POLYGONS AND QUADRILATERALS; SEQUENCES

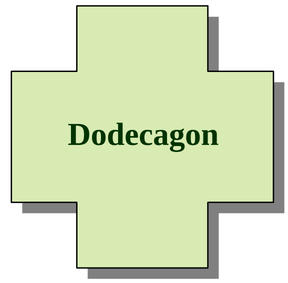
In the unit, you will examine polygons and their characterisitics including interior and exterior angles. You will take a closer look at the quadrilaterals and their relationships with each other. In the final part of the unit, you will study arithmetic and geometric sequences.

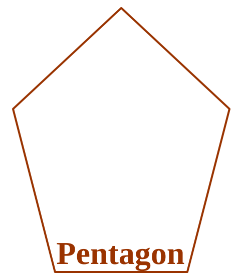
Polygons

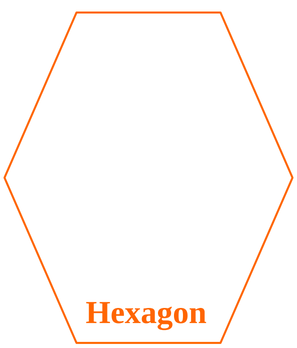
polygon – A polygon is a closed figure made up of three or more line segments.

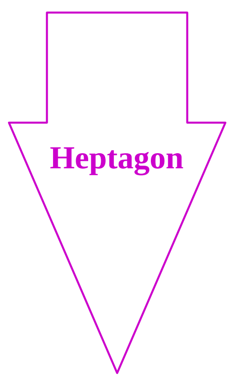
vertex (vertices – plural) – The vertex of a polygon is the point of intersection where two segments meet.

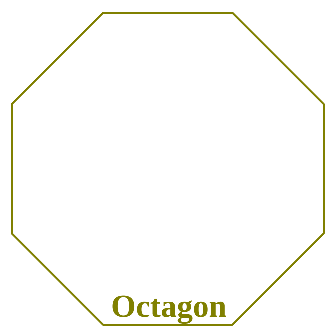
Polygon	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Hendecagon	11
Dodecagon	12

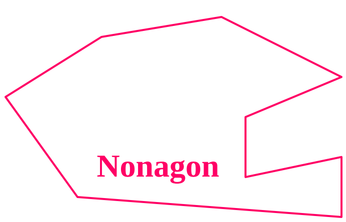


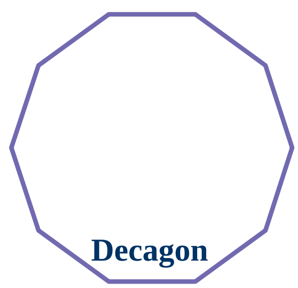












*Note: Some of the polygons are referenced by different names. The heptagon may be called a “septagon”, the nonagon may be call an “enneagon”, and the dodecagon may be called a “duodecagon”.

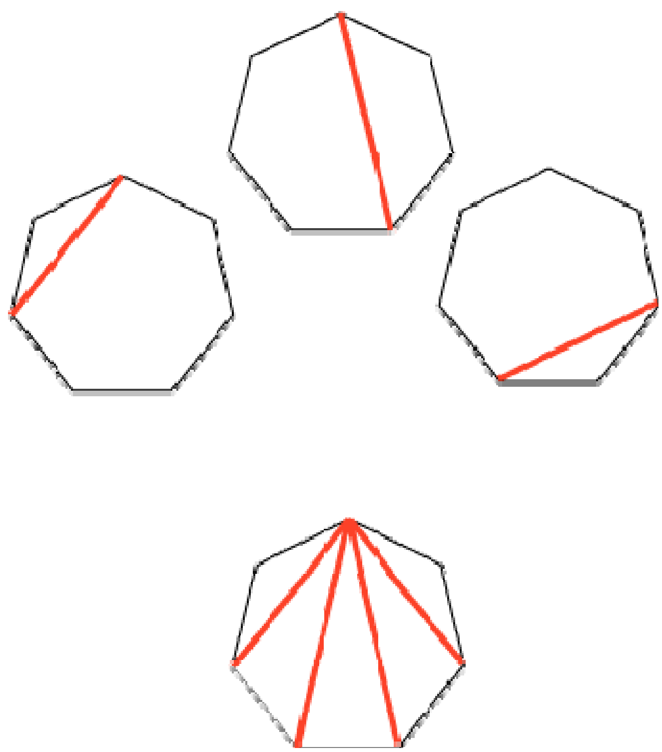
regular polygon – A regular polygon is a polygon that has all sides and all angles congruent.

> *Example*: Which polygons, shown above, are regular polygons?
>
> *Solution*: Determine which polygons have congruent sides. Both the octagon and the decagon are examples of regular polygons.

diagonal – The diagonal of a polygon is a line segment that joins two nonconsecutive vertices of the polygon.

Various diagonals of a regular heptagon are shown below.

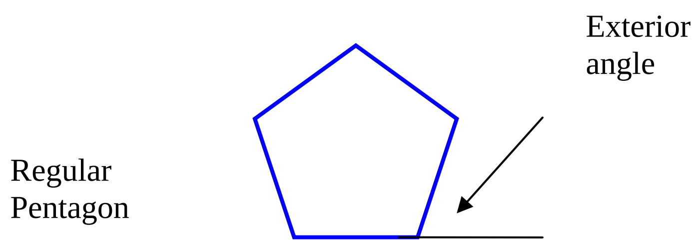
*Note: Four different diagonals may be drawn from one vertex in a heptagon.



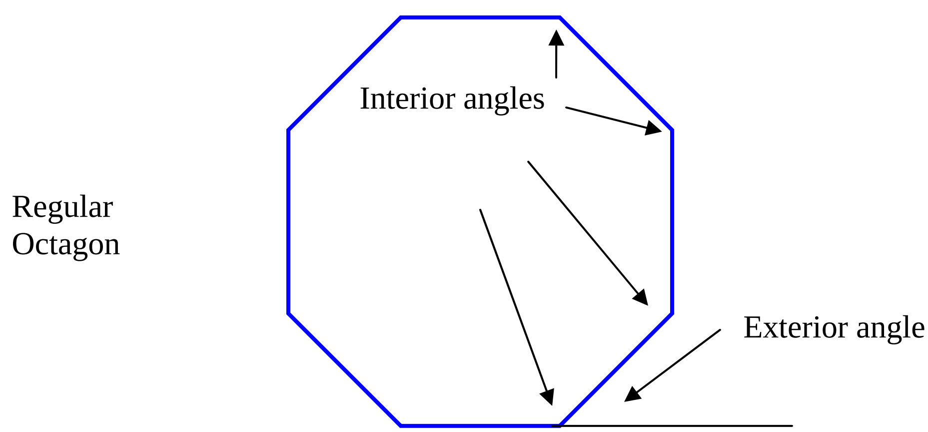
Interior and Exterior Angles of Polygons

Take a look at the two polygons shown below.

Notice that an **exterior angle** of a polygon is formed when a side is extended out from polygon as shown below. .



An **interior angle** of a polygon is any angle that is formed by two adjacent sides of a polygon and located in the interior of the polygon. In this example, some of the interior angles are labeled along with one exterior angle.



There are established formulas to find the measurements of angles of regular polygons.

> **Theorem 1**: The sum of the angles of a convex polygon with n sides is $180(n-2)$.

Theorem 2: The sum of the measures of the exterior angles of a convex polygon (one at each vertex) is 360°.

Example 1: Find the sum of the interior angles of a regular hexagon.

Number of sides in a hexagon: 6

$180(n-2)$	Formula for Sum of Angles in a Polygon
$180(6-2)$	Substitute (A hexagon has 6 sides. $n=6$)
$180(4)$	Simplify
720	Simplify

The sum of the interior angles in a hexagon is 720°.

Example 2: Find the sum of the exterior angles of a hexagon.

Theorem 2 states the sum will be 360°.

This is true of any convex polygon.

The sum of the exterior angles of a hexagon is 360°.

Example 3: Find the measurement of one interior angle of a REGULAR hexagon.

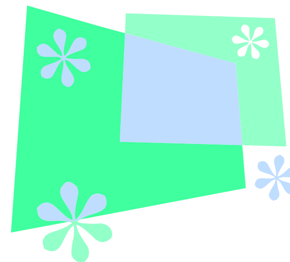
In the first example, we found that the sum of the interior angles of a hexagon is 720°.

Since the polygon is a regular polygon, we know that the sides and angles are congruent. Therefore, we can divide the sum by six to find the measurement of one interior angle.

$$720^\circ \div 6 = 120^\circ$$

One interior angle of a regular hexagon measures 120°.

Quadrilaterals



quadrilateral - A quadrilateral is a polygon with four sides.

Some quadrilaterals are given other names because of the special angles and line segments that make up the shape.

Parallelogram
- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are congruent.

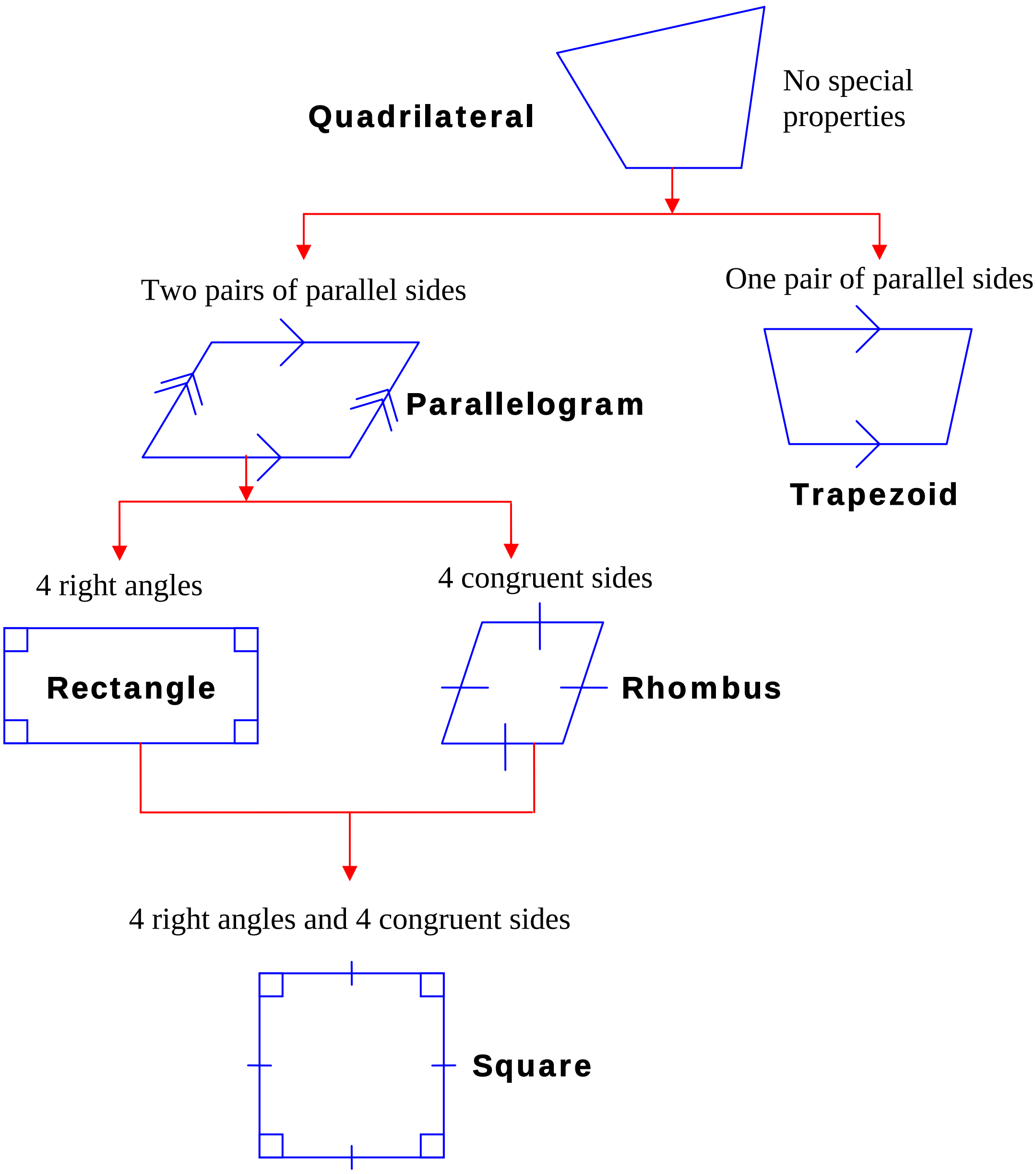
Trapezoid
- One pair of opposite sides are parallel.

Rhombus
- Both pair of opposite sides are parallel.
- All sides congruent.

Rectangle
- Both pair of opposite sides are parallel.
- Both pair of opposite sides are congruent.
- All four angles are right angles.

Square
- Both pair of opposite sides are parallel.
- All four sides are congruent.
- All four angles are right angles.

This chart shows the relationship of the quadrilaterals.



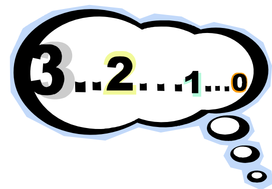
Example 1: True or False. All squares are rectangles.

True. Since a rectangle is a parallelogram with four right angles and since a square is also a parallelogram that has four right angles, then all squares are rectangles.

Example 2: True or False. All rectangles are squares.

False. Rectangles and squares both have right angles, but not all rectangles have four congruent sides.

Sequences



sequence – A sequence is a list of numbers in a certain order connected through a pattern.

term – A term is any of the numbers in a sequence.

arithmetic sequence – An arithmetic sequence is a sequence in which the difference between any two consecutive terms is the same.

{5, 10, 15, 20, ...}

Notice that each number in the above sequence is 5 more than the number before it.

+5 +5 +5

{5, 10, 15, 20, ...}

To find the next three numbers in the sequence, **follow the pattern** and add five each time.

+5 +5 +5

{5, 10, 15, 20, 25, 30, 35,...}

geometric sequence – A geometric sequence is a sequence in which the ratio between any two successive terms is the same.

{6, 12, 24, 48, ...}

Notice that each number in the above sequence is twice as much as the number before it. Each successive term has a ratio of 2.

x2 x2 x2

{6, 12, 24, 48 ...}

To find the next three numbers in the sequence, **follow the pattern** and multiply by two each time.

x2 x2 x2

{6, 12, 24, 48, 96, 192, 384...}

The "nth" Term of a Sequence

Geometric Sequences

geometric sequence – A geometric sequence is a sequence in which the ratio between any two successive terms is the same.

Consider finding the 20th term for this sequence: 3, 18, 108, 648 …

If you know the first term of a geometric sequence and the common ratio, you can find any other term of the sequence.

The first term is 3 which we will call "*a*". The common ratio between the terms is 6 which we will call "*r*". Study the table below.

Term	**1st**	**2nd**	**3rd**	**4th**	**nth**
Sequence	3	18	108	648	...
Factors of term	3	$3 \cdot 6$	$3 \cdot 6 \cdot 6$	$3 \cdot 6 \cdot 6 \cdot 6$	$\frac{3 \cdot 6 \cdot 6 \cdot 6 \cdot ... \cdot 6}{6}$
Term using exponents	3	$3 \cdot 6^1$	$3 \cdot 6^2$	$3 \cdot 6^3$	$3 \cdot 6^{n-1}$
Variables	a	$a \cdot r^1$	$a \cdot r^2$	$a \cdot r^3$	$a \cdot r^{n-1}$

The *n*th term of a **geometric sequence** can be found using the following formula:

$$a_n = a_1 r^{n-1}$$

($n =$ term, $a_n =$ *n*th term, $a_1 =$ first term, $r =$ common ratio)

Take one more look at the chart.

*The number of times 6 is multiplied by the first term is one less than the term number $(n-1)$.

Example 1: Explain how the fourth term in the geometric sequence above can be determined using the formula.

$a_n = a_1 r^{n-1}$	Formula for Determining the *n*th Term of a Geometric Sequence
$a_4 = 3(6^{4-1})$	Substitute ($n = 4,\ a_1 = 3,\ \ r = 6$)
$a_4 = 3(6^3)$	Simplify
$a_4 = 648$	Simplify

Now, let's revisit finding the 20th term of the geometric sequence using the formula, not listing out all of the 20 terms.

Example 2: What is the twentieth term of the geometric sequence shown above?

$a_n = a_1 r^{n-1}$	Formula for Determining the *n*th Term of a Geometric Sequence
$a_{20} = 3(6^{20-1})$	Substitute ($n = 20,\ a_1 = 3,\ \ r = 6$)
$a_{20} = 3(6^{19})$	Simplify
$a_{20} = 1{,}828{,}079{,}220{,}031{,}488$	Simplify

*The computer's scientific calculator can handle large numbers without rounding them and expressing them in scientific notation.

Here are the steps to use the scientific calculator that part of the Windows XP operating system.

Step 1: Make sure you are in scientific view (View/Scientific)

Step 2: Select 6, $\boxed{\text{x\^{}y}}$, then 1 and 9 for 19.

Step 3: Select $\boxed{=}$.

Step 4: Multiply that answer times three ($\boxed{*}$3).

Step 5: Select $\boxed{=}$.

To copy and paste an answer from the computer's calculator, select Edit / Copy (you do not need to select the number first).

Then, Edit / Paste into the document you wish to display the results.

Arithmetic Sequences

arithmetic sequence – An arithmetic sequence is a sequence in which the difference between any two consecutive terms is the same.

Consider finding the 100th term for this arithmetic sequence: 2, 6, 10, 14, 18, 22 …

If you know the first term of an arithmetic sequence and the common difference, you can list the terms of the sequence.

The first term is 2 which we will call "*a*". The common difference between the terms is 4 which we will call "*d*". Look at the pattern below.

1st term	a	2	2
2nd term	$a + d$	2 + 4	6
3rd term	$a + d + d$ or $a + 2d$	2 + 4 + 4 or 2 + 2(4)	10
4th term	$a + d + d + d$ or $a + 3d$	2 + 4 + 4 + 4 or 2 + 3(4)	14
.	.	.	.

. . . .
. .
*n*th term $a + (n - 1)d$

*Note: The number of 4's added is one less than the term number.

The *n*th term of an **arithmetic sequence** can be found using the following formula:

$$a_n = a_1 + (n-1)d$$

($n =$ term, $a_n =$ *n*th term, $a_1 =$ first term, $d =$ common difference)

Example 3: Explain how the third term in the arithmetic sequence above can be determined using the formula.

$a_n = a_1 + (n-1)d$	Formula for Determining the *n*th Term of an Arithemtic Sequence
$a_3 = 2 + (3-1)4$	Substitute ($n = 3$, $a_1 = 2$, $d = 4$)
$a_3 = 2 + (2)4$	Simplify
$a_3 = 2 + 8$	Simplify
$a_3 = 10$	Simplify

Now, let's revisit finding the 100th term of the arithmetic sequence using the formula, not listing out all of the100 terms.

Example 4: What is the 100th term of the arithmetic sequence shown above?

$a_n = a_1 + (n-1)d$	Formula for Determining the *n*th Term of an Arithemtic Sequence
$a_{100} = 2 + (100-1)4$	Substitute ($n = 100$, $a_1 = 2$, $d = 4$)
$a_{100} = 2 + (99)4$	Simplify
$a_{100} = 2 + 396$	Simplify
$a_{100} = 398$	Simplify