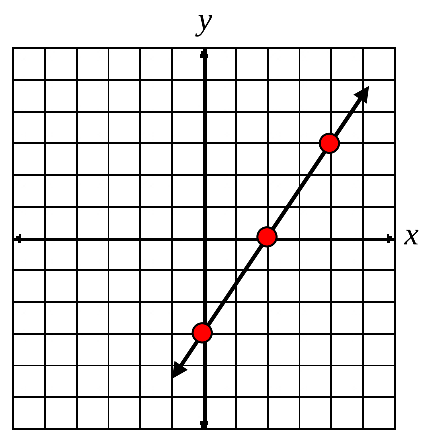
2) Plot the y-intercept, (0, –3).



3) Use the ratio of $\frac{rise}{run}$ and the slope $\frac{3}{2}$ to plot more points.

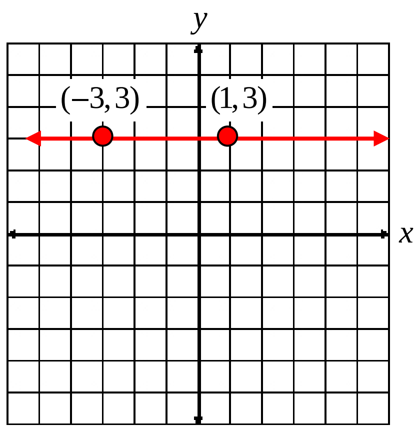


4) Draw a line through the points with a straight edge.



There are two more types of lines that we need to discuss. They are the **vertical** and **horizontal** lines that have special types of slopes.

Example 7: Find the slope of the given horizontal line.



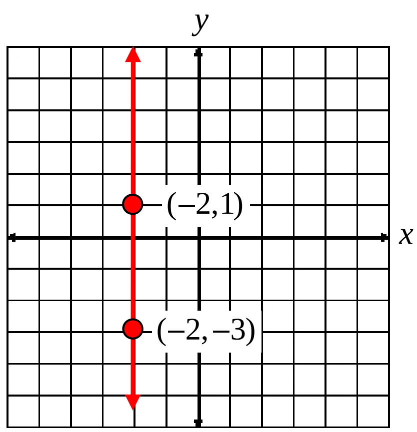
The y-coordinate for every point on a horizontal line is the same.

Choose two points on the line and use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to determine the slope.

$$m = \frac{3-3}{1-(-3)} = \frac{0}{4} = 0$$

The slope of this horizontal line and every horizontal line is 0.

Example 8: Find the slope of the given vertical line.



The x-coordinate for every point on a vertical line is the same.

Choose two points on the line and use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to determine the slope.

$$m = \frac{1-(-3)}{-2-(-2)} = \frac{4}{0} = \text{ undefined or no slope.}$$

Note: Division by zero is undefined.

The slope of this vertical line and every vertical line is undefined.

Equations of Horizontal and Vertical Lines

In a previous section you were introduced to the slopes of horizontal and vertical lines.

Horizontal lines have a **slope of 0**.

Vertical lines have an undefined slope or **no slope**.

In this section we will discuss the equation of both of these lines and how to graph each.

The equation of a horizontal line is $y = b$, where b is the y-intercept.

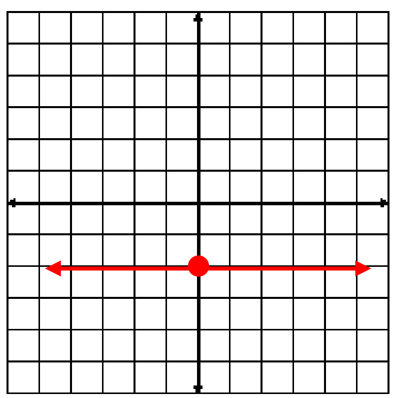
The equation of a vertical line is $x = a$, where a is the x-intercept.

Example 1: Graph each equation.

a.) $y = -2$

-This is a horizontal line with a y-intercept of $(0, -2)$.

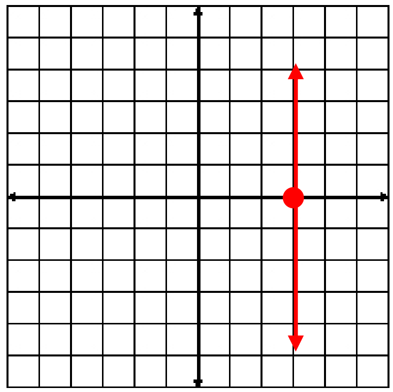
-Plot the point $(0, -2)$ and draw a horizontal line through it.



b.) $x = 3$

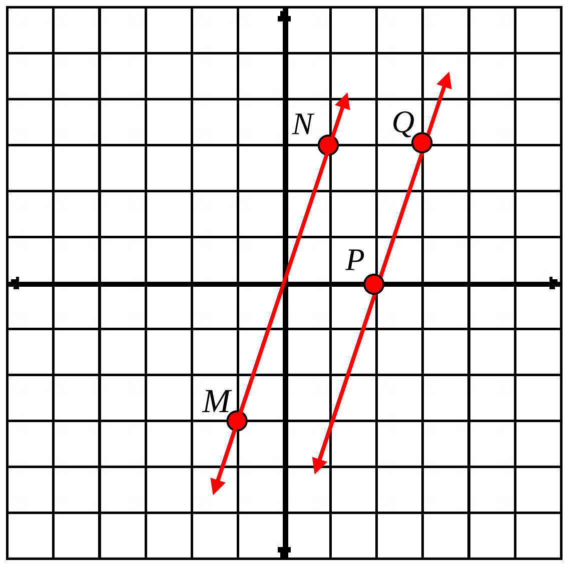
-This is a vertical line with an x-intercept of (3, 0).

-Plot the point (3, 0) and draw a vertical line through it.



Parallel and Perpendicular Lines

Lines that never intersect are called parallel lines. There is a special relationship between parallel lines. Study the graph below and see if you can figure out what the special relationship is.



Do the lines have the same slope? It appears that they do. Let's see if this conjecture (guess) is true.

1. The two points on the line *MN* are *M*(–1, –3) and *N*(1, 3). Find the slope of the line *MN* by using the two points on the line.

$$m = \frac{3-(-3)}{1-(-1)} = \frac{6}{2} = 3$$ The slope of the line containing *M* and *N* is 3.

2. The two points on the line *PQ* are *P*(2, 0) and *Q*(3, 3). Find the slope of the line *PQ* by using the two points on the line.

$$m = \frac{3-0}{3-2} = \frac{3}{1} = 3$$ The slope of the line containing *P* and *Q* is 3.

By finding the slope of both lines, this shows that **parallel lines** have the **same** slope.

Parallel lines have the same slope.

Example 1: Are the graphs of the equations, $y=-\frac{4}{5}x-5$ and $y=-\frac{4}{5}x+2$, parallel lines?

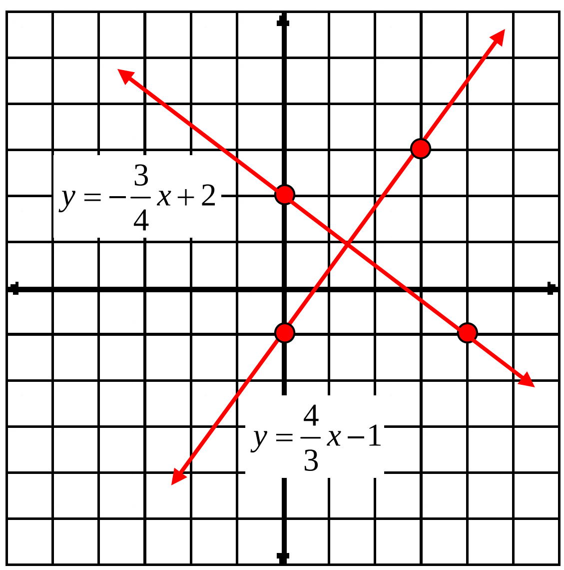
The graphs of the equations are parallel lines because each equation has a slope of $-\frac{4}{5}$.

Perpendicular lines have opposite reciprocal slopes.

Example 2: Are the graphs of the equations, $y=-\frac{3}{4}x+2$ and $y=\frac{4}{3}x-1$, perpendicular lines?

The graphs of the equations are perpendicular lines because $-\frac{3}{4}$ and $\frac{4}{3}$ are opposite reciprocals.

Let's take a look at the graphs of the two lines.



By knowing this you will be able to write equations of lines that are parallel to or perpendicular to the lines of given equations and containing a certain point.

Example 3: Write an equation, in slope-intercept form, of a line that is parallel to $y = 2x + 3$ and passes through point (–1, 4).

a) Determine the slope of the given equation.

$y = 2x + 3$ has a slope of 2 so $m = 2$

Since we want a line parallel to this line, we will use the same slope, $m = 2$, because parallel lines have the same slope.

b) Use the slope-intercept form and substitute the slope you found into the equation.

$$y = mx + b$$

$$y = 2x + b$$

c) Now find the y-intercept (b) by substituting the given point's coordinates $(-1, 4)$ into the equation in the previous step.

$$y = mx + b$$

$$y = 2x + b$$

$$4 = 2(-1) + b$$

$$6 = b$$

Therefore, $y = 2x + 6$, is the equation of a line that is parallel to $y = 2x + 3$ and passes through the point (–1, 4).

Example 4: Write the equation of a line perpendicular to $5x + 2y = 10$ and passes through point (3, –5).

a) Solve the equation $5x + 2y = 10$ for y and determine the slope.

$$5x + 2y = 10$$

$$2y = -5x + 10 \qquad \text{-subtract } 5x \text{ from both sides}$$

$$y = -\frac{5}{2}x + 5 \qquad \text{-divide all terms by 2}$$

The slope of this line is $-\frac{5}{2}$.

We want to use the opposite reciprocal of this value because we want the equation of a line that is **perpendicular t**o the given equation.

b) Use $\frac{2}{5}$ as the slope in the slope-intercept form.

$$y = mx + b$$

$$y = \frac{2}{5}x + b$$

c) Substitute the given point (3, –5) and the slope you determined above, $\frac{2}{5}$, into the slope-intercept form to solve for the y-intercept (*b*).

$$y = mx + b$$

$$y = \frac{2}{5}x + b \qquad m = \frac{2}{5}$$

$$-5 = \frac{2}{5}(3) + b \qquad \text{Point } (3, -5)$$

$$-5 = \frac{6}{5} + b$$

$$\frac{-25}{5} = \frac{6}{5} + b$$

$$\frac{-31}{5} = b$$

Therefore, $y = \frac{2}{5}x - \frac{31}{5}$ is the equation of a line that is perpendicular to 5*x* + 2*y* = 10 and passes through point $(3, -5)$.

Graph Paper